AD NUMBER
ADC001996
CLASSIFICATION CHANGES
TO: unclassified
FROM: confidential
LIMITATION CHANGES
TO: Approved for public release, distribution unlimited
FROM: Distribution authorized to U.S. Gov't. agencies only; Test and Evaluation; 28 JAN 1975. Other requests shall be referred to Office of Naval Research, Arlington, VA 22203.
AUTHORITY
ONR ltr 8 Dec 1977; ONR per DTIC form 55

UNCLASSIFIED

AD C061996L

CLASSIFICATION CHANGED
TO: UNCLASSIFIED
FROM: CONFIDENTIAL
AUTHORITY:
ONR Notice
8 Dec 77

UNCLASSIFIED

CONFIDENTIAL

Naval Warfare Research Center *December 1974*
Technical Report NWRC-TR-10

MODELS OF ACOUSTIC DECEPTION AND ASW SUPPORT IN A TASK GROUP OPERATING AREA (U)

By J R OLMSTEAD, T R ELFERS and G W BLACK

Prepared for

NAVAL ANALYSIS PROGRAMS (Code 431)
OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY
800 NORTH QUINCY STREET
ARLINGTON, VIRGINIA 22217

CONTRACT N00014-71-C-0419

TASK NR 274-008-54

STANFORD RESEARCH INSTITUTE
Menlo Park, California 94025 · U.S.A.

CONFIDENTIAL

TECHNICAL REPORT

A Technical Report is a document of Stanford Research Institute that presents results of work directed toward specific research objectives. The report is a comprehensive treatment of the objectives, scope, methodology, data, analyses, and conclusions, and presents the background, practical significance, and technical information required for a complete and full understanding of the research activity. Technical Reports are reviewed and approved by a division executive director or higher official of the Institute.

RESEARCH MEMORANDUM

A Research Memorandum is a document of Stanford Research Institute that presents the results of work in progress. The purpose of the Research Memorandum is to invite comment on research in progress. It is a comprehensive treatment of a single research area or of a facet of a research area within a larger field of study. The Memorandum presents the background, objectives, scope, summary, and conclusions, as well as method and approach, in a condensed form. The report is reviewed and approved by a department director or higher official of the Institute.

TECHNICAL NOTE

A Technical Note is a working paper that presents the results of research related to a single phase or factor of a research problem. The purpose of the Technical Note is to instigate discussion and criticism. It presents the concepts, findings, and/or conclusions of the author. The report is reviewed by a project leader or higher official of the Institute.

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER SRI-75-0244	2 GOVT ACCESSION NO	3 RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MODELS OF ACOUSTIC DECEPTION AND ASW SUPPORT IN A TASK GROUP OPERATING AREA (U)		5 TYPE OF REPORT & PERIOD COVERED Technical Report NWRC-TR-10
		6 PERFORMING ORG. REPORT NUMBER SRI-75-0244, 1318-25-54
7. AUTHOR(s) J.R Olmstead, T.R. Elfers, G.W. Black		8 CONTRACT OR GRANT NUMBER(s) N00014-71-C-0419
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Warfare Research Center Stanford Research Institute Menlo Park, California 94025		10 PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 65152N R 9103 NR 274-008-54
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Analysis Programs (Code 431) Office of Naval Research Arlington, Virginia 22217		12 REPORT DATE December 1974 / 13. NO. OF PAGES 126
		15 SECURITY CLASS. (of this report) CONFIDENTIAL
14 MONITORING AGENCY NAME & ADDRESS (if diff from Controlling Office)		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE GDS 31 December 1980

16. DISTRIBUTION STATEMENT (of this report)

Distribution limited to U.S. Government agencies only: Test and Evaluation, 28 January 1975. Other requests for this document must be referred to Office of Naval Research (Code 431).

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20 if different from report)

18. SUPPLEMENTARY NOTES

Reproduction in whole or in part is permitted for any purpose of the United States Government.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Acoustic Countermeasures
ADD
Acoustic Deception Device
Uptide
Markov Model

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Three Markov models are developed and applied to some of the problems involving acoustic deception decoys and ASW forces. The scenario involves an enemy submarine searching for a high value unit in a large operating area. Several acoustic deception devices are randomly deployed in the area to draw the submarine's fire. One of the major conclusions of the study is that the effectiveness of acoustic decoys depends on whether or not the submarine expects deception to be used. Another conclusion is that the deployment of the ASW forces (deployed with the high value unit or divided among the acoustic deception units) is not a critical problem.

19 KEY WORDS (Continued)

20 ABSTRACT (Continued)

STANFORD RESEARCH INSTITUTE
Menlo Park, California 94025 · U.S.A.

Naval Warfare Research Center *December 1974*

Technical Report NWRC-TR-10

MODELS OF ACOUSTIC DECEPTION AND ASW SUPPORT IN A TASK GROUP OPERATING AREA (U)

By: J. R OLMSTEAD, T R ELFERS, and G. W. BLACK

Prepared for:

NAVAL ANALYSIS PROGRAMS (Code 431)
OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY
800 NORTH QUINCY STREET
ARLINGTON, VIRGINIA 22217

CONTRACT N00014-71-C-0419

TASK NR 274-008-54

SRI Project 1318-25

NATIONAL SECURITY INFORMATION
Unauthorized Disclosure Subject
to Criminal Sanctions

Approved by:

LAWRENCE J. LOW, *Director*
Naval Warfare Research Center

GEORGE D. HOPKINS, *Executive Director*
Engineering Systems Division

Copy No. ....25....

SRI-75-0244
1318-25-54

"Confidential classified by Assistant Chief for Research, Office of Naval Research, Subject to General Declassification Schedule of Executive Order 11652, automatically downgraded at two-year intervals and declassified 31 Dec. [redacted] 1980

CONFIDENTIAL

(This page is UNCLASSIFIED)

PREFACE

The work reported in this document was performed for the Office of Naval Research, Naval Analysis Programs (Code 431), directed by Mr. R. J. Miller. The project monitor was Mr. J. G. Smith. This report is the latest of several SRI studies on the subject of acoustic countermeasures--a complete bibliography is included at the end of the report.

SUMMARY (U)

(U) This report describes the design and application of three Markov-type models. The first model determines the basic interaction of the most important acoustic deception parameters; the second model extends the basic ideas to include the effects of antisubmarine warfare (ASW); and the third model extends the second by including the effects of attrition on submarine weapons and ASW units.

(C) All three models are based on a scenario similar to the Uptide Two series of exercises. An autonomous cruise-missile submarine is assumed to be searching for a high value unit (HVU) in a large operating area. Several acoustic deception units (ADUs) are deployed in the area to lure the submarine into making an attack on a false target. (The ADUs include an acoustic deception device which acoustically mimics the HVU.) The various units are assumed to be randomly distributed in a uniform manner over the operating area. The objective of the HVU is to survive a given number of days without being detected and correctly classified (and therefore attacked).

1. Basic Model (U)

(U) The basic model resulted in a closed form solution relating the probability of HVU survival as a function of ten scenario parameters. These parameters have to do with detection ranges, speeds, submarine density, time in the area, and classification probabilities.

(U) The most significant methodological result was that the final equation could be written as a function of just two nondimensional variables. (See Figure 4 on page 11.)* These two variables were named

*(U) References to pages in the main body of the report are added to this summary for the convenience of the reader. They may be ignored, if desired.

the "blue deception factor" and the "red coverage factor" and are the basic measures for tactical doctrine in this "pure" acoustic deception scenario. (We suspect, but have not shown, that estimates of these two tactical measures would play an important role in making decisions in a real acoustic deception environment.)

(U) The blue deception factor is a product of parameters relating to ADUs divided by a product of parameters relating to the HVU:

the number of ADUs, times
the probability of misclassifying an ADU, times
the search width of the ADU, times
the average relative speed between the ADU and submarine;

divided by:

the probability of correctly classifying the HVU, times
the search width of the HVU, times
the average relative speed between the HVU and the submarine.

(U) The red coverage factor is a product of parameters relating the HVU and submarine (the first three are the same as the denominator of the deception factor):

the probability of correctly classifying the HVU, times
the search width of the HVU, times
the average relative speed between the HVU and submarine, times
the submarine density, times
the time in the operating area.

(U) The importance of reducing the input parameters into two factors is that broad tactical principles can be reached by considering just the value of the factors without assigning values to each of the parameters. The various conclusions derived by this method are given below.

(1) If the red coverage factor is less than 0.2, then acoustic decoys are not needed.

(2) If the red coverage factor is between 0.2 and 2.0, then ADUs should be deployed, and emphasis should simultaneously be placed on reducing the HVU related parameters: the probability of correctly classifying the HVU, the HVU sweep width, and the HVU/sub average relative speed.

(U)

(3) If the red coverage factor is between 0.2 and 2.0, then it is more important to make the HVU sound like a nontarget than it is to make the ADU sound like the HVU.

(4) If the red coverage factor is greater than 2.0, then a reduction of the time in the operating area has little effect.

(5) If the red coverage factor is greater than 2.0, then increasing the probability of misclassifying the ADU is of equal effect to decreasing the probability of correctly classifying the HVU. Likewise, increasing the detection range of all the ADUs is of equal effect to decreasing the detection range of the HVU. This symmetry of ADU and HVU parameters holds only for a red coverage factor greater than 2.0; at lower values, the HVU parameters are more important.

(6) If the blue deception factor is greater than 4.0, then additional deception devices are not needed.

(7) Numerous short-range deception devices are more cost/effective than a few long-range devices. This result is independent of the submarine density or the time spent in the operating area.

2. Acaso Model (U)

(U) The basic Markov model was extended to model acoustic countermeasures and antisubmarine warfare in the objective area (Acaso). Two additional outcome states were added to the three states of the basic model, making a total of five outcomes that can occur during a given time period in the operating area: (1) the submarine attacks the HVU, (2) the submarine attacks an ADU, (3) the submarine is killed by the HVU's ASW force, (4) the submarine is killed by the ADU's ASW force, and (5) the submarine is in search at the end of the time period. (See Figure 6 on page 22.) The Acaso model consists of equations to compute the probabilities of the five outcomes. All the necessary equations are given in this report so that the model can be easily programmed and used by others. (See Tables 1 through 7 on pages 24-27 and pages 39-41.)

(U) The novel feature of the Acaso model is the classification algorithm. Classification is represented by a Bayesian decision process based on probability distributions of a random feature that quantifies

(U)

the information provided by the submarine's acoustic sensors. The input parameters used in the classification algorithm are: (1) the submarine's estimate of the probability that the next contact will be the HVU (equivalent to estimating the number of ADUs); (2) the maximum allowed classification error, (3) and (4) the true and the estimated probabilities that the HVU produces an "HVU-like" feature, and (5) and (6) the true and estimated probabilities that the ADU produces an "ADU-like" feature. The last four parameters represent the "fidelity" of the HVU and ADU--both the real unit's fidelity and the submarine's estimates of the real unit's fidelity.

(U) The maximum classification error parameter is used to derive two thresholds which are applied to the HVU and ADU submarine-estimated distributions. These two thresholds relate to the submarine-estimated probabilities of two kinds of classification errors: (1) identifying an ADU as a valid target, and (2) identifying an HVU as a nontarget. The two thresholds are then applied to the true HVU and ADU distributions to calculate the probabilities that the submarine will close the HVU or ADU for a positive visual classification.

(U) The Acaso model can quantify, with the aid of the classification algorithm and the ASW kill probabilities, a basic tradeoff faced by the submarine. The tradeoff is: (1) should the submarine acoustically classify at long range, thus avoiding the ASW threat but with a risk of making a classification error? or (2) should the submarine close to a short range for visual classification but with a risk of being detected and attacked by the ASW force? The basic input parameter that represents this tradeoff is the maximum allowed classification error. (See Figure 18 on page 55.)

(U) A parameter sensitivity analysis was performed on the Acaso model. (See Figure 12 on page 49.) Two base cases were examined: (I) the submarine expects acoustic deception, and (II) the submarine does not expect acoustic deception. Both of these cases are run in a basic scenario in which acoustic deception units are deployed. The probability of HVU survival is lower in Case I because the submarine expects the deception that is used. When the HVU does survive in Case I, the reasons

(U)

for survival are about 60 percent deception (submarine attack on an ADU or submar[illegible] searching) and 40 percent kill (submarine killed by [illegible] forces deployed with the HVU or ADUs). This is in contrast to Case II: almost always the reason for survival is due to an attack on an ADU. (See Figures 10 and 11, [illegible] 47.)

(U) The major conclusions concerning the range and speed parameters are:

(1) The HVU's detection range and speed are very important parameters. (See Figure 13, page 50.)

(2) The ADU's detection range and speed are of less importance--increasing the ADU's detection range is more important when the submarine does not expect deception than when he does. (See Figure 14, page 51.)

(3) Submarine search speed becomes important when the ADU detection range is small. (See Figure 15, page 52.)

(U) The major conclusions concerning the submarine's classification parameters are:

(1) Massive ASW strength is required before the tradeoff between visual versus acoustic classification is reversed--the submarine is better off classifying visually, whether or not he expects deception. (See Figures 20 and 21, pages 57 and 58.)

(2) The submarine's estimates of the HVU's and ADU's fidelity are not important parameters. (See Figures 22 and 23, pages 59 and 60.)

(U) The major conclusions concerning the acoustic deception parameters are:

(1) When the submarine expects deception, the number of ADUs is not important--in fact, the use of ADUs does not help much at all. (See Figure 24, page 62.)

(2) Acoustic deception works well when the submarine does not expect it.

(3) When the submarine can accurately estimate the fidelity of the HVU or ADU, the actual fidelity is not too important.

(U)

(4) Fairly high gains in survivability can be made by making the HVU sound like a nontarget, providing that (a) the submarine expects acoustic deception, (b) the submarine expects the HVU to sound like an HVU, and (c) deception is actually used.

(5) High gains in survivability can be made by making the ADU into a high fidelity decoy, providing that the submarine does not expect decoys. (See Figures 25 and 26, pages 63 and 64, to supplement the last four conclusions.)

(U) The major conclusions concerning the ASW related parameters are:

(1) It is only slightly better to keep the total ASW force with the HVU rather than dividing the ASW forces among the ADUs. (See Figure 27, page 65.)

(2) It is more important to have a strong ASW force with the HVU if the submarine expects deception. (See Figure 29, page 67.)

(3) The strength of ASW is not nearly as important when the submarine does not expect deception as when it does.

(U) The major conclusions concerning the interaction of deception and ASW parameters are:

(1) When the submarine expects deception, a strong ASW force is needed to increase the HVU's survivability, but when the submarine does not expect deception, a strong deception effort (in terms of numbers of many high fidelity devices) is needed more than a strong ASW force. (See Figure 32, page 70.)

(2) For a given level of survival, a modest increase in the ADU's fidelity is "worth" a very large increase in the ASW force --this effect is more pronounced for the case when the submarine does not expect deception. (See Figure 35, page 73.)

3. <u>Event Step Simulation Model</u> (U)

(U) The Acaso model was extended by adding two more states: the submarine depletes his missiles, and the ASW forces around the acoustic decoy are attrited by missiles fired at the decoy. Therefore, four events are allowed to occur <u>after</u> the submarine attacks an ADU:

(U)

(1) the submarine is killed by the ADU's ASW forces, (2) the submarine runs out of missiles, (3) the submarine escapes and begins searching again, and (4) the submarine's attack kills part or all of the ASW force and then the submarine escapes and begins searching again. This model was replicated many times to derive average answers; the simple Markov methodology could not be used because the ASW kill probabilities and submarine's missile load change each time an ADU is encountered.

(U) The model was exercised to investigate: (1) the effect of the submarine's saving a portion of his missiles for a reattack capability, and (2) the effect of the ASW forces being attrited during each attack on an ADU. The scenario was similar to the base case in the Acaso model: there was one HVU protected by three ASW elements, and three ADUs each consisting of one decoy and one ASW element. The basic measure of effectiveness was the probability that the HVU remained unattacked for seven days in a 400-nmi diameter operating area.

(U) The results were:

(1) The submarine's reattack capability is not very important when the submarine expects acoustic deception; however, if no deception is expected, then the submarine is better off not to launch all his missiles in one attack.

(2) The HVU's survivability is not very sensitive to whether or not attrition of the ADU's ASW forces is assumed. The reasons for HVU survival can change from ASW kills, when no attrition is assumed, to missile depletion when attrition of the ADU's ASW force is assumed. (See Tables 10 and 11 on pages 78 and 79.)

4. General Comments (U)

(U) In an attempt to integrate the results of the three models, there is one fundamental point that should be first brought into focus: tactics that are effective in one situation are not necessarily effective in another situation. Therefore, statements such as "employment of acoustic deception devices will be effective in helping the HVU survive" should be avoided unless they are qualified by describing the scenario in which the devices are to be employed.

(U) The present study has highlighted two conditions under which acoustic deception seems to lose its effectiveness: (1) when the area is too large or the time too short for the submarine to find the HVU, or (2) when the submarine expects deception and therefore closes for visual classification. There is also a third case that is not covered by the analyses: (3) when the submarine does not depend on acoustic information to find and classify contacts.

(U) Do the above considerations mean that an active program in acoustic deception has little potential? Definitely not! The following reasons in favor of deception are founded on the idea of what will happen if deception devices are not available. The first reason for deception is that there will be situations in which the submarine must use his own acoustic information; also there will be situations in which the operating area is small and the time long enough for the submarine to find the HVU. But more importantly, the second reason in favor of deception is that only the existence and previous use of deception devices will cause the submarine to expect deception tactics. Thus, even though actual use of the devices may not increase survivability, the potential for employment must exist before the submarine is driven to visually classify. Is this desirable? Yes, because it is easier to find the submarine in a relatively small visual zone (about a 10-mile radius circle about the HVU or deception device) than in a much larger acoustic zone. In summary, the best of worlds is to use the deception devices without the submarine's expecting it; second best is to use (or have the potential to use) deception but with the knowledge that the defense burden falls on the ASW forces when the submarine closes to check for deception; and worst of all is not having deception devices available at all, thus allowing the submarine to make a standoff cruise-missile attack without ever being subjected to a high ASW threat.

(U) There still remains a question as to the deployment of deception devices and ASW forces in a situation where it is probable that the submarine expects deception. The study showed that the actual use of the devices was not particularly helpful, and that it is somewhat better to keep the total ASW force around the HVU. In the past, the idea

(U)

was that the devices would be beneficial because they would cost the submarine time. This contention has not been substantiated by the present study. It is also claimed that the devices confuse the submarine because of multiple contacts and deny information to the submarine because of masking effects. The present study cannot be used to judge these ideas because the only aspect of acoustic countermeasures that was modeled was the effect of luring a submarine into making an attack on a decoy. Therefore, the limitation of this study (or any study) should be kept in mind before categorically deciding for or against the actual deployment of the devices.

(U) Even though the results of the study indicate the non-deployment of the devices, we argue in favor of deployment of deception devices with ASW forces near them. The first reason is that the submarine might not be expecting the devices; if so, the devices will be effective. Secondly, even if the submarine does expect deception, the HVU survivability will not be particularly jeopardized by deploying the devices, including ASW forces. Third, and most important, the engagement between task group elements and the submarine, if it happens, will be more likely to occur away from the HVU. Thus, the place and time of the fighting should be taken into account, even though the measure of effectiveness used in this study is not very sensitive to use of deception or allocation of ASW.

(U) As a final comment, this study should not in any way be interpreted as being in favor of trading ASW hard-kill potential for deception devices. A strong ASW capability is mandatory, especially in the case where the submarine expects deception. Therefore, in a way, the use of acoustic deception requires an even stronger hard-kill capability, rather than replacing it.

CONTENTS

ILLUSTRATIONS

TABLES

I A BASIC ACOUSTIC DECEPTION MODEL

The purpose of this section is to quantitatively describe a basic acoustic deception process. Although acoustic deception has been the subject of numerous studies in the past several years, we believe that the basic relationships and interactions of parameters are still not well understood. By simplifying the acoustic deception problem to the bare essentials, we hope to describe the heart of the process. As the ideas about acoustic deception have evolved, the attention has been focused too strongly on the hardware of the deception devices themselves; the objective of acoustic deception has become diminished due to a lack of an overall perspective. It is hoped that this introduction will help clarify the basic tradeoffs involved in acoustic deception.

A. The Problem

The problem we choose for modeling is the "objective area" scenario: one high value unit (HVU) randomly cruises in a large fixed area and one submarine randomly searches in this objective area for the HVU. When the submarine makes contact, the target is classified as to whether or not it is worth attacking; if not, the submarine continues searching. Acoustic deception units (ADUs) are also randomly deployed throughout the objective area; these units may be stationary or moving, as desired. The ADUs acoustically simulate the HVU and may be attacked by the submarine. The mission objective of the HVU is to survive for a given period of time without being attacked. If the submarine attacks an ADU, then the HVU is assumed to survive; this represents the case of either: (1) total weapon expenditure so that no more weapons exist to use against the HVU, or (2) immediate counterattack and submarine kill by ASW forces after the submarine exposes his location by attacking the ADU.

The problem is designed so that it is interesting, but not so detailed that it requires a complicated model. Acoustic deception is limited entirely to luring the submarine into an attack on a false target. Other aspects of acoustic deception, such as delaying the submarine and masking the HVU, are not represented. Also, the search for and ASW attrition of the submarine are not modeled. The submarine operates alone and searches with passive sonar; there is no air surveillance support. Both ship radiated noise and ASW sonar sources may be represented by the magnitude of their detection ranges. Although only one submarine is assumed for the problem, any number of submarines can be represented by a relatively simple manipulation of the model.

B. The Model

The type of problem outlined in the preceding section is conveniently handled by a Markov model. Figure 1 depicts a five state Markov model of the phases of the engagement: search, contact, and attack. The first state on the left represents the submarine's searching for targets; an average time between contacts of "T" hours is indicated. (The self-loop arrow indicates that the submarine stays in the search state for a random amount of time.) When a contact does occur, it can be either the HVU or an ADU; the probability that the contact is the HVU is "P." The submarine then classifies the contact: (1) if the contact is the HVU, the probability of correctly classifying and attacking the HVU is "p"; (2) if the contact is an ADU, the probability of misclassifying and attacking the ADU is "q." (Note that q is not 1-p.) The arrows that return to the search state show that, if the submarine misclassifies the HVU or correctly classifies the ADU, search starts again. Note that no time is spent to classify the contact. (If self-loop arrows were on the contact states, a classification duration would have been indicated.) The self-loop arrows or the attack states mean that once these states are entered they cannot be left.

This simple Markov model can be solved in closed form. The solution is an equation for the probability that the "attack HVU" state is entered before a given period of time expires. Only the parameters shown on

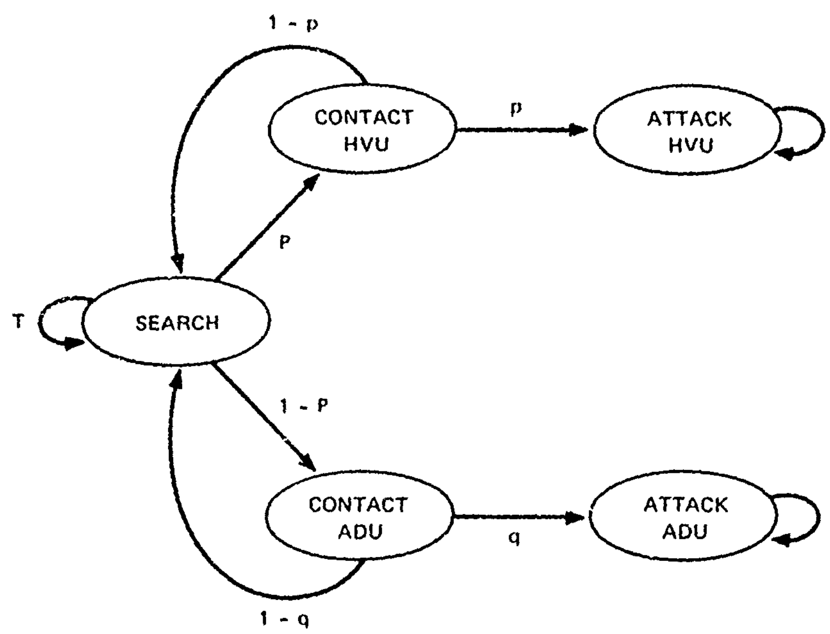


FIGURE 1 MARKOV MODEL OF A BASIC DECEPTION PROCESS

Figure 1 (T,P,p,q) are needed to derive the probability that the HVU is attacked, but T and P are not in terms of tactically interesting quantities. Therefore, more basic input parameters are used.

C. Input Parameters

The parameters that replace T and P are speeds, sweep widths, submarine density, and number of acoustic deception units. The following is a complete list of the input parameters.

1) a = sweep width against the HVU (nmi)
2) b = sweep width against an ADU (nmi)
3) s_0 = average speed of the submarine (kt)
4) s_1 = average speed of the HVU (kt)
5) s_2 = average speed of the ADU (kt)
6) n = number of ADUs
7) σ = submarine density (number/nmi^2)

8) t = time in the objective area (hr)

9) p = probability of correctly classifying the HVU

10) q = probability of misclassifying the ADU.

Sweep width is defined in Naval Operations Analysis[1] as the area under the "lateral range curve." The lateral range curve is, in turn, defined as the cumulative probability of detection as a function of the closest point of approach. (The cumulative probability is found by integrating along a straight line track relative to the target.) Sweep width is not twice the "fifty percent" detection range, but is a considerably more general concept. However, to facilitate intuition, sweep width can be thought of as twice the "cookie cutter" detection range.

The input parameters are not all independent. For example, the sweep width depends on speed of both the target and submarine. If detection is made on the target's radiated noise, and the noise level increases with target speed, then the sweep width increases with target speed. An increase in submarine speed will generally decrease the sweep width because the submarine's self-noise will increase.

Another effect of target and submarine speed works on the lateral range curve. The faster either vehicle goes, the lower the cumulative probability of detection because the time spent at any given range is less. Therefore, increased target speed decreases the sweep width. As to whether the sweep width actually grows or diminishes with target speed will depend on the details of the individual case. However, it is expected that, when radiated noise is the only contributing factor to sweep width, increased target speed will increase sweep width.

Another example of parameter interdependence is between the probability of misclassifying the ADU and the speed of the ADU. The submarine may be able to discriminate on kinematic information if the ADU is not actually moving at the speed that it sounds as though it is moving.

The three speed parameters are basic tactical parameters but are not directly useful in the model. It is the average relative speed between the target and submarine that is of interest. If the courses of the HVU and submarine are uniformly distributed, then the average relative speed is given by the integral:

$$u = \frac{1}{2\pi} \int_0^{2\pi} \sqrt{s_0^2 + s_1^2 - 2 s_0 s_1 \cos\theta}\; d\theta \quad ,$$

where θ is the angle between the two speed vectors. This integral can be numerically integrated or tables of elliptical integrals may be used. A simple equation that closely approximates the integral is shown on Figure 2. (The largest error is about 1 percent.)

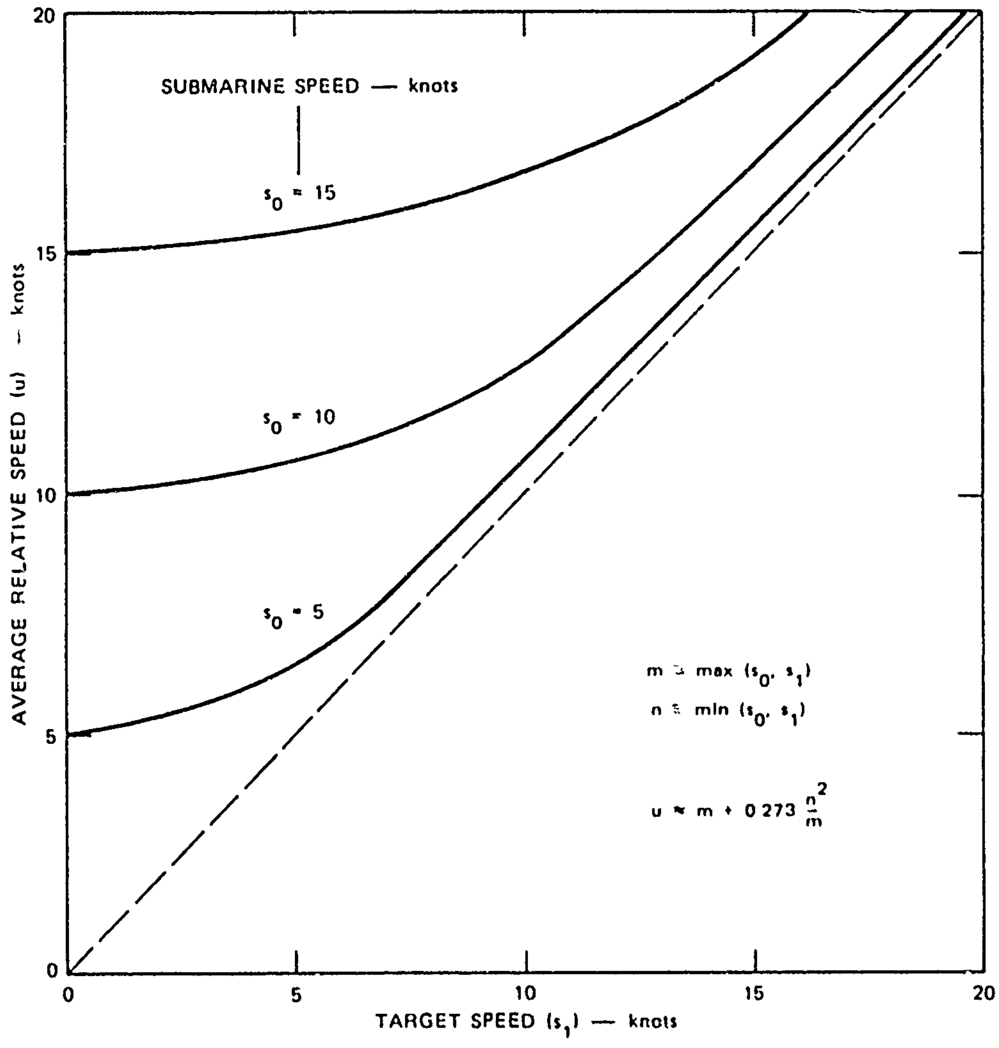


FIGURE 2 AVERAGE RELATIVE SPEED FOR UNIFORMLY RANDOM COURSES OF SUBMARINE AND TARGET

Two new parameters, u and v , are used in place of s_0 , s_1 , and s_2 in the rest of the analysis:

1) u = average relative speed between the HVU and submarine
2) v = average relative speed between the ADU and submarine.

D. The Derivation

The derivation of the equations describing the model is straightforward and therefore is presented here instead of being relegated to an appendix. Besides, a certain amount of insight into the model can be gained at each stage of the derivation.

The rate at which the submarine encounters the HVU or ADUs is needed. These rates are identical to the rates at which the HVU encounters the submarine and the ADUs encounter the submarine. The sweep width times the relative speed produces area per unit time; this multiplied by the submarine density (number per unit area) yields the required search rates:

$$\lambda_1 = au\sigma$$

$$\lambda_2 = nbv\sigma \quad ,$$

where λ_1 is the number of HVU contacts per hour, and λ_2 is the number of ADU contacts per hour. (There are n ADUs and any one can be contacted, therefore the factor n is included.)

Given that a contact is made, what is the probability that it is the HVU? In one hour λ_1 HVUs are detected and a total of $\lambda_1 + \lambda_2$ contacts are made. Therefore,

$$p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

is the probability that the HVU is contacted given that a contact is made.

With the above probability P and the classification probabilities, the conditional probability that the HVU is attacked, given that an attack occurs, can be calculated. Referring to Figure 1, it is seen that the number of cases ending in the "attack HVU" state is proportional to pP, whereas the total number of cases ending in either "attack" state is proportional to pP + q(1-P). Therefore, the conditional probability is:

$$C = \frac{pP}{pP + q(1-P)} \quad .$$

This equation can be simplified by defining a new parameter, the "blue deception factor":

$$x = \frac{q(1-P)}{pP} = \frac{nqbv}{pau} \quad .$$

Therefore,

$$C = \frac{1}{1 + x} \quad .$$

The adjective "blue" is used to describe x because the friendly forces benefit when x increases. The adjective "deception" is used to describe x because this factor contains all the parameters that define the acoustic deception units. When there are no deception devices (n=0), there is no deception (x=0) and the conditional probability of attack on the HVU, given an attack, is certain (C=1).

The average time between contacts is by definition the reciprocal of the total contact rate:

$$T = \frac{1}{\lambda_1 + \lambda_2} = \frac{P}{\lambda_1} \quad .$$

The average number of contacts before an attack occurs (including the contact that is attacked) is given by:

$$N = \frac{1}{pP + q(1-P)} \quad .$$

The last equation can be deduced by an analogy to tossing a coin of probability "h" of coming up heads. The average number of tosses (contacts) before heads (an attack) is one over h (one over pP + q(1-P)).

The average time before an attack occurs is just:

$$\tau = NT = \frac{C}{p\lambda_1} .$$

The ratio t/τ, which is needed later, can be written in terms of a new parameter, the "red coverage factor":

$$y = p\lambda_1 t = pau\sigma t ,$$

where t is the time spent in the objective area. Therefore,

$$\frac{t}{\tau} = (1+x)y .$$

The adjective "red" is used to describe y because the enemy force (the submarine) benefits when y increases. The adjective "coverage" is used because it represents the percent of the objective area that the submarine can cover in a time t while looking for the HVU.

Now that the average time before an attack τ is known, the probabilities of ending in an attack state can be calculated as a function of time. To do this, the five state model of Figure 1 is replaced by the two state model of Figure 3.

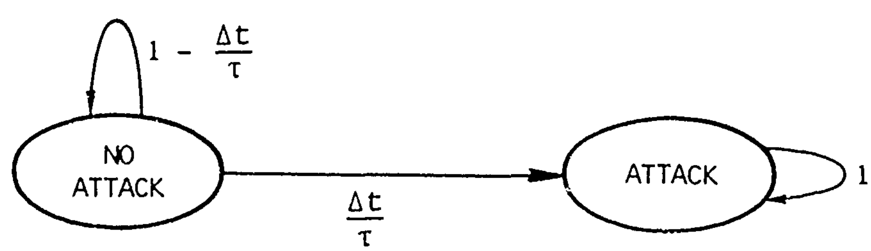


FIGURE 3 TWO STATE MODEL

This figure shows that the probability of transitioning from the "no attack" state to the "attack" state is $\Delta t/\tau$, where Δt is a small increment of time that is much smaller than τ. The expression for the transition probability is almost intuitive: if an attack is equally likely anywhere in the interval of τ, then the probability of attack in an interval Δt is just $\Delta t/\tau$.

The probability of being in the "no attack" state after "m" transitions is $(1 - \Delta t/\tau)^m$. Replacing Δt by t/m and letting m approach infinity, the probability of being in the "no attack" state at time t approaches $e^{-t/\tau}$. This result shows that the two state Markov process reduces to a Poisson process as the time for one transition approaches zero.

The probability that an attack has occurred by time t is $1 - e^{-t/\tau}$. The probability that the HVU is attacked by time t is the conditional probability of attack, C, times the probability of an attack occurring by time t . Finally, the probability of survival is defined as the probability that the HVU is <u>not</u> attacked by time t ; therefore, the probability of survival is:

$$P_s = 1 - C(1 - e^{-t/\tau}) \quad .$$

And this reduces immediately to the final solution:

$$P_s = 1 - \frac{1 - e^{-(1+x)y}}{1 + x} \quad ,$$

where $x = \dfrac{nqbv}{pau\sigma}$, blue deception factor

$y = pau\sigma t$, red coverage factor ,

and where n = number of ADUs

q = probability of misclassifying the ADU

b = search width of the ADU (nmi)

v = relative speed between ADU and submarine (kt)

p = probability of correctly classifying the HVU

a = sweep width of the HVU (nmi)

u = relative speed of HVU and submarine (kt)

σ = submarine density (number/nmi^2)

t = time in area (hr) .

E. The Results

The probability of HVU survival is plotted in Figure 4 as a function of the deception and coverage factors, x and y. Note that the effect of varying any parameter can be deduced from Figure 4; this is due to the inclusion of all the parameters in just two nondimensional factors. This factorization is very important to analyzing the model because it allows parameter comparisons by inspection only, rather than by a cumbersome numerical sensitivity analysis. Also notice that, for the case in which there is no deception (x=0), the model reduces to the common random search model (except that the usual search rate, $au\sigma$, is multiplied by the probability of correctly classifying the HVU).

Several important results and interactions can be discerned from Figure 4. First, there are cases in which deception devices should not be used. When the red coverage factor is less than about y = 0.2, HVU survival does not change as deception increases; therefore, it is pointless to deploy deception devices in a low coverage situation. The reason for this statement is that the submarine will have a difficult time finding the HVU in the allotted time, regardless of the number of decoys.

When the red coverage factor is between y = 0.2 and y = 2, deception devices will have a beneficial effect; however, the biggest payoff will be from reducing the three parameters, pau. The probabilities of correctly classifying the HVU (p), the HVU sweep width (a), and the HVU/submarine relative speed (u), are the most important parameters because they occur in both the coverage factor and in the deception factor. To illustrate this let us determine which is better: to double the number of ADUs or to cut the HVU sweep width in half? Assume that the initial operating point is at x = 1 and y = 1; then

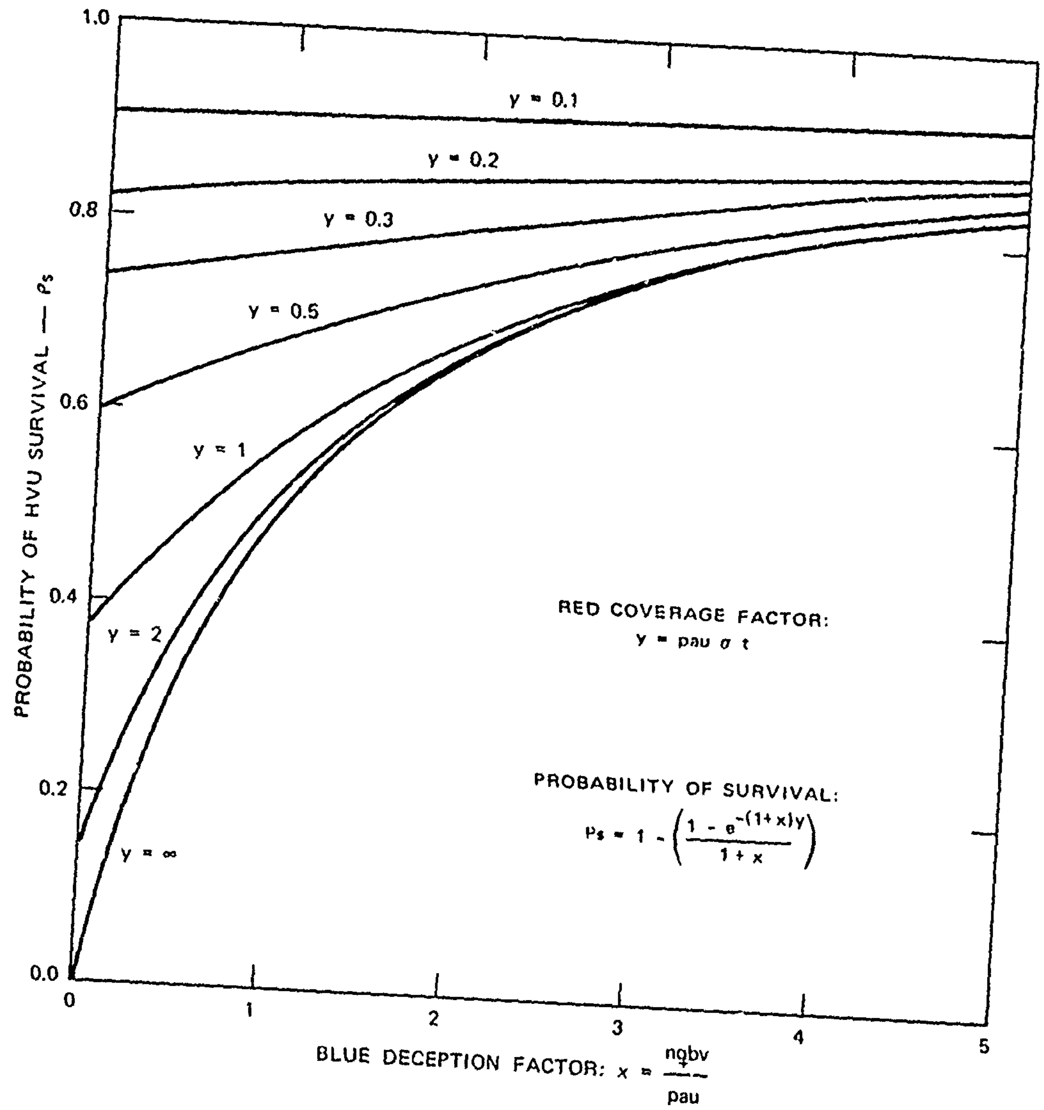


FIGURE 4 EFFECT OF COVERAGE AND DECEPTION ON SURVIVAL

from Figure 4, the probability of HVU survival is $P_s = 0.57$. If the number of devices is doubled, $x = 2$, $y = 1$, and $P_s = 0.68$. If, instead, the HVU sweep width is cut in half, $x = 2$, $y = 0.5$, and $P_s = 0.74$. A smaller sweep width increases the probability of survival 17 points, whereas adding more devices results in only an 11-point increase.

The HVU sweep width is largely under control of the HVU. Radiated noise can be reduced by going slower. This, in turn, reduces the relative speed, which also decreases the "pau" factor. An active sonar may be deployed with the HVU; if it were turned off, the survivability of the HVU would increase because the sweep width is smaller. However, there may be a way to keep the active sonar on to enhance ASW protection. If the parameter "p" can be lowered the same amount that "a" is increased, then the probability of survival will remain the same. This might be accomplished by equipping all of the ADUs with ASW sonar simulators so that all the units will look alike and therefore the submarine will have a difficult time correctly classifying the HVU. It is expected, however, that only modest increases in "a" could be allowed because of both the difficulty of lowering "p", and because of the uncertainty of its value.

For very large coverage factors, greater than about y = 2, the use of deception devices will be most beneficial. For this coverage region the importance of the "pau" factor is reduced because the payoff from reducing the coverage factor is not nearly so great as is the payoff from increasing the deception factor. Therefore, all seven parameters in the blue deception factor (nqbv/pau) are of equal importance when the red coverage factor is large. Also, the submarine density (σ) and the time in area (t) are of less importance since they do not affect the deception factor.

An implication of the above statements is that for $y > 2$, the reduction of p (the probability of correctly classifying the HVU) is equally important to increasing q (the probability of misclassifying the ADU). This is not true when $y < 2$: in this case it is more important to decrease p than it is to increase q . In other words, it is more important to make the HVU look like some other target than it is to make the ADU look like the HVU, when the coverage factor is less than y = 2.

Another kind of constraint region on Figure 4 is when the deception factor is large. The addition of more deception devices is not needed if the deception factor is greater than about x = 4. The reason is that the change in the probability of survival is very small for a unit change in deception, regardless of the coverage factor.

The fact that only the ADU parameters are included in the deception factor allows for simple tradeoffs to be investigated by fixing the value of x . A good value to choose is x = 4, since it is a kind of maximum deception value. An interesting tradeoff is between numbers of decoys and their detection range.

Figure 5 shows the number of decoys needed to make x = 4 as a function of the ratio of ADU-over-HVU sweep width. Three curves are shown for the likely range of values for the remaining four parameters (qvpu). Note that these tradeoff curves do not depend on the submarine density nor the time in the area.

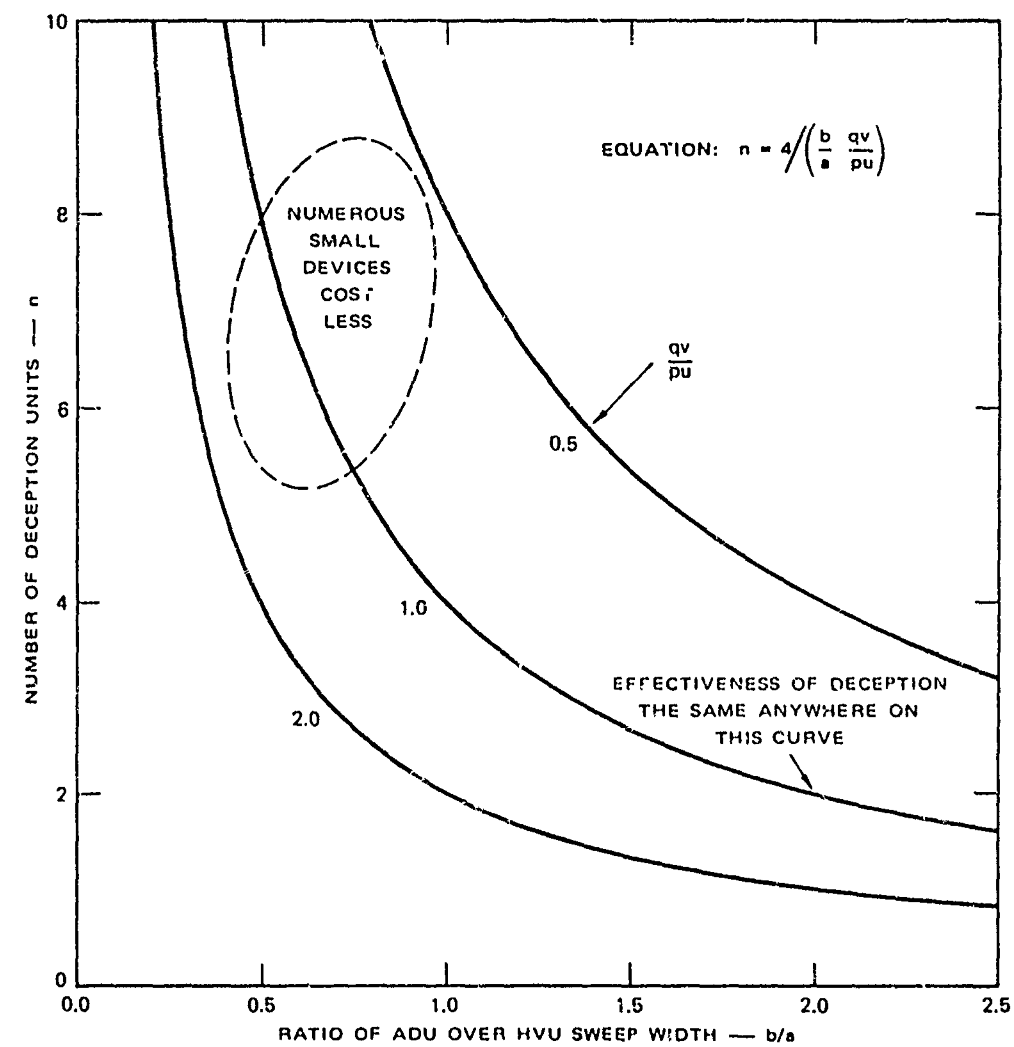


FIGURE 5 TRADEOFF BETWEEN NUMBER OF DECEPTION UNITS AND DETECTION RANGE

Since there is a tradeoff between numbers of devices and detection range, is it better to have numerous small devices or a few large devices? The operational value of the deception is fixed (x = 4); therefore, the choice should be made on the cost of the deception. Assume that the cost is proportional to: (1) the number of devices, and (2) the power required for each device. Assume further that the required power is proportional to: (1) the detection range-squared for nonconvergence zone environment, and (2) the detection range for convergence zone environments. (This assumes that the transmission loss is approximately spherical spreading for nonconvergence zones and cylindrical spreading for convergence zones.) With these simplifying assumptions, the cost of deception is proportional to nb^2 or nb. But from Figure 5, the detection range is inversely proportional to the number of devices for a fixed deception value. Therefore, the conclusion is that, for a given level of deception: (1) the cost of deception is inversely proportional to the number of devices in a nonconvergence zone environment, and (2) the cost of deception is independent of the number of devices in a convergence zone environment. This means that the cost goes down if more devices are used in the nonconvergence zone environment, but that there is no cost tradeoff for the convergence zone environment. However, the convergence zone environments do not always exist, therefore it is better to have numerous small devices than a few large devices.

F. More Submarines

The model of the simple acoustic deception process was developed with one submarine as the threat. More submarines can be included in the model in two basically different ways, depending on the scenario. First, if the submarines operate independently in the area, then the probabilities of surviving each submarine are independent, and they can be multiplied together to give a final probability of surviving all submarines. The submarine density (σ) for this case is calculated as though there were only one submarine.

If instead, the submarines divide the area into zones, one submarine to a zone, then the probability of surviving all submarines depends on what the HVU does if a submarine is neutralized by attacking an ADU. If the HVU stays in the neutralized zone, then the probability of survival is given by the present model, but with the submarine density calculated as the total number of submarines divided by the operating area. In other words, the HVU must survive only one zone.

If however, the HVU moves to zones in which submarines are still a threat (or if the submarines divide the area anew after an unsuccessful attack on an ADU), then the calculation of the probability of survival is more complicated. Assume that two submarines divide the area into two equal zones. If one of the submarines is neutralized by attacking an ADU, assume that the engagement continues with just one submarine. The probability of survival can be written as one minus the total probability of attacking the HVU. The probability of attacking the HVU is composed of two parts: when both submarines are present (P_2), and when only one submarine is present (P_1):

$$P_2 = C(1 - e^{-t/\tau_2})$$

$$P_1 = C(1 - e^{-t/\tau_1}) \quad ,$$

where τ_2 is calculated with a density of two submarines in the area, and τ_1 with one submarine in the area. The probability that one or the submarines attacks an ADU is:

$$A = (1-C)(1 - e^{-t/\tau_2}) \quad .$$

Therefore, the probability of survival is:

$$P_s = 1 - (P_2 + A\,P_1) \quad .$$

G. <u>An Example Calculation</u>

As an example, assume that one submarine is randomly searching in a circular area 400 nmi in diameter and that the HVU must survivie for seven days. The input parameters are as follows:

1) $a = 60$ nmi

2) $b = 60$ nmi

3) $s_0 = 10$ kt

4) $s_1 = 10$ kt

5) $s_2 = 0$ kt

6) $n = 3$ ADUs

7) $\sigma = 1/(\pi\ 200^2)$ number/nmi^2

8) $t = 7(24)$ hr

9) $p = 0.8$

10) $q = 0.6$.

The relative speeds are calculated as:

1) $u = 12.7$ kt

2) $v = 10.0$ kt.

If contact is made, the probability that it is the HVU is:

$$P = \frac{\lambda_1}{\lambda_1 + \lambda_2} = \frac{60\ 12.7}{60\ 12.7 + 3\ 60\ 10} = 0.30 \quad .$$

The conditional probability that the HVU is attacked, given that an attack occurs is:

$$C = \frac{pP}{pP + q(1-P)} = \frac{.8\ .30}{.8\ .30 + .6\ .70} = 0.36 \quad .$$

The average time between contacts is:

$$T = \frac{1}{\lambda_1 + \lambda_2} = \frac{3.14\ 200\ 200}{60\ 12.7 + 3\ 60\ 10} = 49.0 \text{ hours} \quad .$$

The average number of contacts before an attack occurs is:

$$N = \frac{1}{pP + q(1-P)} = \frac{1}{.8\ .30 + .6\ .70} = 1.52 \quad .$$

The average time before an attack is:

$$\tau_1 = NT = 1.52\ \ 49.0 = 74.5 \text{ hours} \quad .$$

The blue deception factor is:

$$x = \frac{nqbv}{pau} = \frac{3\ .6\ 60\ 10.0}{.8\ 60\ 12.7} = 1.77 \quad .$$

The red coverage factor is:

$$y = pau\sigma t = \frac{.8\ 60\ 12.7\ 7\ 24}{3.14\ 200\ 200} = 0.81 \quad .$$

The probability that the HVU is attacked is:

$$P_1 = C(1 - e^{-t/\tau_1}) = 0.36\ (1 - e^{-168/74.5}) = 0.32 \quad .$$

The probability of surviving the one-submarine threat is:

$$P_s = 1 - P_1 = 0.68 \quad .$$

If two submarines operate independently, the probability of surviving the two-submarine threat is:

$$P_s = (0.68)^2 = 0.46 \quad .$$

The probability of an attack on the HVU when two submarines operate in zones is:

$$P_2 = C(1 - e^{-t/\tau_2}) = 0.36\ (1 - e^{-168/37.3}) = 0.36 \quad .$$

If the HVU must survive only one zone, then the probability of surviving the two-submarine threat is:

$$P_s = 1 - P_2 = 0.64 \quad .$$

The probability that one of the submarines attacks an ADU is:

$$A = (1-C)\ (1 - e^{-t/\tau_2}) = 0.64\ (1 - e^{-168/37.3}) = 0.64 \quad .$$

If the HVU must survive both zones, then the probability of surviving the two-submarine threat is:

$$P_s = 1 - (P_2 + A\ P_1) = 1 - (.36 + .64\ .32) = 0.44 \quad .$$

Of the various ways to calculate the two-submarine threat, the best for the HVU is the one-zone survival (0.64), the next is the random-submarine survival (0.46), and the worst is the two-zone survival (0.44). All of these probabilities can be compared with the one-submarine threat survival (0.68).

H. Limitations

Although a good deal of information can be gleaned from this simple deception model, its limitations should be kept in mind. First, the scenario is limited: an objective area engagement with one autonomous submarine and two kinds of friendly units, one high value unit, and several acoustic deception units.

Second, the definition of "survival" includes the neutralization of the submarine if it attacks a decoy. No ASW protection, missile defense, or missile reliability is included in the survival probability. "Survival," as used in the model, is not "real" survival.

Third, the only kind of deception represented in the model is the "luring" type, in which the submarine attacks the decoy and is neutralized. Degradation of the submarine threat by the confusion induced by multiple contacts is not modeled.

Fourth, the complicated aspects of classification are ignored and replaced with two input parameters. Also, the classification, approach, and attack phases of the engagement are assumed to take no time at all.

Fifth, the model is a continuous time Markov process; this means that the time in-state is exponentially distributed. These kinds of models are very similar to random search models, so the assumptions of randomness must be obeyed. The positions of the friendly units cannot be in a pattern. The randomness requirement is broken when the search width is about equal to the dimensions of the operating area. Then the boundaries in which the participants must stay induce a kind of pattern. The model assumed no boundary conditions.

The model investigated in the next section is designed to reduce some of the above limitations. It includes ASW attrition of the submarine and a much closer look at the classification problem.

II ACASO MODEL DESCRIPTION

To study the basic tradeoffs between acoustic countermeasures (ACM) and ASW in the objective area, a nine state Markov model was developed that relates ACM effectiveness and ASW capabilities to high value unit survival. This model is an expansion of the basic model described in the previous section. There is one high value unit (HVU) and a number of acoustic deception units (ADU) moving randomly and independently in an objective area. Associated with each unit is a designated level of ASW protection against a submarine that is searching for the HVU. Five mutually exclusive outcomes can occur for a given duration in the operating area:

- Submarine attacks the HVU
- Submarine attacks an ADU
- Submarine is killed by the HVU
- Submarine is killed by an ADU
- Submarine is in search at end of the time period.

The measure of effectiveness calculated by the Acaso* model is the probability that the HVU survives for a given time period.

The state diagram for the model is shown in Figure 6. The submarine is assumed to be initially in the search state and then encounters the HVU or an ADU. Then the submarine decides to close the target for an acoustic or visual classification. If the submarine successfully penetrates the target's ASW defenses, a classification decision is made and the submarine either attacks the target or resumes search for the HVU. It is assumed that the submarine launches all of his weapons on an attack; therefore, if the submarine mistakenly attacks an ADU, the HVU will survive.

---

*Acaso: acoustic countermeasures and antisubmarine warfare in the objective area.

PRECEDING PAGE BLANK-NOT FILMED

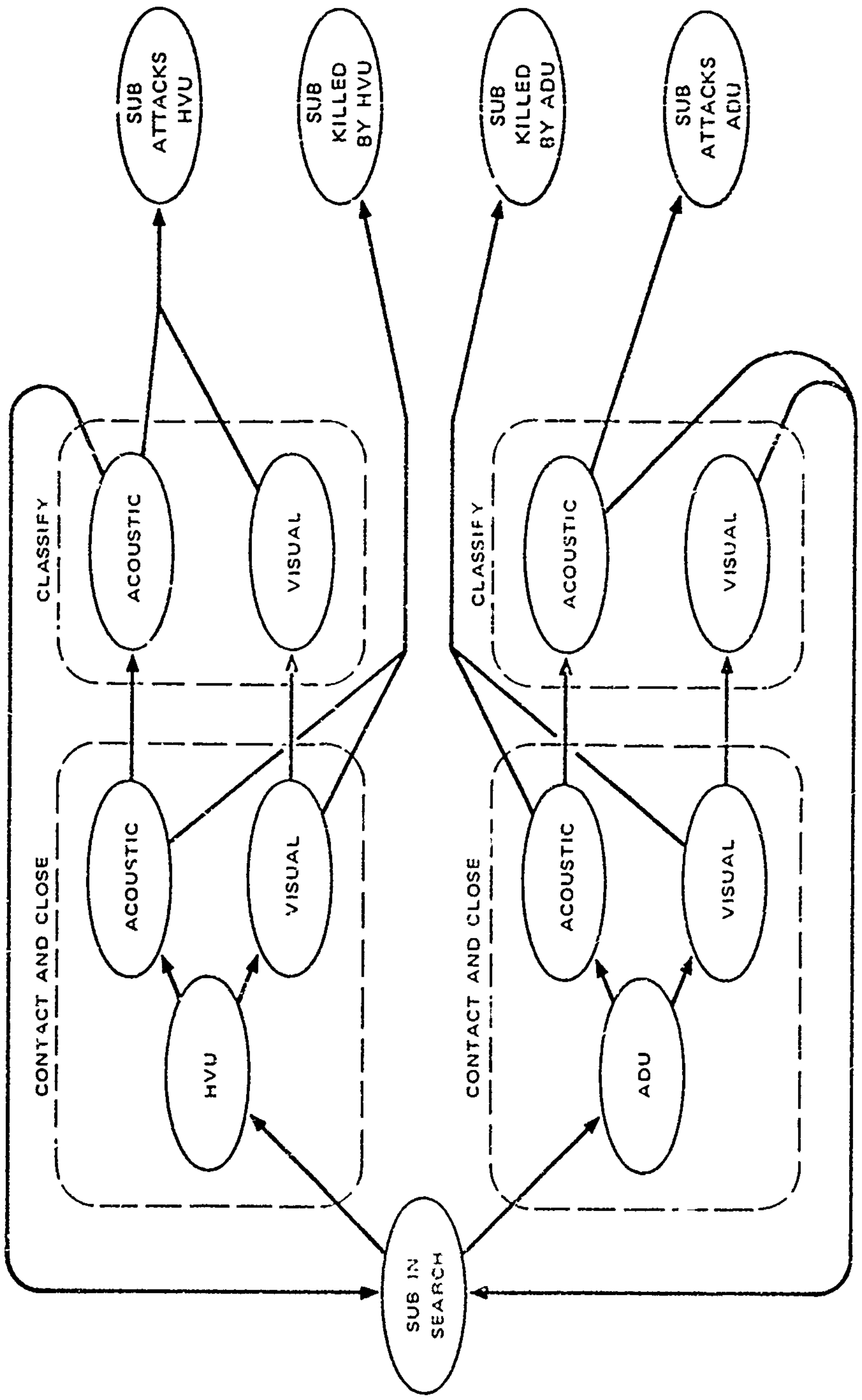


FIGURE 6 DIAGRAM OF THE ACASO MODEL

The diagram of Figure 6 can be collapsed into a nine state Markov model as shown in Figure 7. For the ease of calculation, the equations developed by Operations Research Incorporated (ORI)[2] are used for the final calculations in the model. These equations are shown in Appendix A to be nearly equivalent to the Markov model of Figure 7.

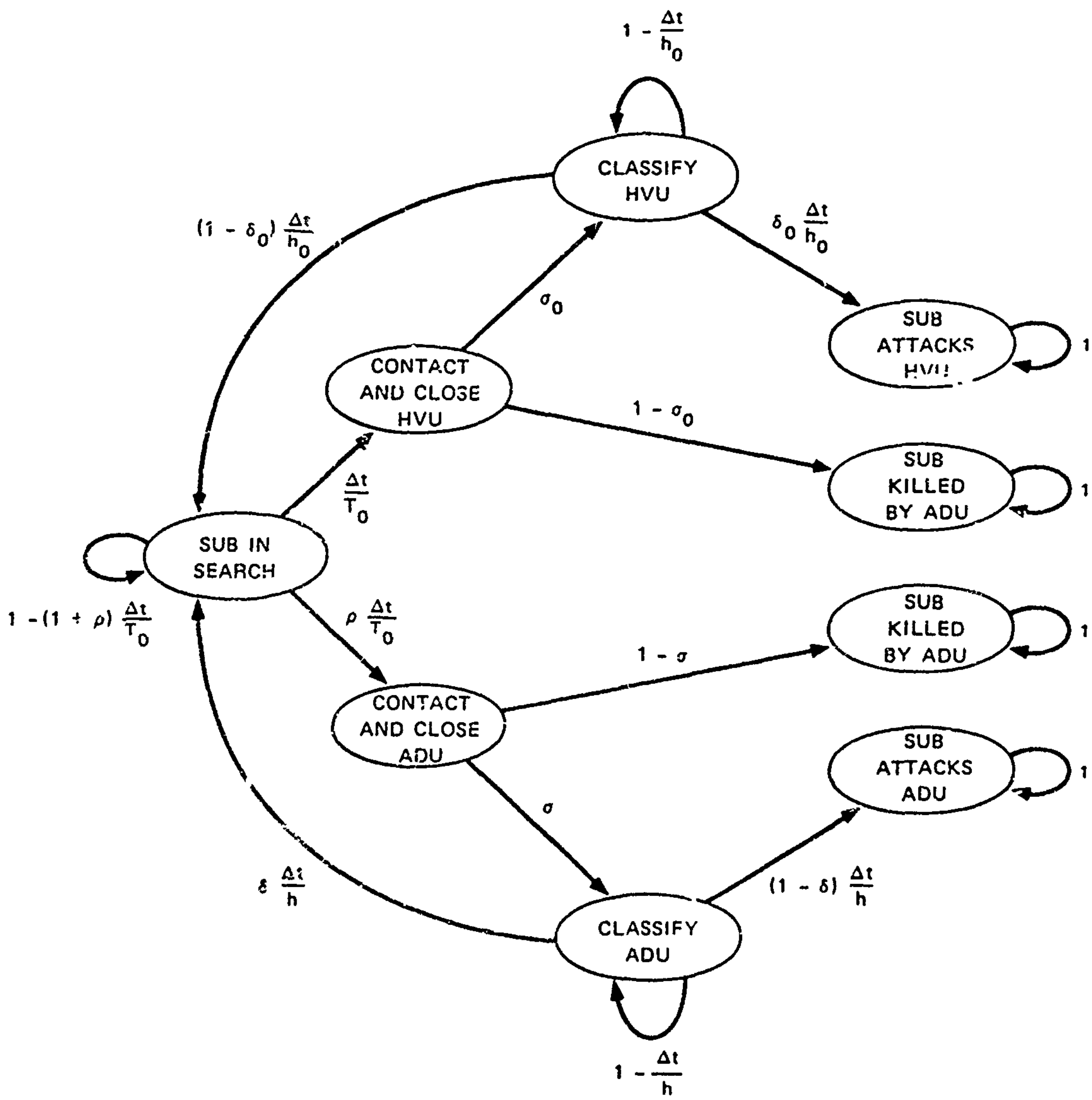


FIGURE 7 NINE-STATE MARKOV MODEL

The transition probabilities shown next to the arrows on Figure 7 are constructed from the ORI parameters as defined in Table 1. In order

Table 1

ORI's PARAMETERS

T_o	Average time required to detect the HVU in the absence of any ADUs
ρ	Ratio of ADU-over-HVU search rates (the HVU search rate is $1/T_o$)
σ_o	Probability that the submarine can penetrate the HVU's ASW defense
σ	Probability that the submarine can penetrate the ADU's ASW defense
δ_o	Probability that the submarine will correctly classify the HVU, given that it survives the local ASW defense
δ	Probability that the submarine will correctly classify an ADU, given that it survives the local ASW defense
h_o	Mean time to classify the HVU
h	Mean time to classify the ADU
t	Total time in the operating area

to use the ORI equations to calculate the probability of HVU survival for various tactical situations, it is necessary to convert a tactical description of the scenario to these nine ORI parameters. The Acaso model determines these parameters from a more basic set of 17 tactical parameters.

Tables 2, 3, and 4 list the input parameters to the model, the intermediate parameters used within the model, and the output parameters. As shown in Table 2, the input parameters can be classified into four groups: the parameters which define the scenario, submarine, ACM capabilities, and ASW effectiveness. The model converts these input

Table 2

INPUT PARAMETERS

Scenario	
r_o	HVU detection range (nmi)
r	ADU detection range (nmi)
v_o	HVU speed (kt)
v	ADU speed (kt)
u	Submarine speed (kt)
t	Time in the operating area (hr)
A	Operating area (nmi^2)
Submarine	
$\hat{P}_o$	The submarine's estimate of the probability that the next contact will be the HVU
$\widehat{PH}$	The submarine's estimate of the probability that the HVU will produce an "HVU-like" feature
$\widehat{PA}$	The submarine's estimate of the probability that the ADU will produce an "ADU-like" feature
PE	The maximum allowed expected classification error (PE is set by the submarine)
Acoustic Countermeasures	
N	Number of ADUs in the operating area (there is one HVU in the area)
PH	Probability that the HVU produces an "HVU-like" feature
PA	Probability that the ADU produces an "ADU-like" feature
Antisubmarine Warfare	
F_o	Fraction of ASW forces remaining with the HVU
PKA	Probability that the total ASW force can kill the submarine if the submarine closes to acoustically classify
PKV	Probability that the total ASW force can kill the submarine if the submarine closes to visually classify

Table 3

INTERMEDIATE PARAMETERS

Speed Parameters		
	s_o	Average relative speed between the HVU and submarine
	s	Average relative speed between the ADU and submarine
Classification Parameters		
	PM	Probability of a "missed detection" because the HVU is classified "ADU"
	PF	Probability of "false alarm" because an ADU is classified "HVU" (these interpretations of PM and PF are good under the case I threshold condition only, see discussion)
	$\hat{X}_o$	Separation of the estimated HVU and ADU distributions
	X_m	Missed detection threshold
	X_f	False alarm threshold
	X_o	True separation of the HVU and ADU distributions
	Y	Displacement of the estimated ADU distribution from the true one
	X_1	Lower visual threshold in standard deviations from the mean of true ADU distribution
	X_2	Upper visual threshold in standard deviations from the mean of true ADU distribution
	PV_o	Probability that the submarine chooses to visually classify the HVU
	PV	Probability that the submarine chooses to visually classify the ADU
	PHA_o	Probability that the HVU is called "HVU", given that acoustic classification is used
	PHA	Probability that the ADU is called "HVU", given that acoustic classification is used
Penetration Parameters		
	F	Fraction of the total ASW force that is assigned to one ADU
	PPV_o	Probability that the submarine penetrates to visually classify the HVU
	PPV	Probability that the submarine penetrates to visually classify the ADU
	PPA_o	Probability that the submarine penetrates to acoustically classify the HVU
	PPA	Probability that the submarine penetrates to acoustically classify the ADU
	W_o	Probability that the submarine classifies acoustically, given that it has penetrated the HVU's defenses
	W	Probability that the submarine classifies acoustically, given that it has penetrated the ADU's defenses

Table 4

OUTPUT PARAMETERS

T	Average time of HVU survival, given that the HVU is eventually attacked
A_o	Asymptotic probability that the HVU is attacked
A	Asymptotic probability that an ADU is attacked
K_o	Asymptotic probability that the submarine is killed by the HVU's ASW forces
K	Asymptotic probability that the submarine is killed by the ADU's ASW forces
P_s	Probability that the HVU survives for a given duration
P_1	Probability that the submarine was killed by the HVU's ASW forces, given that the HVU survived
P_2	Probability that the submarine was killed by the ADU's ASW forces, given that the HVU survived
P_3	Probability that the submarine attacked an ADU, given that the HVU survived
P_4	Probability that the submarine was in search or was attempting to classify a contact at the time of HVU survival (Note: $P_1 + P_2 + P_3 + P_4 = 1$)

parameters into the intermediate parameters, and then ORI's parameters are derived and used to calculate the output parameters. (Note: the symbols used in this section for the Acaso model are not the same as used in the previous section for the basic model.)

A. Intermediate Equations

1. Speed Submodel

The average relative speed (s_0) between the HVU and the submarine is:

$$s_0 = \frac{1}{2\pi}\int_0^{2\pi} w\,d\varphi \quad ,$$

where w is the relative speed for a particular track angle φ. The exact solution requires numerical integration; however, in the model s_0 is approximated by the formula:

$$s_0 = \max(v_0, u) + .273 \min^2(v_0, u)/\max(v_0, u) \quad .$$

The value of s is computed by a similar equation with v_0 replaced by v. This relative speed model assumes that the submarine's track direction is uniformly distributed from 0 to 360 degrees.

2. Classification Submodel

The classification submodel is a Bayesian decision process based on probability distributions of the random variable X, the value of a hypothetical feature. This feature is a one-dimensional composite measure of the information provided by the submarine's acoustic sensors. In the model it is assumed that the features generated by the ADUs and HVU are normally distributed with equal variance. The calculations in the classification model require the use of two functions of the Normal distribution:*

$p = G(x)$

where $p = \text{Prob}(-\infty < Z \leq x)$ where Z is the standard normal variable

$x = AG(p)$, the inverse Normal function (if $p = G(x)$, then $x = AG(p)$).

---

*These functions were approximated by using the formulae on pages 932 and 933 of the "Handbook of Mathematical Functions," APS-55, National Bureau of Standards, U.S. Department of Commerce, ninth printing, November 1970.

In order to position the ADU and HVU distributions for further calculations, it is assumed that there is an arbitrary value of the feature, X_T, which is a preliminary classification threshold set by the submarine. Values of the feature to the right of X_T are assumed to be designated by the submarine as "HVU-like" and values to the left of X_T are assumed to be designated by the submarines as "ADU-like." Figure 8 shows the construction of the true ADU and HVU feature distributions. The input parameter PA specifies the amount of area of the ADU feature distribution that is to the left of X_T, and, likewise, PH specifies the amount of area of the HVU feature distribution that is to the right of X_T.

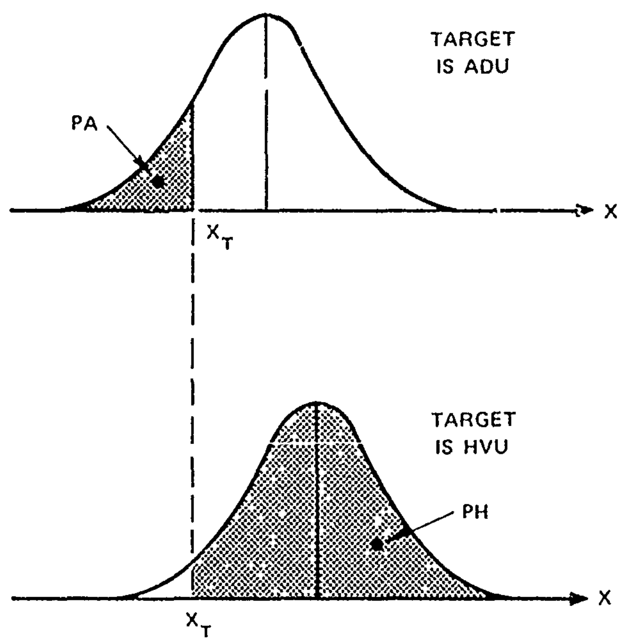


FIGURE 8 TRUE ADU AND HVU FEATURE DISTRIBUTIONS

If the submarine were constrained to acoustic sensor information only, X_T would be the classification decision threshold, and the submarine would have to accept the large errors associated with such a criteria. However, the submarine has more information available to utilize in his classification decision making:

- Estimate of the ADU feature distribution
- Estimate of the HVU feature distribution
- Estimate of the probability that the next contact will be the HVU
- Total expected misclassification error the submarine is willing to accept.

To limit his misclassification errors, the submarine has the choice of closing the target for visual identification (perfect classification). The value of the input parameter PE carries an implicit submarine estimate of the ASW threat. Although the submarine can achieve perfect classification by closing the target for visual identification, he does so at a greater risk than with acoustic classification. Therefore, the submarine should want to close the target for visual classification only when the acoustic information is not sufficient to make a decision (that is, when the expected error is greater than PE). The maximum expected misclassification error, PE, consists of two types of errors:

- Classifying the HVU as "ADU" ("missed detection")
- Classifying the ADU as "HVU" ("false alarm").

It is assumed that the submarine constrains the ratio of these two types of errors in proportion to his estimate of the probability of encountering the HVU ($\hat{P}_O$), therefore:

$$\frac{PM}{PF} = \frac{1 - \hat{P}_O}{\hat{P}_O} ,$$

and the submarine constrains his total expected error by:

$$PE \geq \hat{P}_O PM + (1-\hat{P}_O) PF ,$$

where PM is the probability of "missed detection" and PF is the probability of "false alarm."

Solving for PM and PF:

$$PM = \begin{cases} .5\ PE/\hat{P}_o & .5\ PE \le \hat{P}_o \le 1 \\ 1 & \text{otherwise} \end{cases}$$

and

$$PF = \begin{cases} .5\ PE/(1-\hat{P}_o) & 0 \le \hat{P}_o < 1 - .5\ PE \\ 1 & \text{otherwise} \end{cases} \quad .$$

To relate these errors to the received feature, the submarine constructs estimated feature distributions for the ADU and HVU. These distributions are constructed in a similar manner to the real distribution, utilizing the input parameters $\widehat{PA}$ and $\widehat{PH}$, and the previously discussed threshold, X_T. Also, these estimated distributions are assumed to be normally distributed with equal variance. For calculation purposes, the mean of the estimated ADU feature distribution is defined as zero.

Referring to Figure 9a, the submarine can now set his decision thresholds:

$$X_f = AG(1 - PF)$$

$$X_m = \hat{X}_o + AG(PM) \quad ,$$

where $\hat{X}_o$ is the difference of the means of the estimated ADU and HVU feature distributions:

$$\hat{X}_o = AG(\widehat{PA}) - AG(1 - \widehat{PH}) \quad .$$

If in a particular encounter the received feature X is less than X_m, then the submarine can acoustically classify the target "ADU" and his estimated error will be less than PM, given the HVU was encountered. Similarly, if the received feature is greater than X_f, then the submarine can acoustically classify the target "HVU" and the estimated error will be less than PF, given the ADU was encountered.

If the received feature X is in the interval:

$$X_m \le X \le X_f \quad ,$$

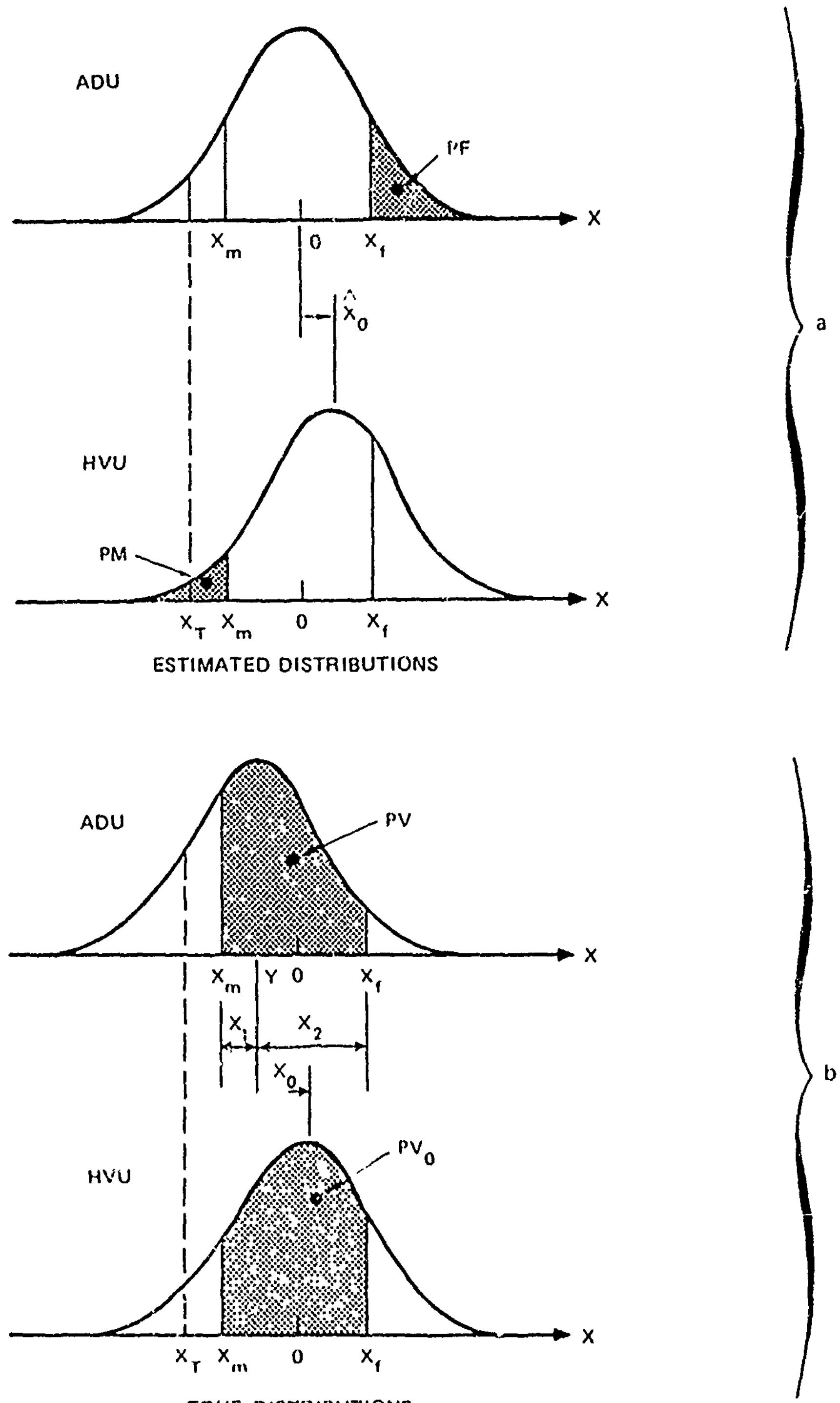


FIGURE 9 ESTIMATED AND TRUE FEATURE DISTRIBUTIONS SHOWING THRESHOLD VALUES

then the submarine must close visually for perfect classification in order to maintain the total expected error less than PE.

Under certain conditions, depending on the difference in the means of the estimated HVU and ADU distribution and $\hat{P}_0$, X_f will be less than X_m. In this case there is no need for the submarine to close for visual classification (at greater risk) in order to maintain his expected error less than PE. The value of X_m is changed to equal X_f if $\hat{P}_0$ is less than 0.5. On the other hand, if $\hat{P}_0$ is greater than 0.5, the value X_f is changed to equal X_m. In other words, both decision thresholds are identical and set to the false alarm value, if the submarine estimates it is more likely to encounter an ADU. Conversely, the thresholds are set to the "missed detection" value, if the submarine estimates it is more likely to encounter the HVU.

In order to compute the true classification probabilities, as shown in Figure 9b, the decision thresholds, which are relative to the mean of the estimated ADU feature distribution, must be translated into standard deviations from the mean of the true ADU distribution. Also, as required for the probability calculations, the difference of the means between the true ADU and HVU feature distribution is calculated:

$$X_0 = AG(PA) - AG(1-PH) \quad .$$

The signed difference between the means of the real and estimated ADU distributions is calculated as:

$$Y = AG(PA) - AG(\widehat{PA}) \quad .$$

Then the decision threshold in standard deviations from the mean of the real ADU distribution can be written:

$$X_1 = X_m + Y$$

$$X_2 = X_f + Y \quad .$$

The probab ity of visually classifying, given that the submarine encountered the HVU, is then calculated as the area under the true HVU feature distribution between the thresholds X_m and X_f. Therefore:

$$PV_0 = G(X_2 - X_0) - G(X_1 - X_0) \quad ,$$

and similarly the probability of visually classifying, given the submarine encountered an ADU is:

$$PV = G(X_2) - G(X_1) \quad .$$

The conditional probability of acoustically classifying the target "HVU," given the submarine encountered the HVU, is calculated as the ratio of area of the HVU distribution to the right of X_f divided by the total "acoustic area" of the HVU distribution:

$$PHA_0 = [1 - G(X_2 - X_0)]/[1 - PV_0] \quad .$$

Similarly, the probability of acoustically classifying the target "HVU," given the submarine encountered an ADU, is:

$$PHA = [1 - G(X_2)]/[1 - PV] \quad .$$

3. Penetration Submodel

The penetration parameters are calculated from the three remaining input parameters F_0, PKA, and PKV; and the previously developed probabilities of visually closing, PV_0 and PV, are also used in the calculation.

The total ASW force is assumed to have two capabilities--(1) when the submarine closes to the acoustic classification zone, and (2) when the submarine closes to the visual classification zone.

For the acoustic zone case, the probability of penetrating the total ASW force can be written:

$$P^M \equiv 1 - PKA \quad ,$$

where M is the total ASW level and P is the probability of penetrating a unit level of ASW.

Since the total ASW force is usually divided among the HVU and ADUs, it is desired to find the probability of penetrating the HVU's ASW force in the acoustic zone. Then:

$$PPA_o = P^m = (1-PKA)^{F_o} ,$$

where m is the ASW level with the HVU. The ratio m/M (i.e., ratio of HVU defenses over the total defenses) is defined as the input parameter F_o. In a similar manner, the probability of the submarine's penetrating the HVU defenses in the visual zone is:

$$PPV_o = (1-PKV)^{F_o} .$$

The equations for the ADU penetration probabilities are similar. It is assumed in the model that the portion of the total ASW force that is not with the HVU is equally divided among the ADUs; therefore F, the proportion of the ASW force with each ADU, is defined as:

$$F \equiv (1-F_o)/N .$$

Then the penetration probabilities for an ADU can be written directly:

$$PPA = (1-PKA)^F$$

and

$$PPV = (1-PKV)^F .$$

Due to submarine attrition prior to the classification decision, two additional probabilities are required for the model's calculations. Upon contact, the submarine decides if he is going to close the target for a visual or acoustic classification. Since each tactic yields a different probability of penetrating the target, the probability of the submarine's completing each tactic is required. The probability of acoustically classifying the HVU, given successful penetration can be written:

$$W_o = (1-PV_o)PPA_o/[(1-PV_o)PPA_o + PV_o PPV_o] ,$$

and similarly for the ADU:

$$W = (1-PV)PPA/[(1-PV)PPA + PV\ PPV] .$$

B. ORI Parameters

In order to calculate the probability of HVU survival, the ORI parameters must be derived from the previously developed intermediate parameters.

1. Search Rate

The first two parameters are functions of the input parameters r_o, r, A, and N; and the relative speed parameters s_o and s:

$$T_o = (A - \pi r_o^2 - N\pi r^2)/(2r_o s_o)$$

$$\rho = (Nrs)/(r_o s_o) \quad .$$

The first equation, T_o, is the reciprocal of the product of: (1) twice detection range on the HVU, (2) the average relative speed between the HVU and submarine, and (3) the submarine density.

The submarine density is the reciprocal of the total area minus the area covered by the HVU and ADUs. This model assumes that: (1) the submarine is not in contact with the HVU or ADUs at the beginning of the engagement, and (2) the HVU and ADU circles do not overlap. The second assumption is necessary because the submarine always returns to search before detecting a new target. Also, the parameters r, r_o, A, N are mutually constrained so that all the detection circles fit into the area without overlapping.

2. Penetration

The penetration probabilities σ_o and σ are composed of two parts--ASW actions in the visual zone and ASW action in the acoustic zone.

By taking the product of the previously developed conditional penetration probabilities and their respective unconditional probabilities of visually closing (or not), the total probability of submarine penetration for each type of unit can be written:

$$\sigma_0 = PV_0 PPV_0 + (1-PV_0)PPA_0$$

$$\sigma = PV\ PPV + (1-PV)PPA \quad .$$

3. Classification

The correct classification probabilities also consist of two components--ability of the submarine to classify acoustically and the ability to classify visually (assumed perfect).

The previously developed classification probabilities, PHA_0 and PHA, are conditioned on the submarine's penetrating the ASW defenses. Since the classification tactics of closing for acoustic or visual classification yield different risks to the submarine (penetration probabilities), the conditional classification probability must be unconditioned by the probability that the submarine completes his intended tactic. Thus the correct classification probability can be written:

$$\delta_0 = W_0 PHA_0 + (1-W_0) \cdot 1$$

$$\delta = 1 - W \cdot PHA \quad .$$

4. Delay Time

The submarine time delay to reach a classification decision upon encountering a target is based on the tactic of the submarine to visually or acoustically classify. It is assumed the submarine closes to half the detection range for acoustic classification, and all the way for visual classification. The time delay is calculated as half the detection range divided by the average relative speed for the acoustic case, and twice this value for the visual case. The average time delay is composed of the weighted average of both the acoustic and visual cases:

$$h_0 = (1-W_0/2)r_0/s_0$$

$$h = (1-W/2)r/s \quad .$$

C. Output Equations

The equations discussed in the previous two sections are summarized in Tables 5 and 6; the output equations are listed in Table 7. The time constant and asymptotic value equations and the probability of HVU survival expression of Table 7 are simplified forms of the ORI equations and their derivation as shown in Appendix A.

Four secondary measures of effectiveness for the model were derived from the asymptotic value equations. These conditional probabilities are used in the analysis to show the relative importance ASW or ACM factors had on the HVU survival. P_1 and P_2, the conditional probabilities that the submarine was killed by the ASW force, measure the relative effectiveness of ASW. P_3 and P_4 measure the effectiveness of ACM; P_3 shows the relative importance of the ADU as an alternate target (i.e., causing the submarine to misclassify and launch), and P_4 is composite measure of the importance of the time delay of the submarine from encountering the ADUs and the search effort required to find and correctly classify the HVU.

A computer program listing of the Acaso model is provided in Appendix B.

Table 5

SUMMARY OF INTERMEDIATE EQUATIONS

1. Speed parameters

$$s_o = \max(v_o,u) + .273 \min^2(v_o,u)/\max(v_o,u)$$

$$s = \max(v,u) + .273 \min^2(v,u)/\max(v,u)$$

2. Classification parameters

G(x) The Normal distribution function

AG(p) The inverse Normal function

$$PM = \begin{cases} .5\ PE/\hat{P}_o & .5\ PE \le \hat{P}_o \le 1 \\ 1 & \text{otherwise} \end{cases}$$

$$PF = \begin{cases} .5\ PE/(1-\hat{P}_o) & 0 \le \hat{P}_o \le 1 - .5\ PE \\ 1 & \text{otherwise} \end{cases}$$

$$\hat{X}_o = AG(\widehat{PA}) - AG(1-\widehat{PH})$$

Case I:

$$X_m = \hat{X}_o + AG(PM)$$

$$X_f = AG(1-PF)$$

Case II: $X_m \ge X_f$

$$X_m = X_f , \quad \text{if } 0 \le \hat{P}_o \le .5$$

$$X_f = X_m , \quad \text{otherwise}$$

2. Classification parameters (Concluded)

$$X_o = AG(PA) - AG(1-PH)$$

$$Y = AG(PA) - AG(\widehat{PA})$$

$$X_1 = X_m + Y$$

$$X_2 = X_f + Y$$

$$PV_c = G(X_2-X_o) - G(X_1-X_o)$$

$$PV = G(X_2) - G(X_1)$$

$$PHA_o = [1-G(X_2-X_o)]\,[1-PV_o]$$

$$PHA = [1-G(X_2)]/[1-PV]$$

3. Penetration parameters

$$F = (1-F_o)/N$$

$$PPV_o = (1-PKV)^{F_o}$$

$$PPA_o = (1-PKA)^{F_o}$$

$$PPV = (1-PKV)^F$$

$$PPA = (1-PKA)^F$$

$$W_o = (1-PV_o)PPA_o/[(1-PV_o)PPA_o + PV_o PPV_o]$$

$$W = (1-PV)PPA/[(1-PV)PPA + PV\ PPV]$$

Table 6

EQUATIONS TO CONVERT TO ORI's PARAMETERS

1. Search Rate

$$T_o = (A - \pi r_o^2 - N\pi r^2)/(2r_o s_o)$$

$$\rho = (Nrs)/(r_o s_o)$$

2. Penetration

$$\sigma_o = PV_o PPV_o + (1-PV_o)PPA_o$$

$$\sigma = PV\ PPV + (1-PV)PPA$$

3. Classification

$$\delta_o = W_o PHA_o + (1-W_o)$$

$$\delta = 1 - W\ PHA$$

4. Time Delay

$$h_o = (1-W_o/2)r_o/s_o$$

$$h = (1-W/2)r/s$$

Table 7

OUTPUT EQUATIONS

Time constant and asymptotic values

$$B = \sigma_o\delta_o + (1-\sigma_o) + \rho(1-\sigma\delta)$$

$$T = [T_o + \rho\sigma\rho h + \sigma_o(1-\delta_o)h_o]/B$$

$$A_o = \sigma_o\delta_o/B$$

$$A = \rho\sigma(1-\delta)/B$$

$$K_o = (1-\sigma_o)/B$$

$$K = \rho(1-\sigma)/B$$

Probability of HVU survival

$$P_s = 1 - A_o(1-e^{-t/T})$$

Given HVU Survival ...

- Probability that the submarine was killed by the HVU's ASW forces:

$$P_1 = K_o(1-e^{-t/T})/P_s$$

- Probability that the submarine was killed by the ADU's ASW forces:

$$P_2 = K(1-e^{-t/T})/P_s$$

- Probability that the submarine attacked an ADU:

$$P_3 = A(1-e^{-t/T})/P_s$$

- Probability that the submarine was still searching:

$$P_4 = e^{-t/T}/P_s$$

III ACASO MODEL ANALYSIS

The Acaso model was exercised to study the basic tradeoffs between Acoustic Countermeasures (ACM) and ASW in the objective area. The input parameters for the model can be classified into four groups (their formal definitions are listed in Table 2 of the model's description).

Scenario:

The scenario parameters basically determine the search rate and time constraints of the submarine against the high value unit (HVU) and acoustic deception units (ADU). The parameters r_0, r, N, and A are all mutually constrained so that the sum of the detection circle areas is less than A.

Submarine:

The submarine parameters describe the submarine's estimate of the tactical situation. $\hat{P}_0$ is related to the submarine's estimate of his search rate against the HVU. $\widehat{PH}$ and $\widehat{PA}$ describe the submarine's estimate of the acoustic signatures generated by the HVU and ADUs. The maximum allowed estimated misclassification error, PE, carries an implicit estimate of the ASW threat. By reducing PE, the submarine would have to close more targets for visual classification at a greater risk (the ASW force is assumed to be more effective in the visual zone).

ACM:

The ACM parameters describe the acoustic countermeasures used by the task force. N is the number of deployed ADUs, and PA describes their fidelity. The parameter PH describes the ACM capabilities of the HVU. A lower value of PH represents a case where turn-count masking, reduced speed, or related tactics were being employed.

ASW:

The ASW parameters describe the distribution and effectiveness of the ASW force. F_0 is the proportion of the force in company with the HVU, and the balance of the ASW force is evenly divided among the ADUs. PKA and PKV describe the ASW effectiveness in the acoustic and visual zones.

A. Base Case

In order to have a standard of comparison for the study of the interaction between the input parameters and the probability of HVU survival, a nominal set of base case parameters was developed. The base case parameter values and the resulting measures of effectiveness are shown in Table 8. The computer program used in the following analysis is reproduced in Appendix B.

In the base case there is one HVU and three ADUs moving randomly and independently in an operating area (200-nmi radius circle). Accompanying each unit is a designated level of ASW protection. There is one submarine, also confined to the operating area, searching for the HVU.

One of the basic questions of ACM effectiveness is--what effect does the submarine's prior knowledge of the use of ADUs have on HVU survivability? $\hat{P}_0$, the submarine's estimate of the probability that the next contact will be the HVU, is the parameter related to this question. If the submarine was perfect in his estimate, $\hat{P}_0$ would equal $\frac{1}{1+\rho}$, where ρ is the ratio of ADU to HVU search rates. In the analysis, values of $\hat{P}_0$ greater than 0.5 indicate the submarine estimates that he is more likely to encounter the HVU next; and, likewise, values of $\hat{P}_0$ less than 0.5 indicate the submarine estimates that he is more likely to encounter an ADU next.

TABLE 8

BASE CASE RESULTS

INPUT PARAMETERS

SCENARIO		
r_0 = 30 nmi	v_0 = 10 knots	A = π 200^2 nmi^2
r = 60 nmi	v = 5 knots	t = 7 days
	u = 10 knots	
ASW	**ACM**	**SUB**
F_0 = 0.50	N = 3.00	$\hat{P}_0$ = *
PKA = 0.10	PH = 0.95	$\hat{PH}$ = 0.95
PKV = 0.40	PA = 0.20	$\hat{PA}$ = 0.20
		PE = 0.10

RESULTS

* BASE CASE	MEASURES OF EFFECTIVENESS				
	P_s	P_1	P_2	P_3	P_4
I $\hat{P}_0$ = 0.3	0.60	0.14	0.24	0.34	0.28
II $\hat{P}_0$ = 0.9	0.76	0.03	0.06	0.90	0.01

Figure 10 shows the effects on the probability of HVU survival (P_s) by varying $\hat{P}_0$ from zero to one. Due to the significance of $\hat{P}_0$, two base cases were established. For Case I, $\hat{P}_0 = 0.3$. This value was chosen because it yields the minimum P_s for the base case parameters. Note that the submarine does better by overestimating $\hat{P}_0$ from the true value $\frac{1}{1+\rho}$. For Case II, $\hat{P}_0$ was set at 0.9. This second base case was considered to determine if the lack of the submarine's prior knowledge of the use of ADUs had any significant effects in the other parameter studies.

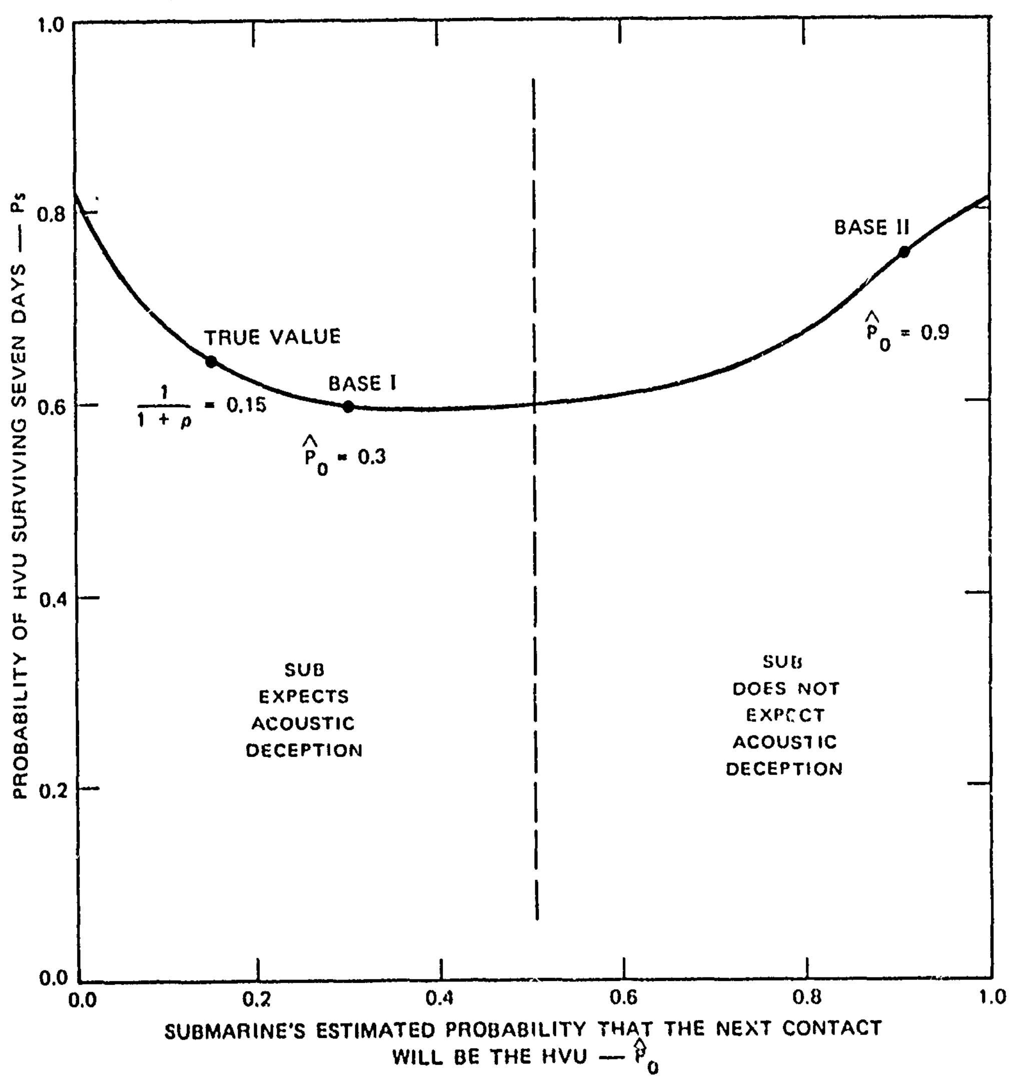


FIGURE 10 CHOICE OF TWO BASE CASES

Figure 11 shows the relative importance of ACM and ASW as a function of $\hat{P}_0$. The secondary measures of effectiveness for the model (P_1, P_2, P_3, P_4) are plotted cumulatively. P_1 plus P_2 show the importance of ASW, and P_3 plus P_4 show the importance of ACM in helping the HVU survive. It is interesting to note that, as $\hat{P}_0$ is increased, the probability that the submarine mistakenly attacks an ADU is predominant. This is due to the fact that, for values of $\hat{P}_0$ close to one, the submarine tends to attack the first target he encounters.

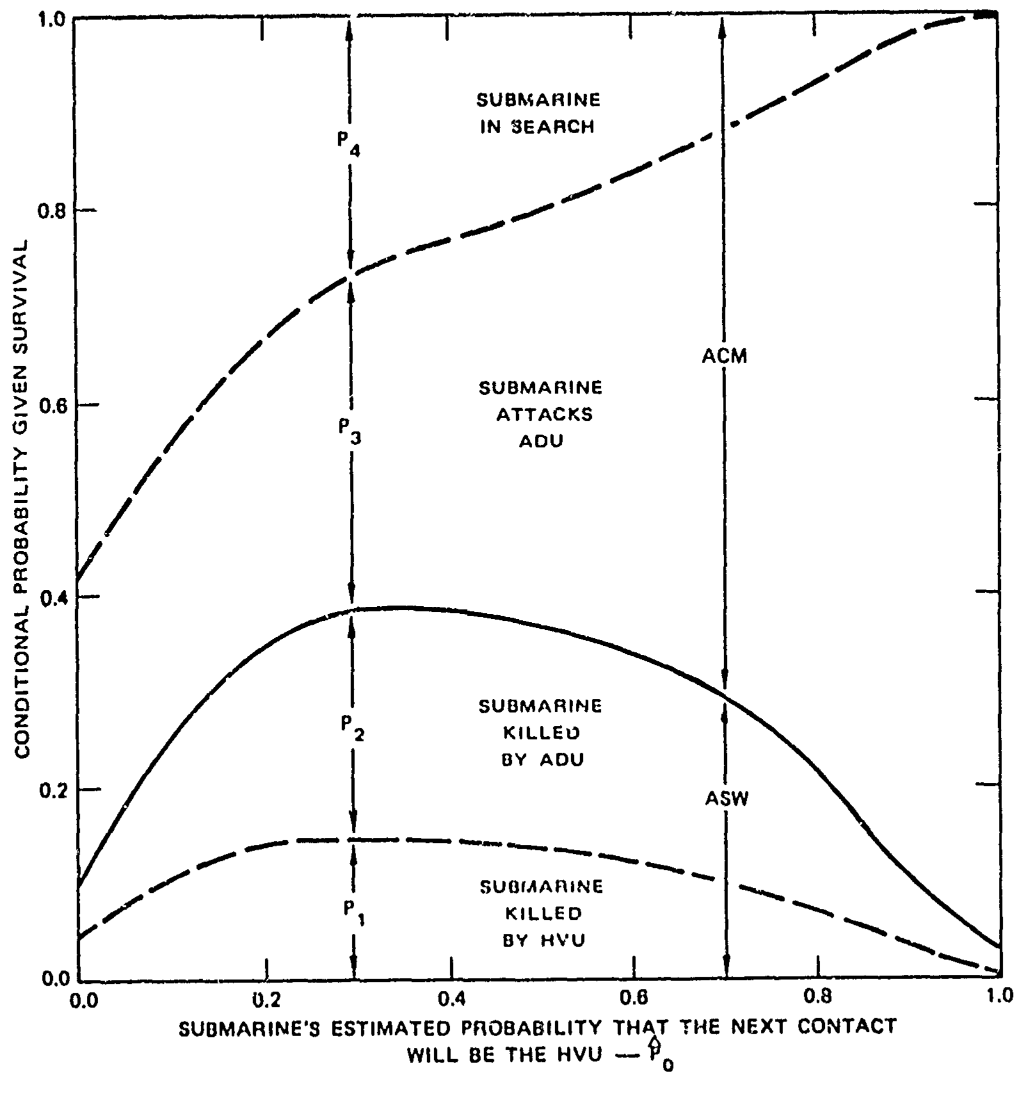


FIGURE 11 FOUR WAYS OF SURVIVING AS A FUNCTION OF THE SUBMARINE'S EXPECTATIONS ABOUT CONTACTS

B. Parameter Analysis

To give an overview of the sensitivity of each parameter, a summary of the parameter variations and the resulting change of P_s is shown in Table 9 and Figure 12. This summary was constructed by varying each parameter, while holding all other parameters constant at their base value. Figure 12 presents a graphical comparison between the base cases. In general, P_s is more sensitive to the ACM parameters in Case II (submarine does not expect ADUs), and P_s is more sensitive to the ASW parameters in Case I (submarine expects acoustic deception).

TABLE 9

PARAMETER VARIATION AROUND BASE CASES

PARAMETER	BASE VALUE	PARAMETER RANGE
SCENARIO:		
r_0	30 nmi	15, 90 nmi
r	60 nmi	15, 90 nmi
v_0	10 knots	5, 25 knots
v	5 knots	0, 20 knots
u	10 knots	5, 20 knots
t	7 days	3, 15 days
$\mathcal{A}$	$\pi\ 200^2$ nmi^2	$\pi\ 150^2, 400^2$ nmi^2
SUBMARINE:		
$\hat{P}_0$	0.3, 0.9	0.0, 1.0
$\hat{PH}$	0.95	0.8, 1.0
$\hat{PA}$	0.20	0.0, 0.5
PE	0.10	0.0, 0.5
ACM:		
N	3	0, 6
PH	0.95	0.8, 1.0
PA	0.20	0.0, 0.5
ASW:		
F_0	0.50	0.0, 1.0
PKA	0.10	0.0, 0.4
PKV	0.40	0.1, 1.0

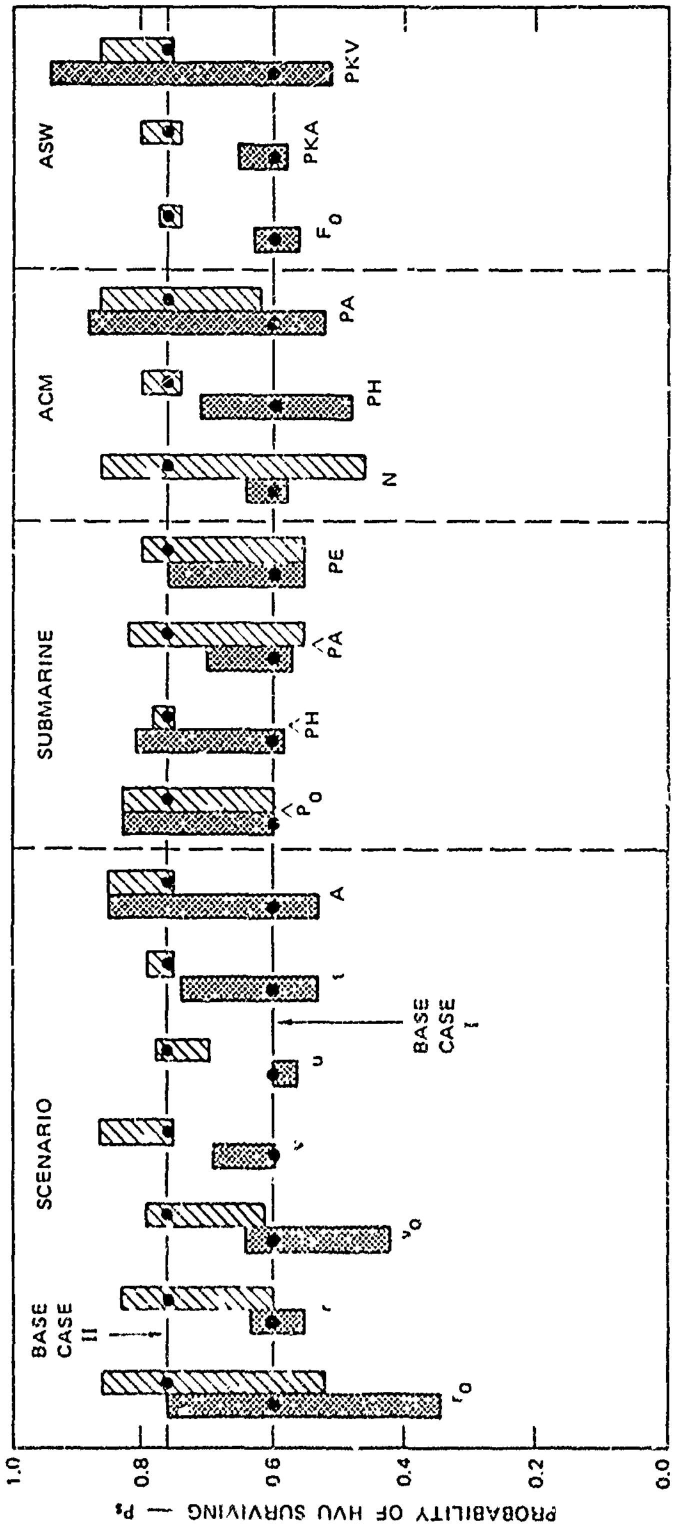


FIGURE 12 PARAMETER SENSITIVITY ANALYSIS

1. Scenario Parameters

The scenario parameters determine the search constraints for the submarine. As r_0 and u_0 increase, the HVU becomes more detectable, and P_s decreases. As shown in Figure 13, the probability of HVU survival is higher in Case II than in Case I. This effect is due to the submarine's attacking more ADUs in Case II, which reduces the effects of increased HVU detectability.

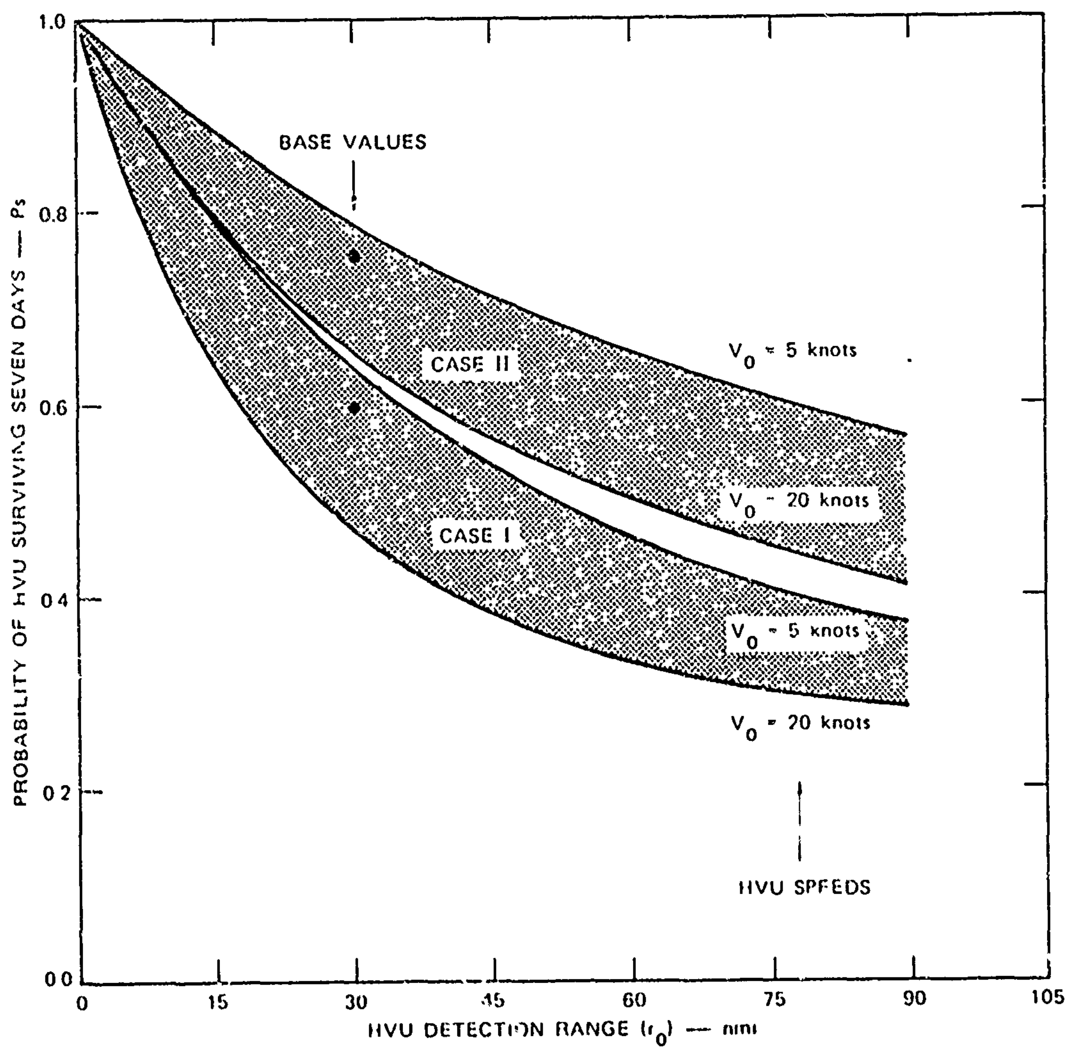


FIGURE 13 EFFECT OF HVU DETECTABILITY AND MOTION ON SURVIVAL (U)

Conversely, increasing ADU detection range and speed (r and v) makes the ADUs more detectable, and this improves the chances of HVU survival. As illustrated in Figure 14, Case II is superior because the submarine tends to attack ADUs when encountered (increasing the detectability of the ADUs increases the chances of the submarine's making this mistake). For Case I the increase in P_s is predominantly due to ASW, since the submarine has better classification capability. Case II crosses below Case I at R ≤ 15 nmi. This effect is from the lower effectiveness of ASW in Case II, since the submarine tends to make his classification decisions in the acoustic region.

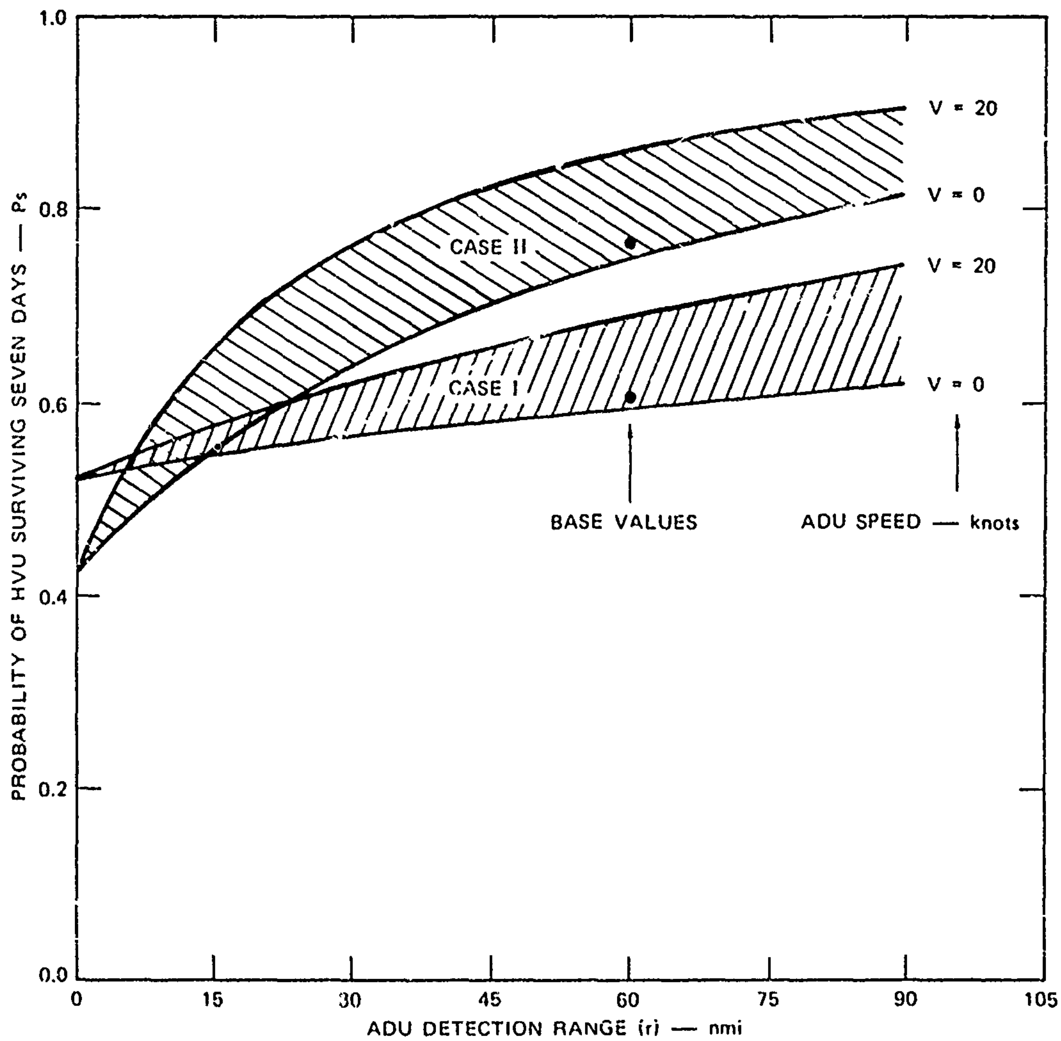


FIGURE 14 EFFECT OF ADU DETECTABILITY AND MOTION ON SURVIVAL

Figure 15 shows the effects of ADU detection range (r) and submarine speed (u) for Cases I and II. An increase in submarine speed increases the submarine search rate against both the HVU and ADUs. For Case I, the increase in submarine speed decreases P_s, since the submarine can detect the HVU sooner. But for Case II, the opposite is true. By increasing his speed the submarine encounters more ADU; the increase in ADU attacks overrides the increased HVU encounter rate. Both sets of curves show a cross-over point. This point is where the tradeoff between ADU and HVU encounter rate occurs.

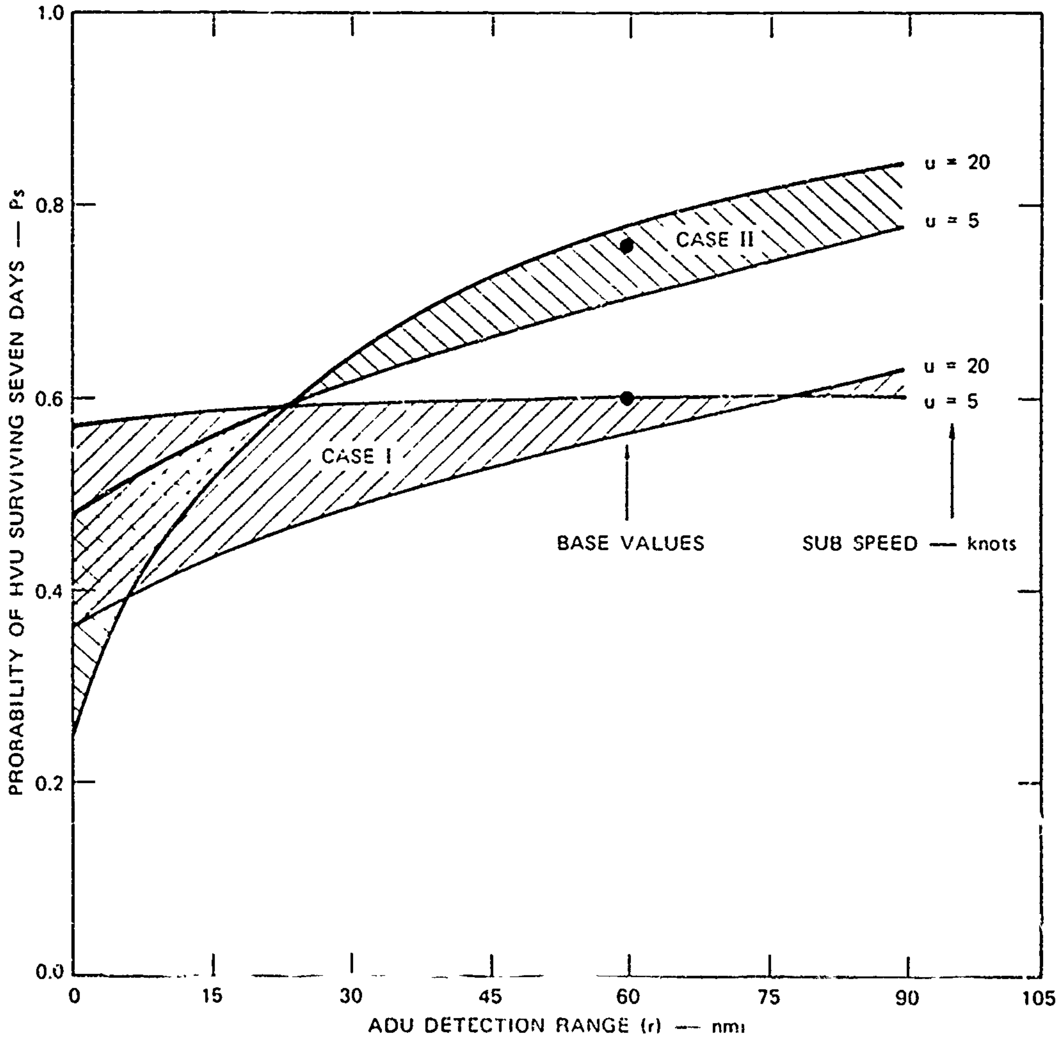


FIGURE 15 EFFECTS OF ADU DETECTABILITY AND SUBMARINE MOTION ON SURVIVAL

Figure 16 illustrates the effects of survival time and operating area size for Case I. In general, the larger the operating area, and the shorter the time, provide the highest probabilities of HVU survival.

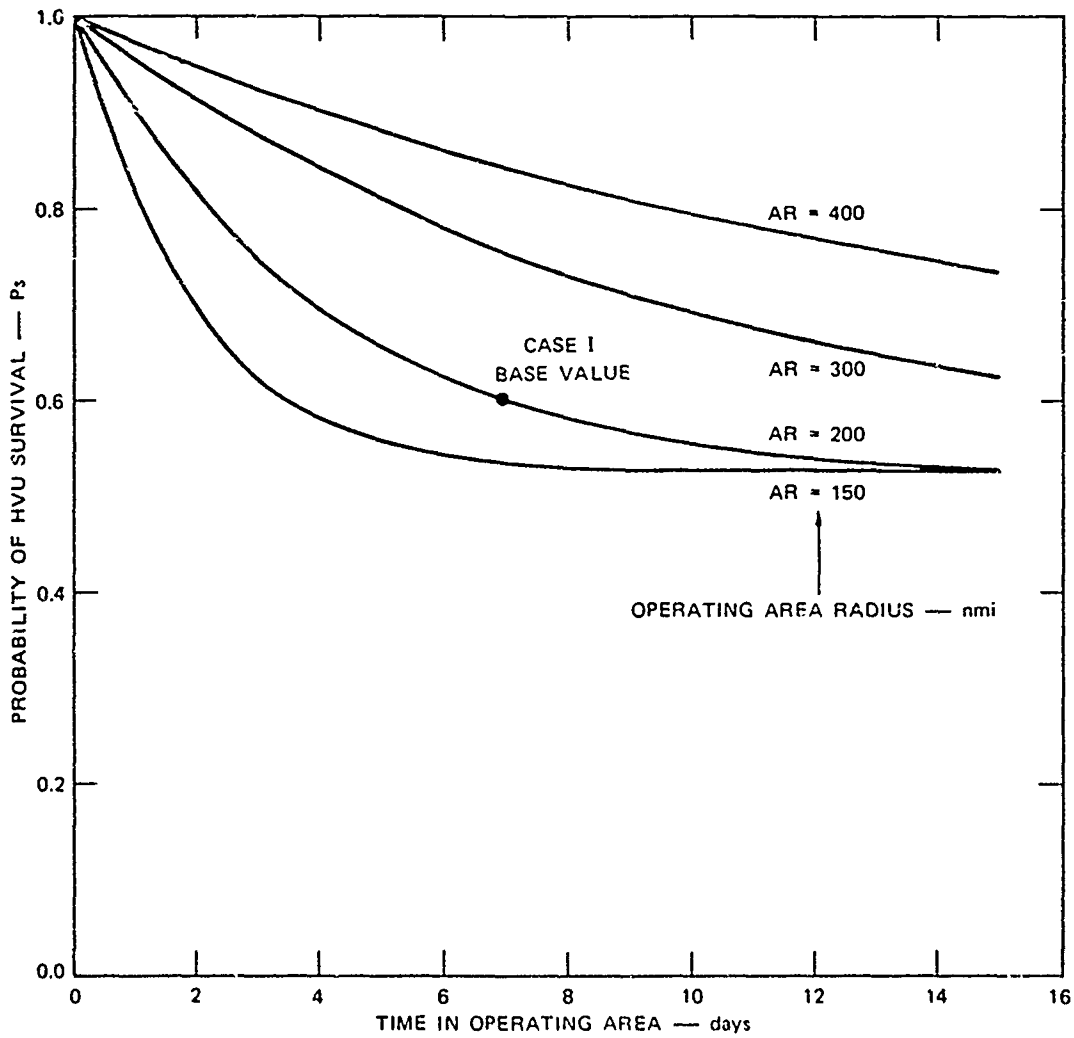


FIGURE 16 EFFECT OF OPERATION DURATION AND SIZE OF AREA ON SURVIVAL (CASE I)

Figure 17 shows the relative importance of ACM and ASW growth as a function of time. For short time periods, HV survival is due primarily to the submarine's inability to find the HVU. But as time increases ACM (submarine's attacking ADUs) and ASW (submarine's being killed) become the primary reasons for HVU survival.

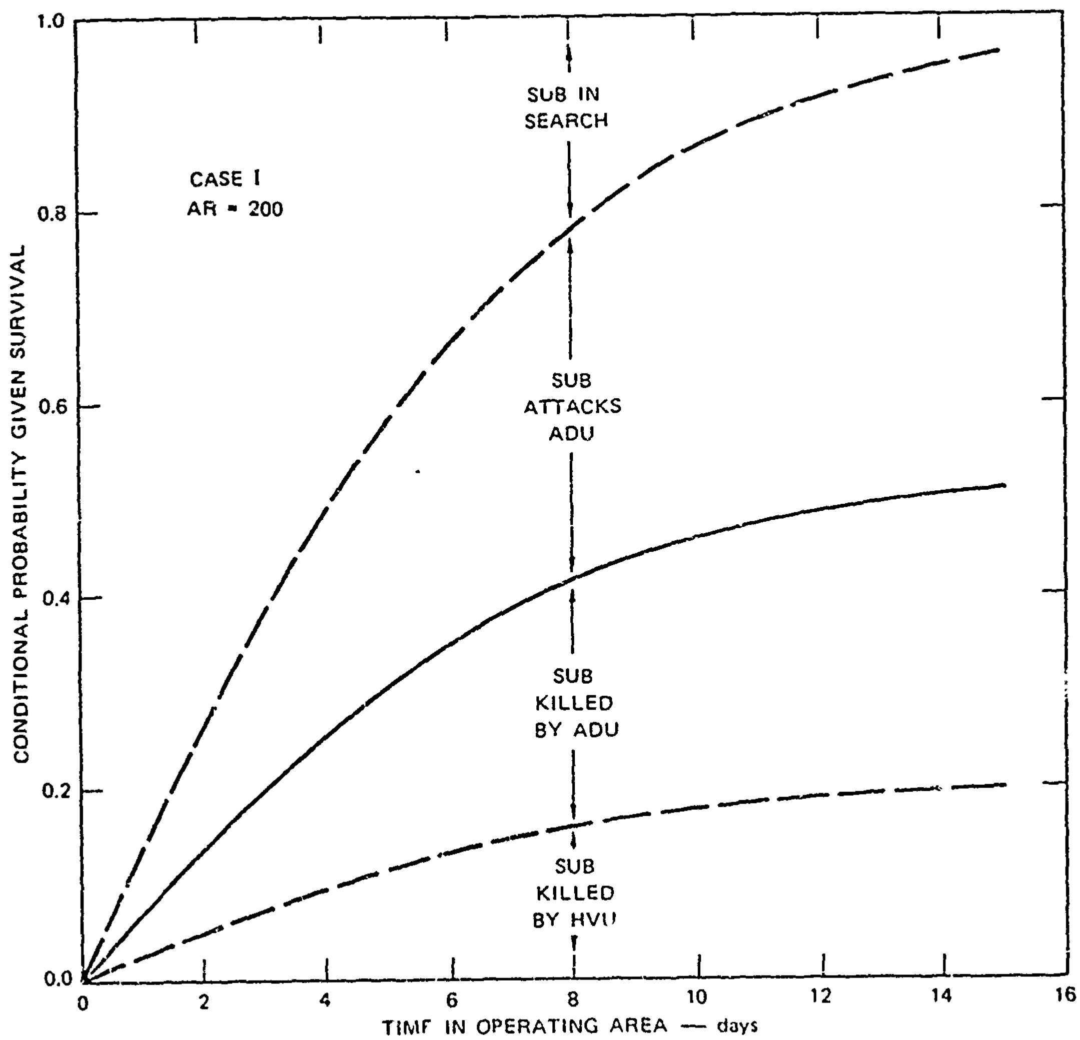


FIGURE 17 TIME DEVELOPMENT OF HVU SURVIVAL (CASE I)

2. Submarine Parameters

The overall purpose of the study is to investigate the effects of ACM and ASW; therefore, the submarine parameters were analyzed so that reasonable submarine parameters would be used, but not to study submarine tactics.

The submarine parameters are primarily input to the classification submodel (see model description). The submarine's estimated probability that the next contact will be the HVU, $\hat{P}_O$, strongly affects the submarine's classification decision process. Figure 18 shows the effects of this parameter on the submarine's decision to close for visual classification, given that the HVU was encountered. In general, the more certain the submarine is that he will encounter one type of target ($\hat{P}_O$ close to 0 or 1), the less it is necessary to close the target for visual classification. Also, the parameter PE determines the visual classification probability. As PE is decreased, the more the submarine will have to close for visual (perfect) classification to maintain his estimated classification errors less than PE.

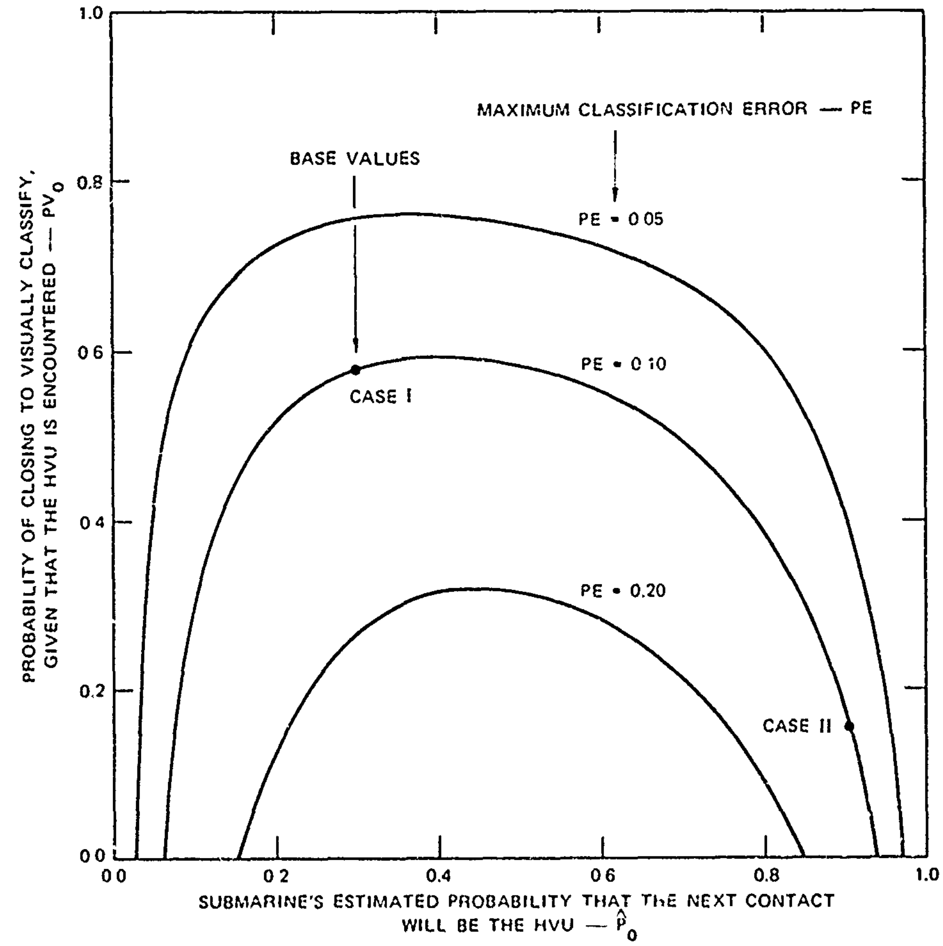


FIGURE 18 EFFECT OF SUBMARINE'S EXPECTATIONS ABOUT CONTACTS AND CLASSIFICATION ERROR THRESHOLDS ON THE PROBABILITY OF VISUAL CLASSIFICATION

The acoustic classification decision is also affected by $\hat{P}_0$. Figure 19 illustrates an acoustic classification "operating characteristic," which is constructed by parameterizing $\hat{P}_0$. In Case II, the submarine is willing to accept a higher probability of misclassifying an ADU (PHA) in order to improve the probability of correctly classifying the HVU (PHA_0). This is because he does not expect to encounter ADU. The combination of the lower visual classification probability and higher ADU acoustic misclassification probability accounts for the increased ACM effectiveness for Case II.

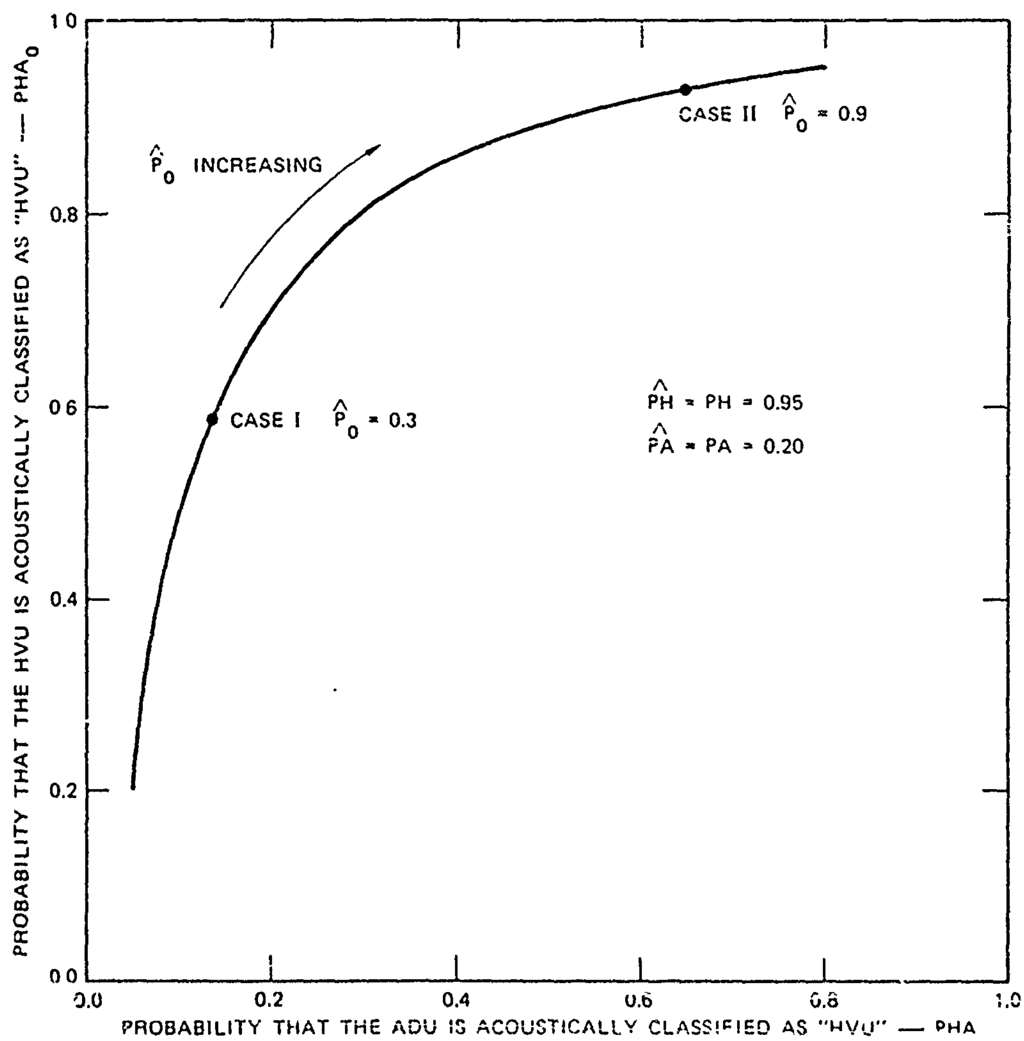


FIGURE 19 EFFECT OF SUBMARINE'S EXPECTATIONS ABOUT CONTACTS ON HVU CORRECT CLASSIFICATION AND ADU MISCLASSIFICATION PROBABILITIES

The effects of the parameter PE are shown in Figures 20 and 21 for Cases I and II, respectively. As PE tends to 0, the submarine must close visually for perfect classification. It is assumed in the model that the ASW force is more effective in the visual zone (PKV > PKA), so that by reducing PE the submarine faces a greater ASW threat. As shown in Figures 12 and 13, as PKV is increased the probability of HVU survival develops local minimums for values of PE close to 10 percent. Therefore, PE = 0.10 is a reasonable value for the submarine to choose.

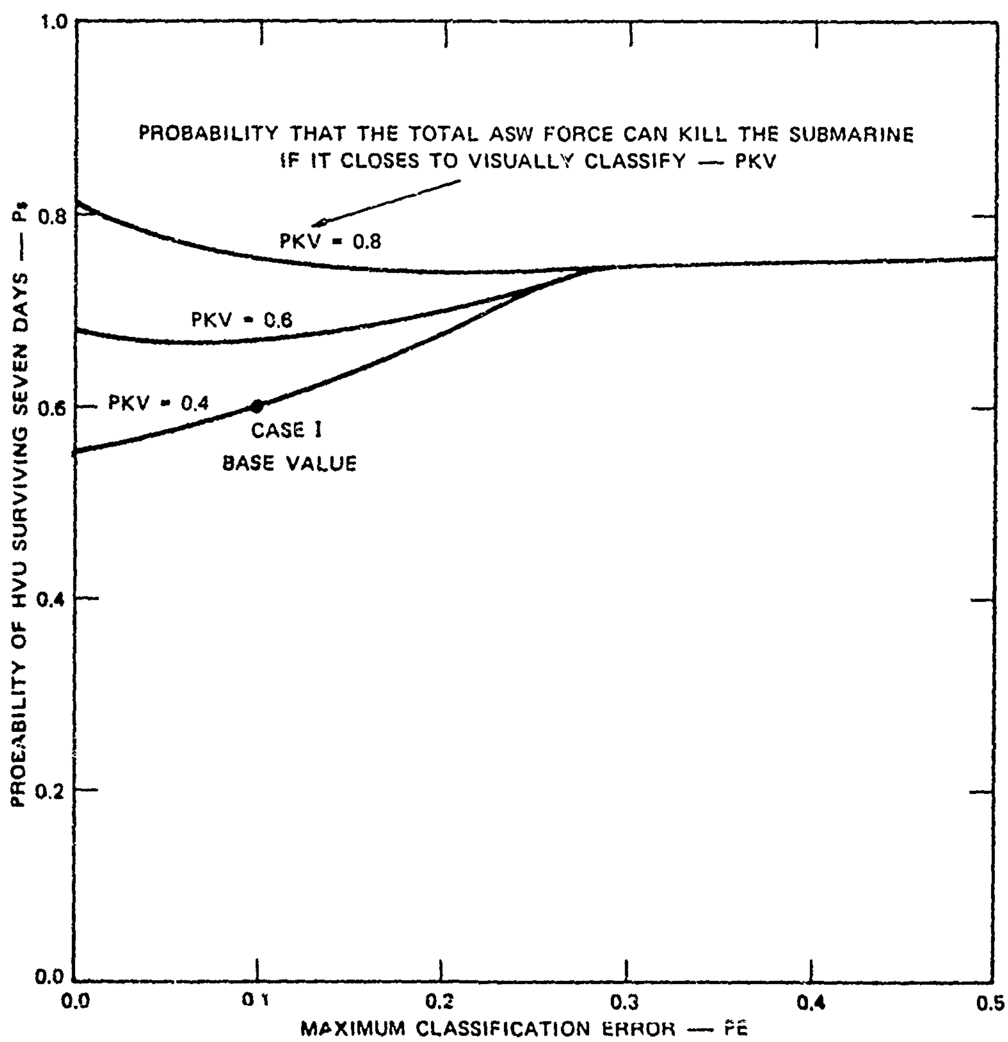


FIGURE 20 EFFECT OF CLASSIFICATION ERROR LIMITS AND CLOSE-IN ASW CAPABILITY ON SURVIVAL (CASE I)

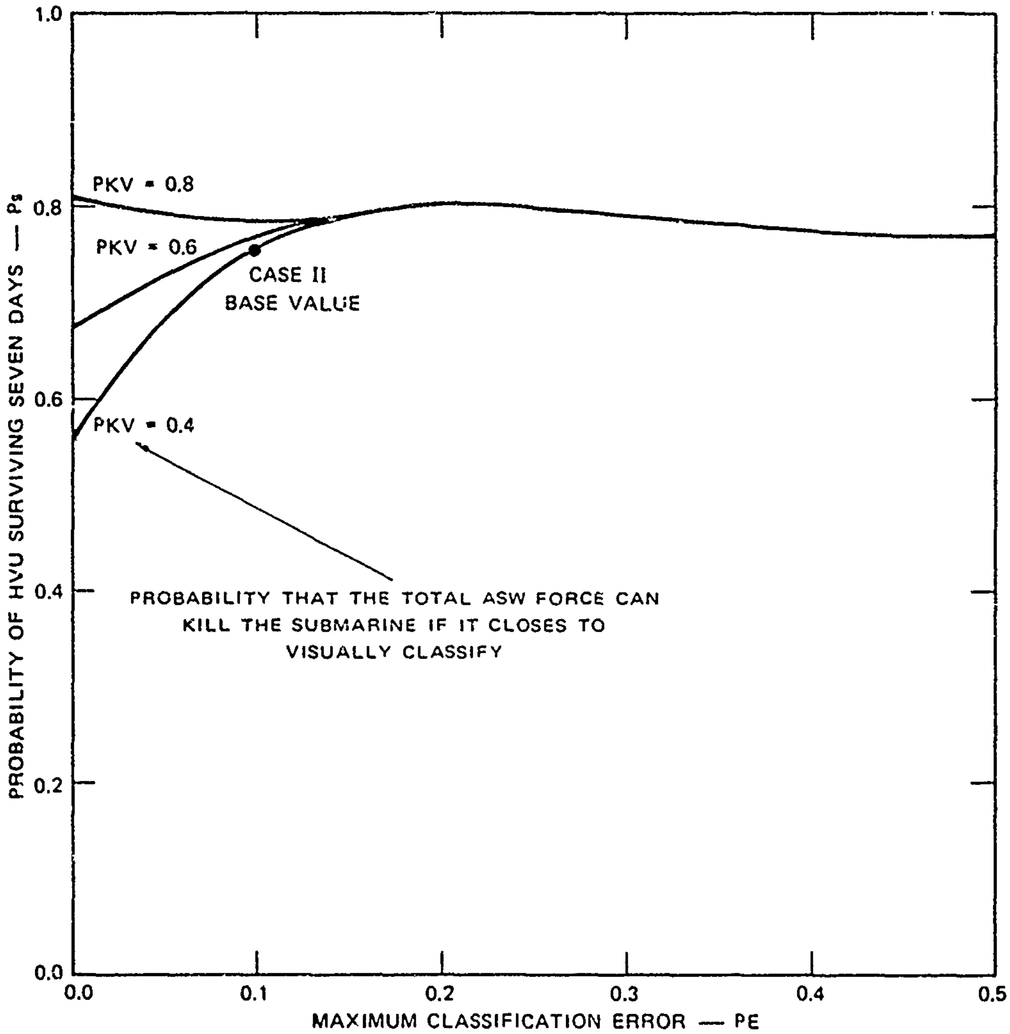


FIGURE 21 EFFECT OF CLASSIFICATION ERROR LIMITS AND CLOSE-IN ASW EFFECTIVENESS ON SURVIVAL (CASE II)

The effects of the last two submarine parameters, $\widehat{PH}$ and $\widehat{PA}$, are illustrated in Figures 22 and 23. Since submarine tactics were not the purpose of this study, the parameters $\widehat{PH}$ and $\widehat{PA}$ were set equal to the real values (PH and PA) for most of the parameter studies. As shown in Figure 23, $\widehat{PH}$ has a small effect on P_s in the range of interest. (The sharp rise of P_s for $\widehat{PH} > 0.99$ is due to the classification sub-model design (feature distribution assumption) and not a real effect.)

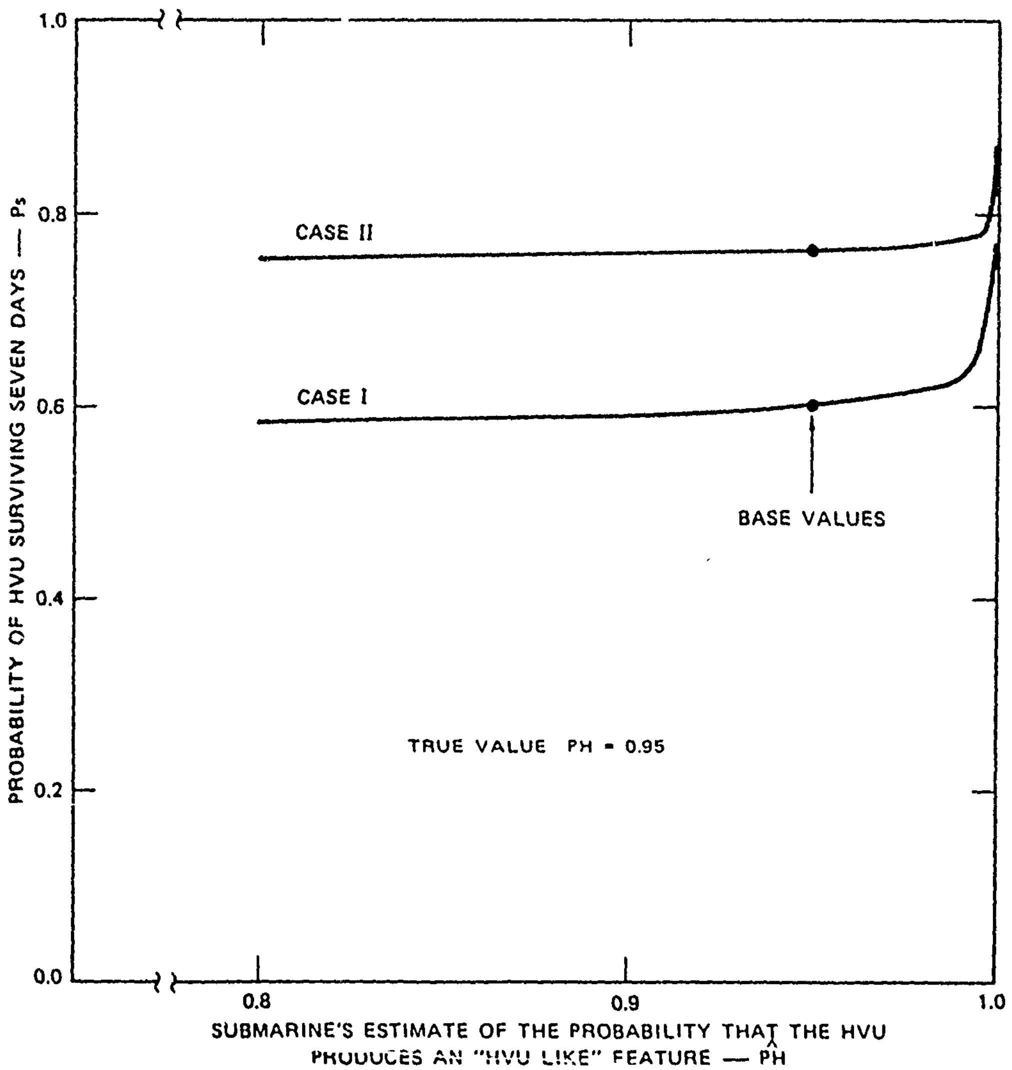


FIGURE 22 EFFECT OF THE SUBMARINE'S EXPECTATIONS ABOUT THE HVU'S FIDELITY ON SURVIVAL

The parameter $\hat{PA}$, as illustrated in Figure 23, has a stronger effect on P_s. This effect is greater for Case II since, as discussed previously, the submarine tends to acoustically classify when he does not expect ADUs to be present.

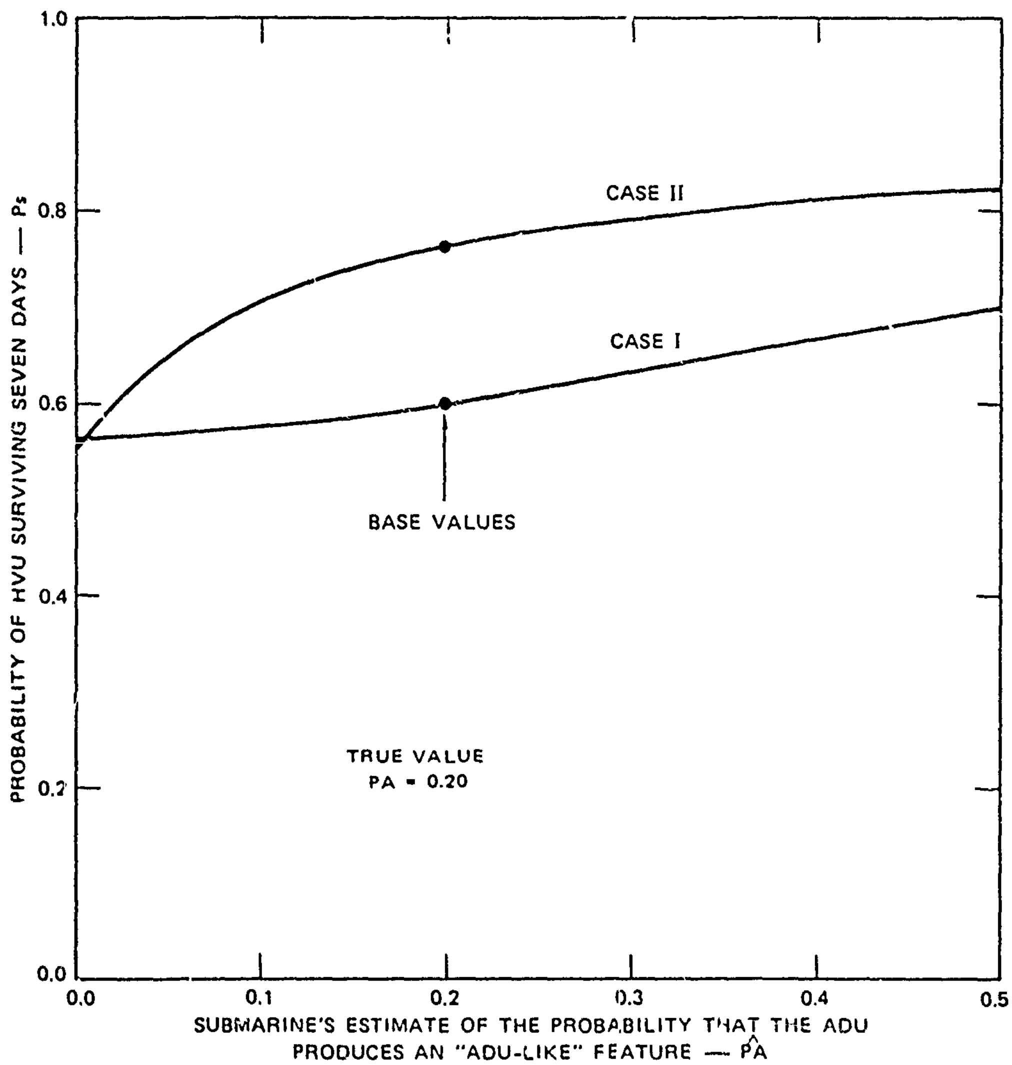


FIGURE 23 EFFECT OF THE SUBMARINE'S EXPECTATIONS ABOUT THE ADU'S FIDELITY ON SURVIVAL

3. ACM Parameters

The acoustic countermeasures parameters determine the extent the task force can confuse or deny information to the submarine's classification process. The deployment of ADUs has potentially these effects on HVU survivability:

- Alternate target (submarine attacks ADUs)
- Delay submarine's search for the HVU
- Submarine attrition (by ASW forces in company with an ADU).

The first effect (alternate target) was the most predominant effect of the ADUs for the base case parameters used in the model. The second effect (delay time) was negligible due to the small durations, relative to HVU search time, that were calculated in the model. (A discussion of the delay time assumptions and calculations is presented in the model descriptions.) The third effect (submarine attrition) will be discussed with the ASW parameters.

The probability of HVU survival as a function of the number of deployed ADUs (N) is shown in Figure 24. For Case I the deployment of additional ADUs had a small effect on P_s. Under the base case parameter assumptions, P_s decreased with the deployment of the first ADU. This was due to the dilution of the ASW force. When no ADUs are deployed, the total ASW force is with the HVU, but with the deployment of the first ADU, 50 percent (F_0) of the ASW force is with the HVU. (In general, as the number of ADUs is increased the ASW effectiveness of each ADU is reduced.) The lack of sensitivity of survival to the number of ADUs is because the submarine visually classifies often enough to negate their influence.

For Case II the deployment of ADUs significantly increased P_s. Under the Case II assumptions, the submarine tends to misclassify and attack the ADUs because he believes there are no acoustic deception devices. Due to saturation of the operating area with ADUs, the marginal effect of each additional ADU is seen to decrease.

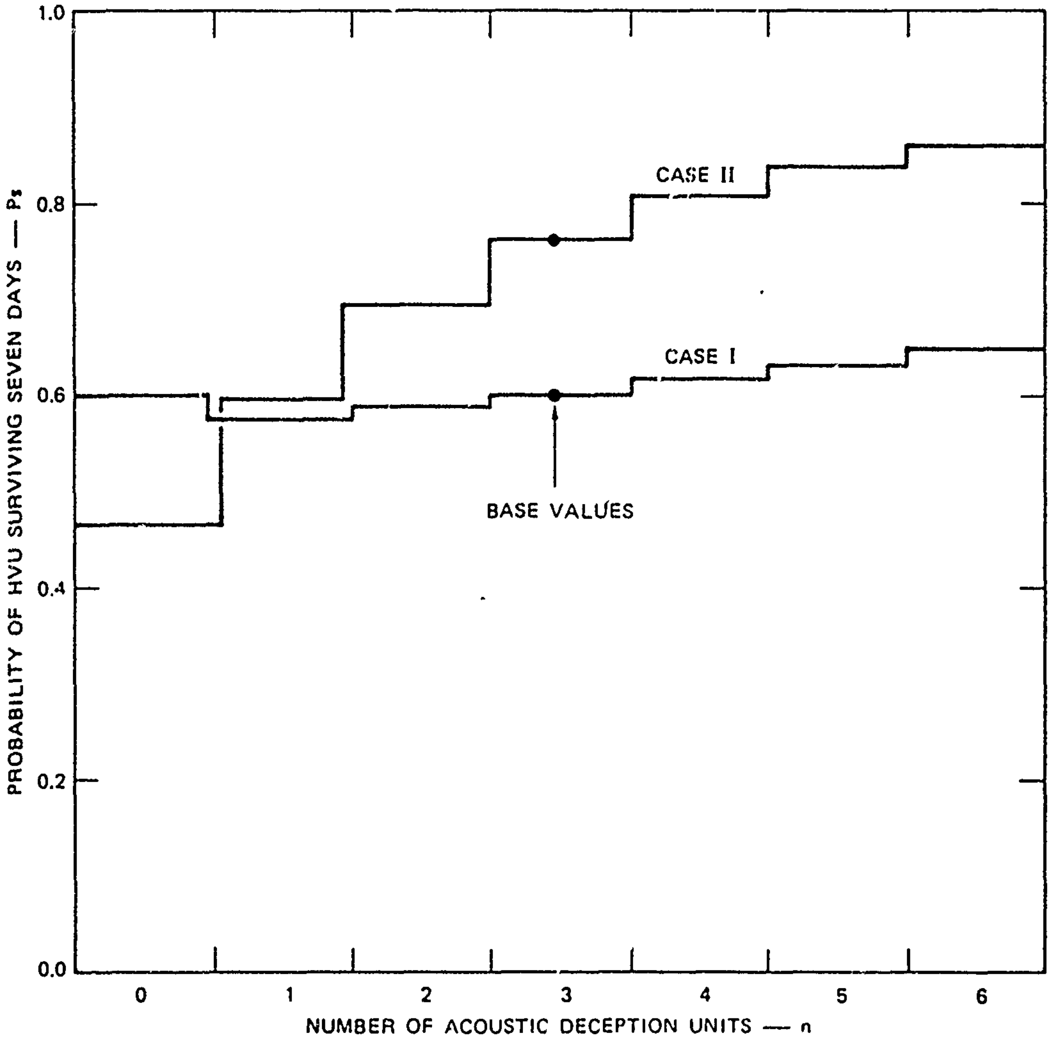


FIGURE 24 EFFECT OF NUMBERS OF ADU'S ON HVU SURVIVAL

The effect of the HVU's fidelity is shown in Figure 25 for Cases I and II. A high value of PH indicates that the received feature from the HVU closely matches the submarine's concept of an HVU feature. Similarly, a high value of $\hat{PH}$ (the submarine's estimate of PH) indicates that the submarine expects to receive a close match when the HVU is in contact.

The HVU employment of turn-count masking, or other tactics to alter the HVU's acoustic signal, would be indicated in the model by a lower value of PH. If the submarine thought that the HVU was employing these tactics, then a lower value of $\hat{PH}$ would also be used in the model.

As shown in Figure 25, if the submarine is perfect in his estimate of PH ($\widehat{PH}$ = PH), then there is only a very small increase in P_s when PH is lowered. However, if the submarine's estimate of PH is fixed ($\widehat{PH}$ = 0.95), then the reduction of PH improves HVU survivability. The increase in P_s is stronger in Case I because the submarine expects to encounter ADUs. Therefore, by lowering PH the submarine will tend to misclassify the HVU and not attack. (The sharp decrease in P_s for PH > 0.99 is due to the model design and is not a real effect.)

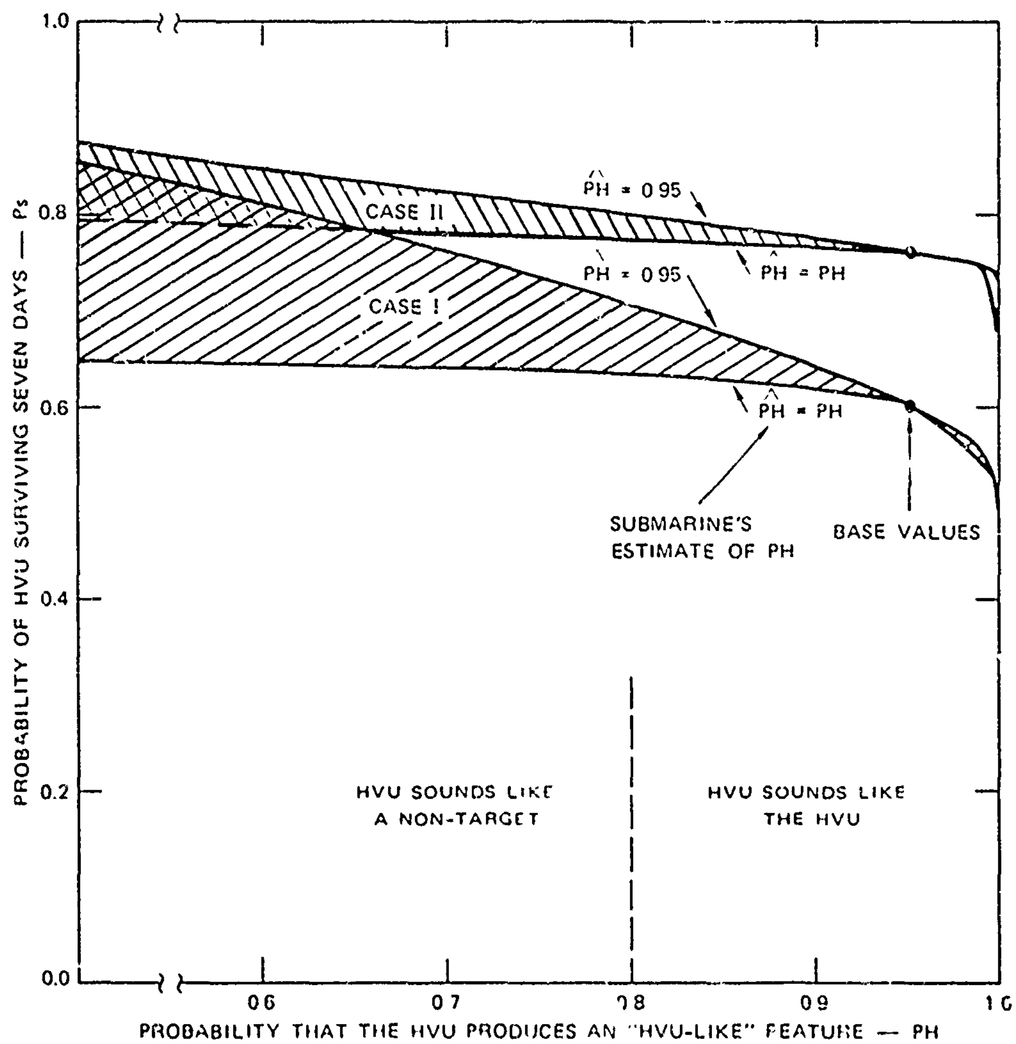


FIGURE 25 EFFECT OF HVU FIDELITY ON SURVIVAL

Figure 26 illustrates a similar analysis for PA and $\widehat{PA}$. A low value of PA indicates an ADU of good fidelity (sounds like an HVU). Similarly, a low value of $\widehat{PA}$ indicates that the submarine expects ADUs to have good fidelity. ($\widehat{PA}$ should not be confused with $\hat{P}_0$. $\widehat{PA}$ is related to the expected ADU fidelity, given encounter, and $\hat{P}_0$ is related to the expected ADU encounter rate.)

As shown in Figure 26, there is a strong increase in P_s for Case I, when the ADU is better than expected ($PA \le \widehat{PA}$). For Case II, there are strong decreases in P_s when the ADUs are worse than expected. However, if the submarine is perfect in his estimate ($\widehat{PA} = PA \le 0.5$), then the fidelity of the ADU is not as important for HVU survival.

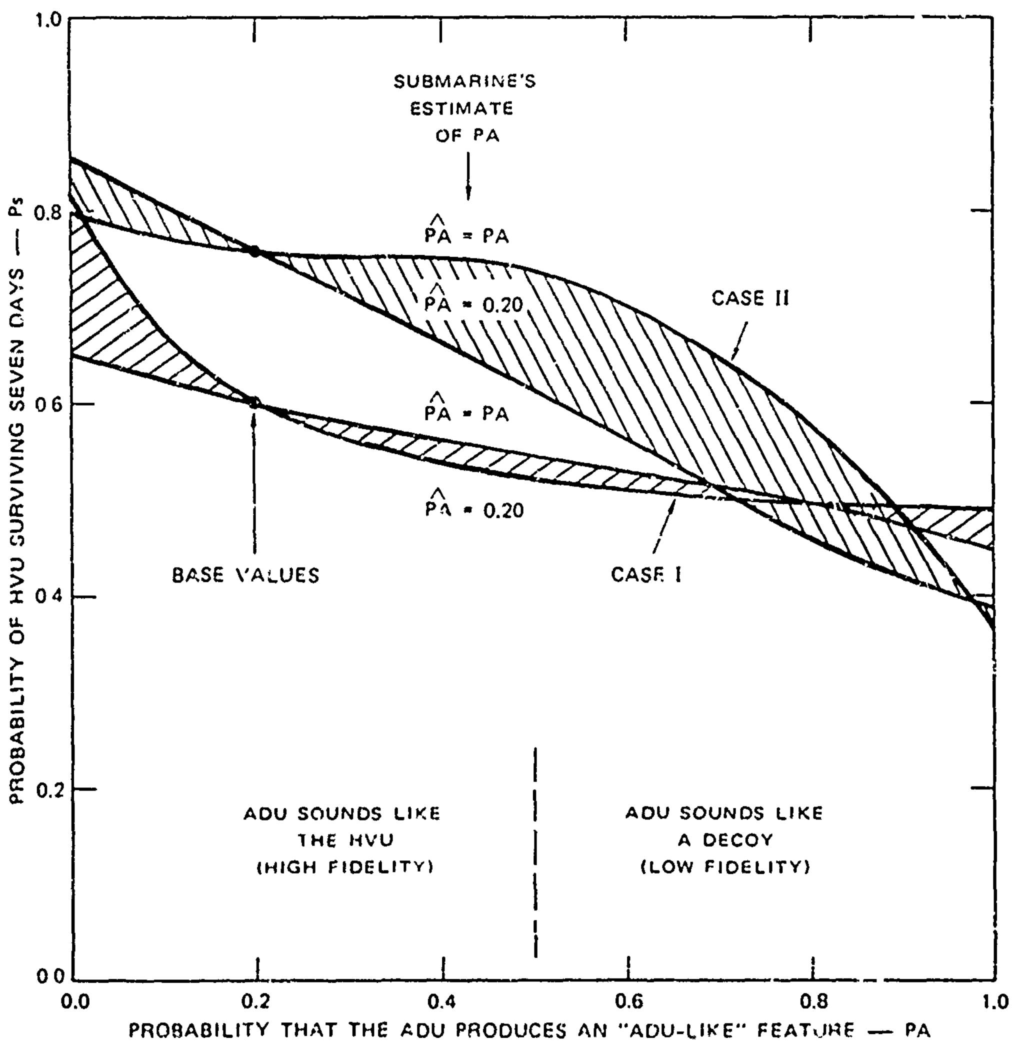


FIGURE 26 EFFECT OF ADU FIDELITY ON SURVIVAL

4. ASW Parameters

The ASW parameters describe the ASW effectiveness of the task force. F_0, the proportion of the total ASW force in company with the HVU, determines the distribution of the ASW force. In the model it is assumed that the ASW force which is not with the HVU is evenly distributed among the ADUs.

Under the Base Case parameter assumptions, as shown in Figure 27, the distribution of the ASW force is not very important for either Case I or Case II. The reasons for this effect can be seen in Figure 28. As F_0 is increased, HVU kills go up and ADU kills go down.

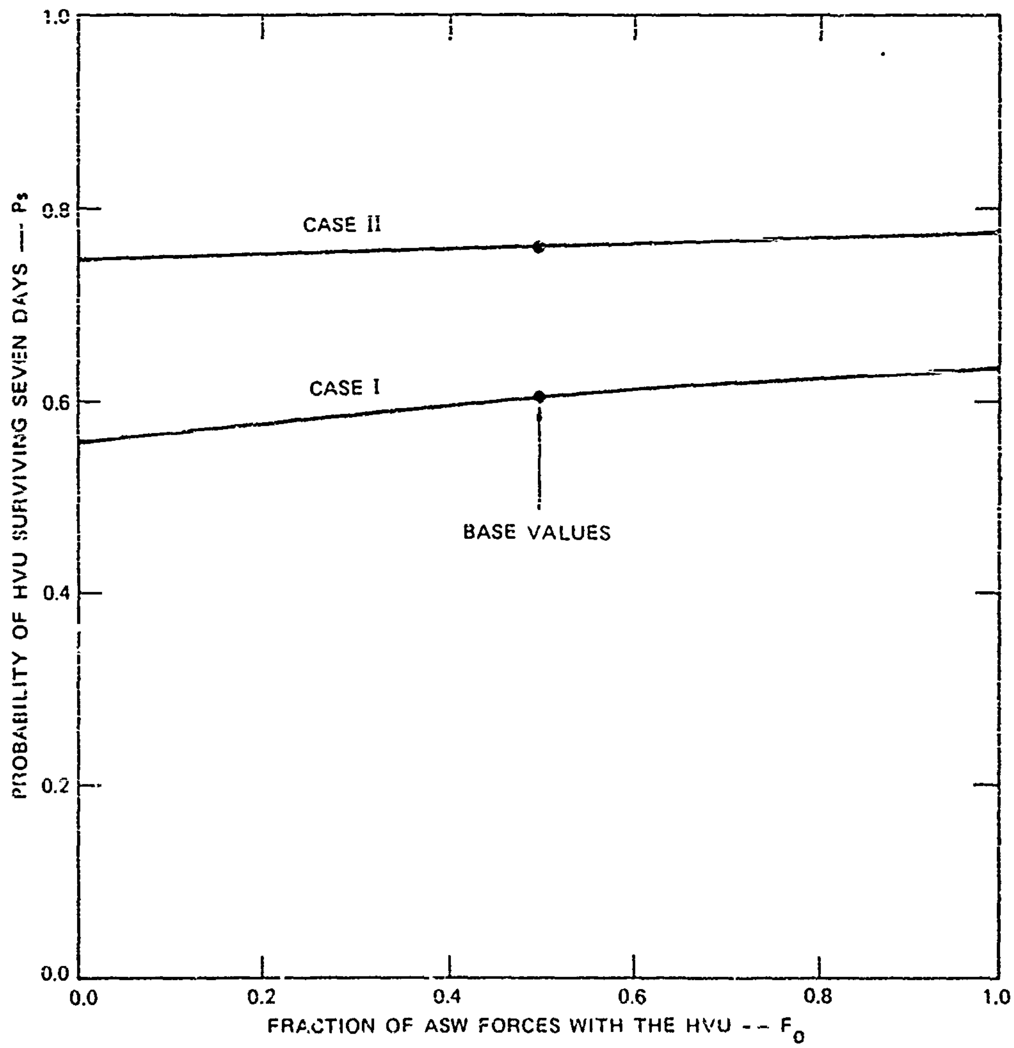


FIGURE 27 EFFECT OF ASW DISTRIBUTION ON SURVIVAL

Although P_s does not change significantly, the reasons for the HVU survival change. When $F_0 = 0$, 45 percent of HVU survival is due to the ASW force, but when $F_0 = 1$, only 28 percent HVU survival is due to ASW.

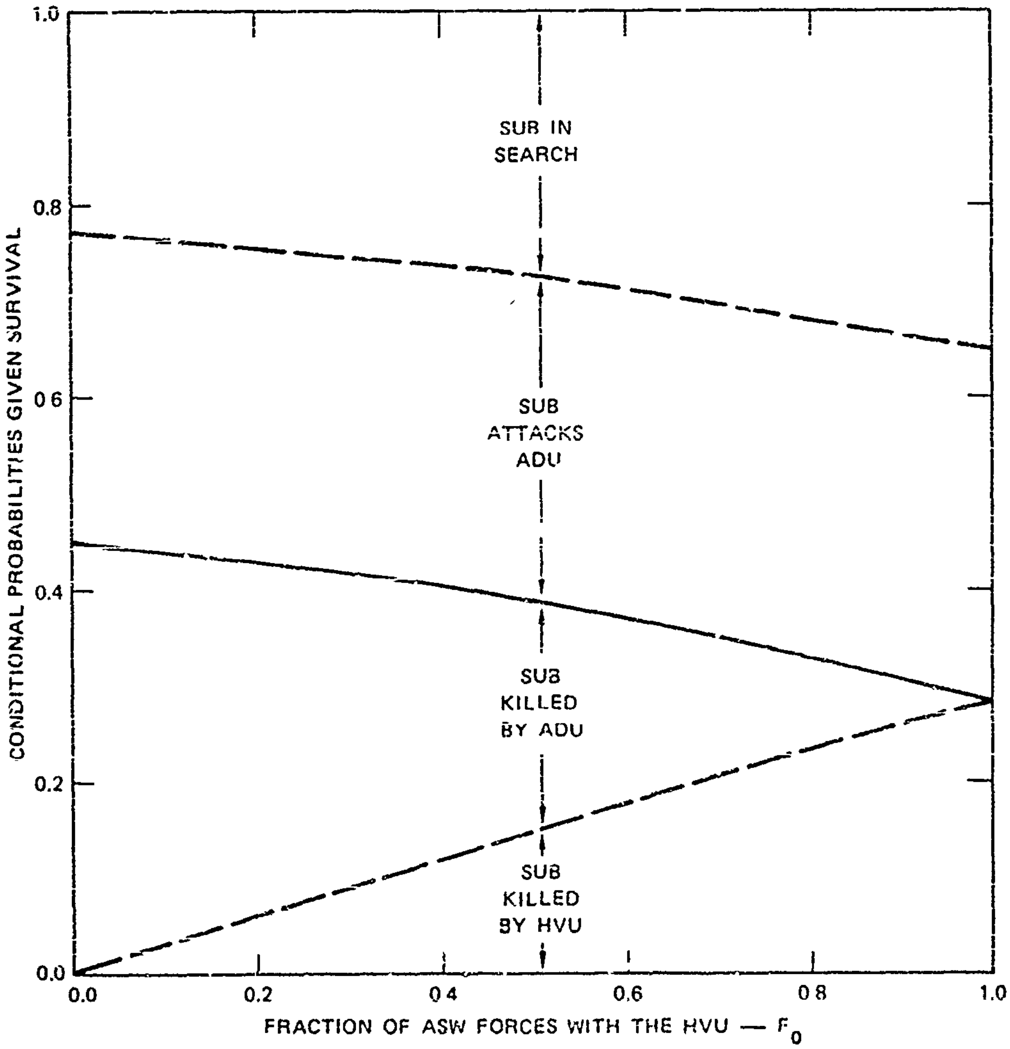


FIGURE 28 FOUR REASONS FOR SURVIVING AS A FUNCTION OF ASW DISTRIBUTION (CASE I)

The significance of the ASW force distribution (F_0) can be increased with higher ASW kill probabilities (PKA and PKV). As shown in Figure 29, increased ASW effectiveness significantly improves HVU survival for Case I, but not for Case II. This is because the probability of visually classifying are higher for Case I than for Case II. The conclusion is that, for Case I assumptions and high levels of ASW protections, it would be better, in terms of survival, to concentrate the ASW force about the HVU. However, in terms of submarine kills, it may be desirable to distribute part of the ASW force among the ADUs.

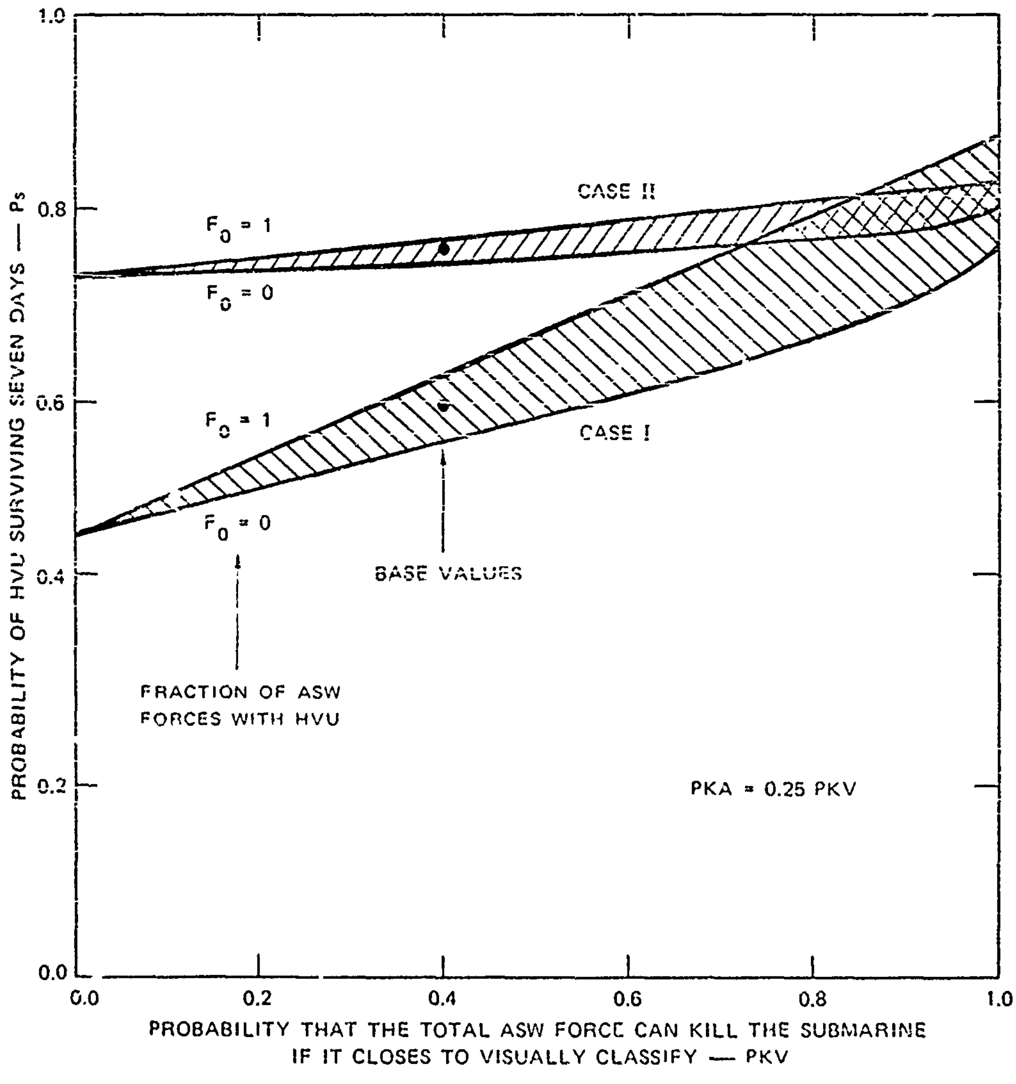


FIGURE 29 EFFECTS OF ASW EFFECTIVENESS AND DISTRIBUTION ON SURVIVAL

Figure 30 shows that, with increased ASW effectiveness, the largest component of HVU survival is ADU kills on the submarine for $F_0 = 0.5$. In summary, the basic tradeoff in the ASW distribution decision is that the ASW force is more effective when deployed with the ADUs because ADUs are encountered more often than the HVU, but the overall probability of HVU survival is less.

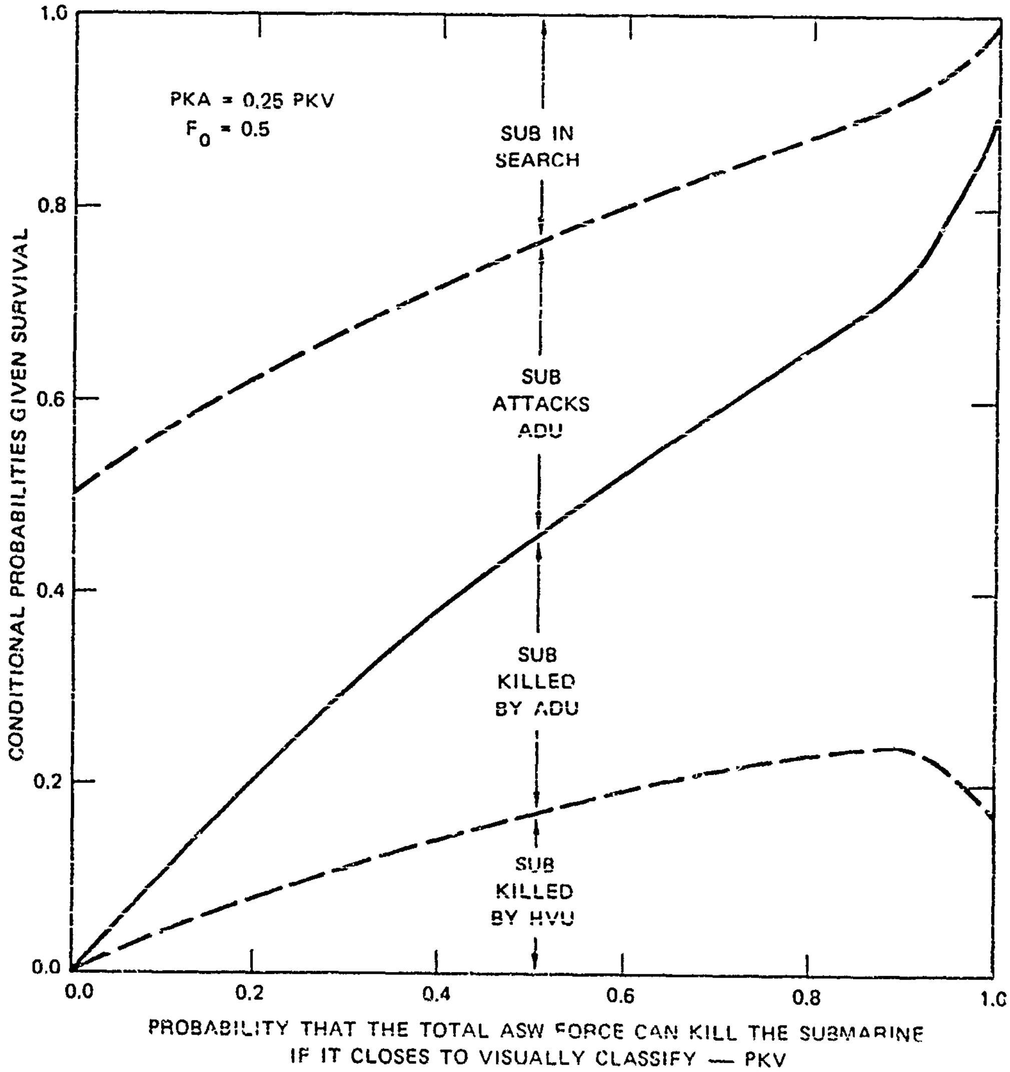


FIGURE 30 FOUR REASONS FOR SURVIVING AS A FUNCTION OF ASW EFFECTIVENESS (CASE I)

C. ACM/ASW Interactions

Throughout the analysis, two base cases have been considered, and, in general, ACM has been more effective for Case II, (submarine does not expect ADUs) than for Case I (submarine expects ADUs). Figure 31 illustrates the interaction of N (number of ADUs) and $\hat{P}_0$ (submarine estimate of the ADUs in terms of HVU survivability). The ADUs are most effective when the submarine does not expect them.

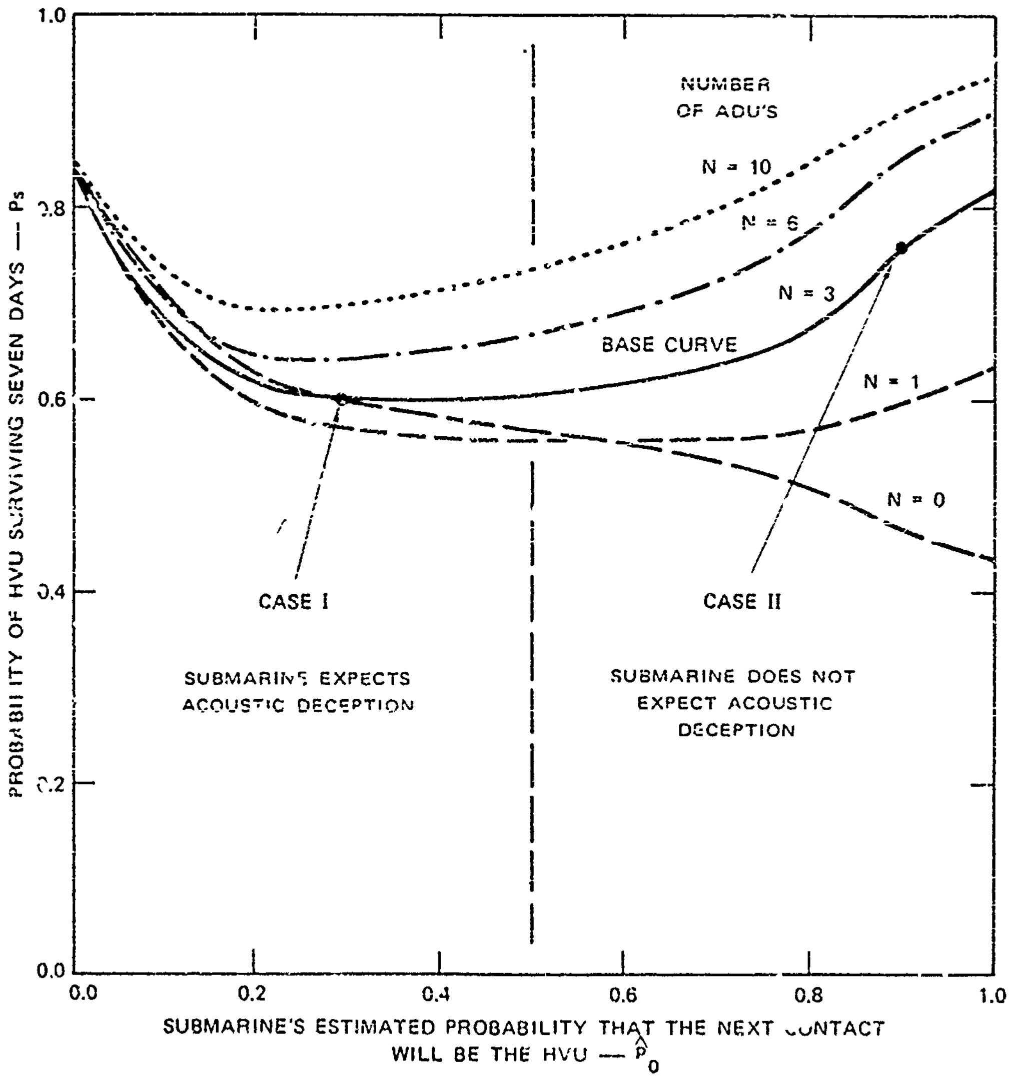


FIGURE 31 EFFECT OF SUBMARINE'S EXPECTATIONS ABOUT CONTACTS AND NUMBERS OF ADU S ON SURVIVAL

Four of the curves contain minima. This effect is due to the visual classification probabilities, which are highest when the submarine is unsure of which type of target to expec ($\hat{P}_0$ close to 0.5).

The reason the N = 0 curve continues to decrease when the other curves climb as $\hat{P}_0$ increases is because the submarine guessed correctly (he did not expect any ADUs and there are none).

Figure 32 illustrates the importance of ACM and ASW for Cases I and II. For Case I there is a small difference between 0 and 3 ADUs, and the slope of the shaded area is due to increasing ASW effects. However, for Case II, the number of ADUs is the primary factor for HVU survival. This figure also illustrates the importance of the submarine's prior estimate of the use of ADUs ($\hat{P}_0$). Under the base case parameter assumptions, it requires about twice the level of ASW (0.8 vs 0.4) for

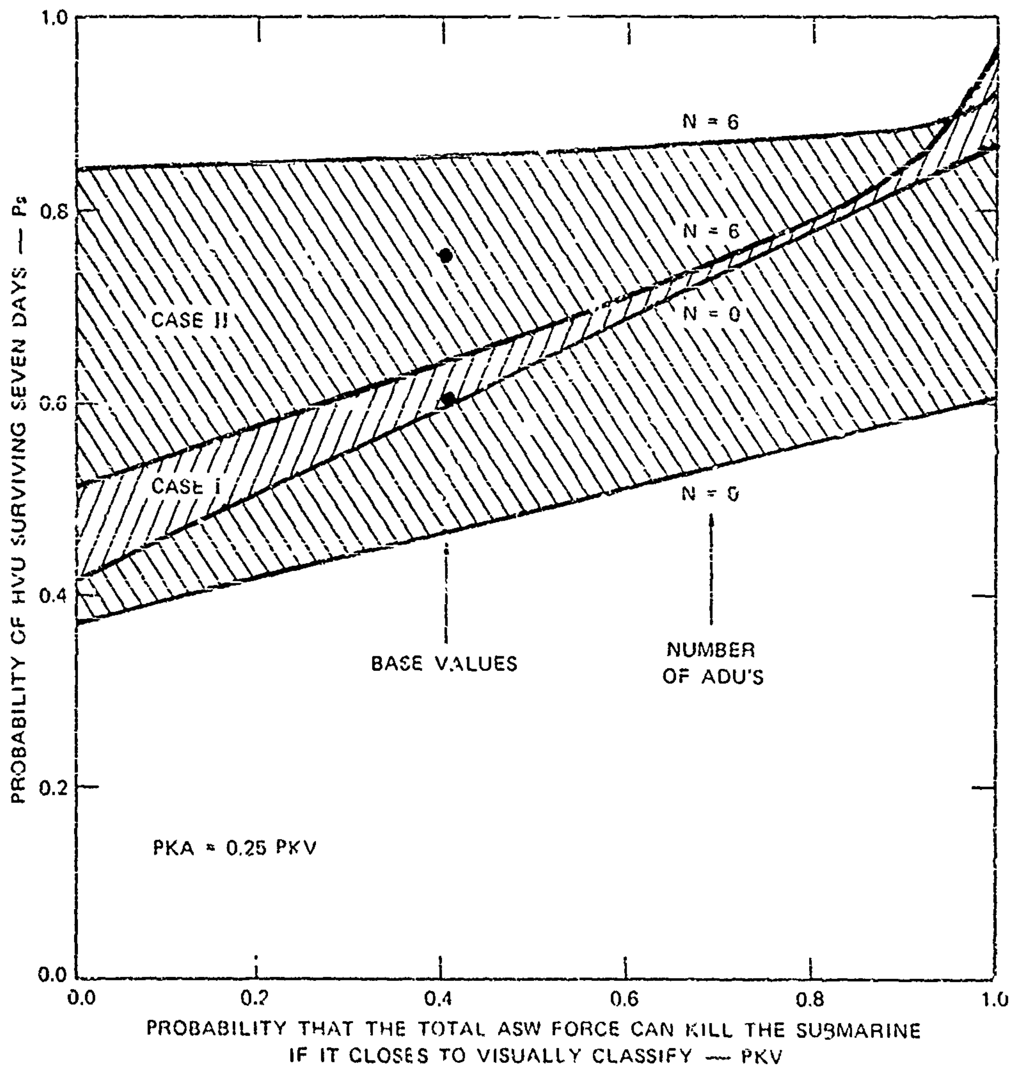


FIGURE 32 EFFECT OF ASW EFFECTIVENESS AND NUMBER OF ADU'S ON SURVIVAL

Case I (submarine expects ADU) to achieve the same probability of survival as for Case II (submarine does not expect ADUs).

Figure 33 shows the conditional probabilities of ACM or ASW being the reason for HVU survival. It is seen that in Case I ASW plays a larger relative role than in Case II. The submarine expects ADUs to be used in Case I and therefore must close to visually classify; thus the ASW forces can be more effective. Also notice the width of the two cases; Case I is insensitive to the number of ADUs as was seen on the

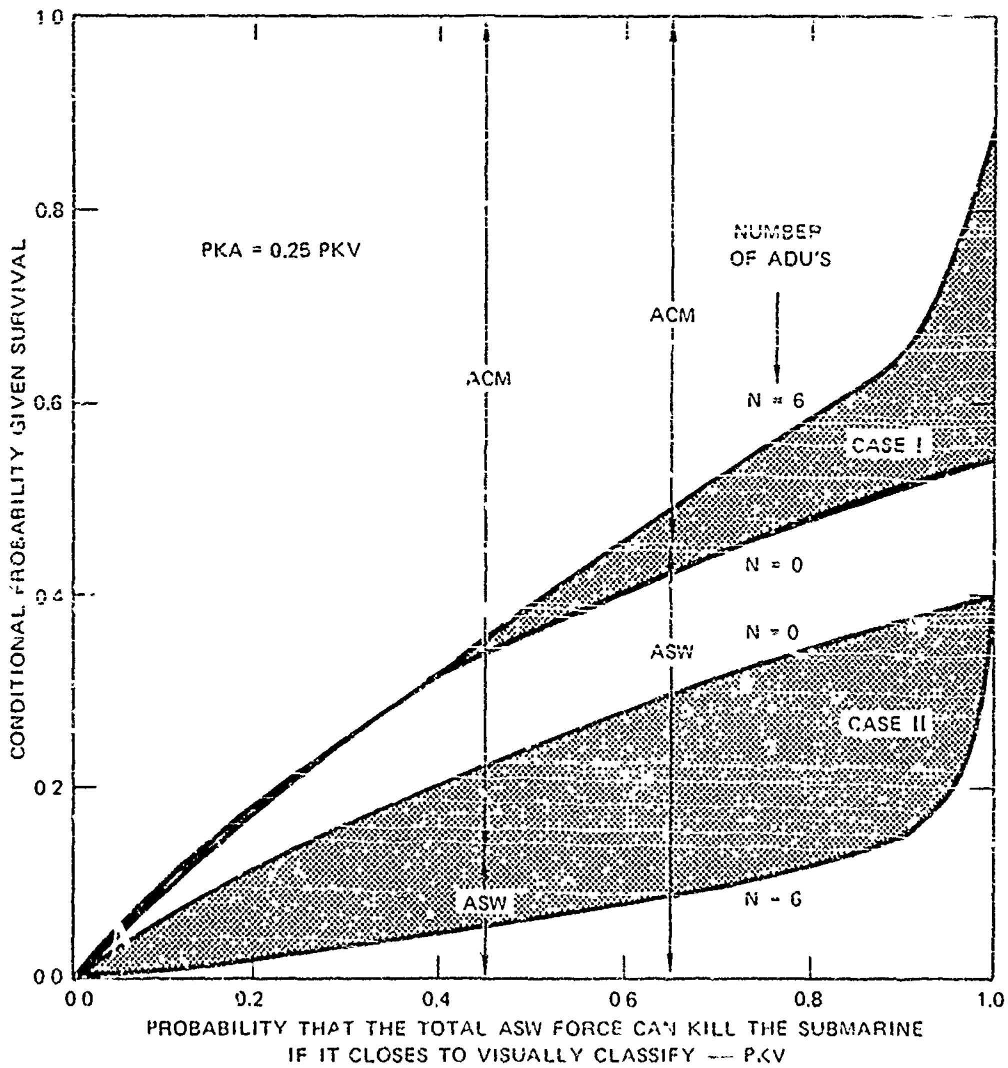


FIGURE 33 ACM AND ASW REASONS FOR SURVIVING AS A FUNCTION OF ASW EFFECTIVENESS AND NUMBERS OF ADU'S

previous figure. Case II is, however, sensitive to the number of ADUs; when a large number of ADUs is used, ASW plays only a small role in helping the HVU survive.

Figures 34 and 35 show the interaction of the remaining ACM parameters, PH and PA, with ASW effectiveness. Again for Case II, ASW makes only a small effect on survival; but for Case I, ASW is more important. In both cases the effects of ACM are almost constant (width of Case I and Case II bands). Figure 26 is similar analysis for PA.

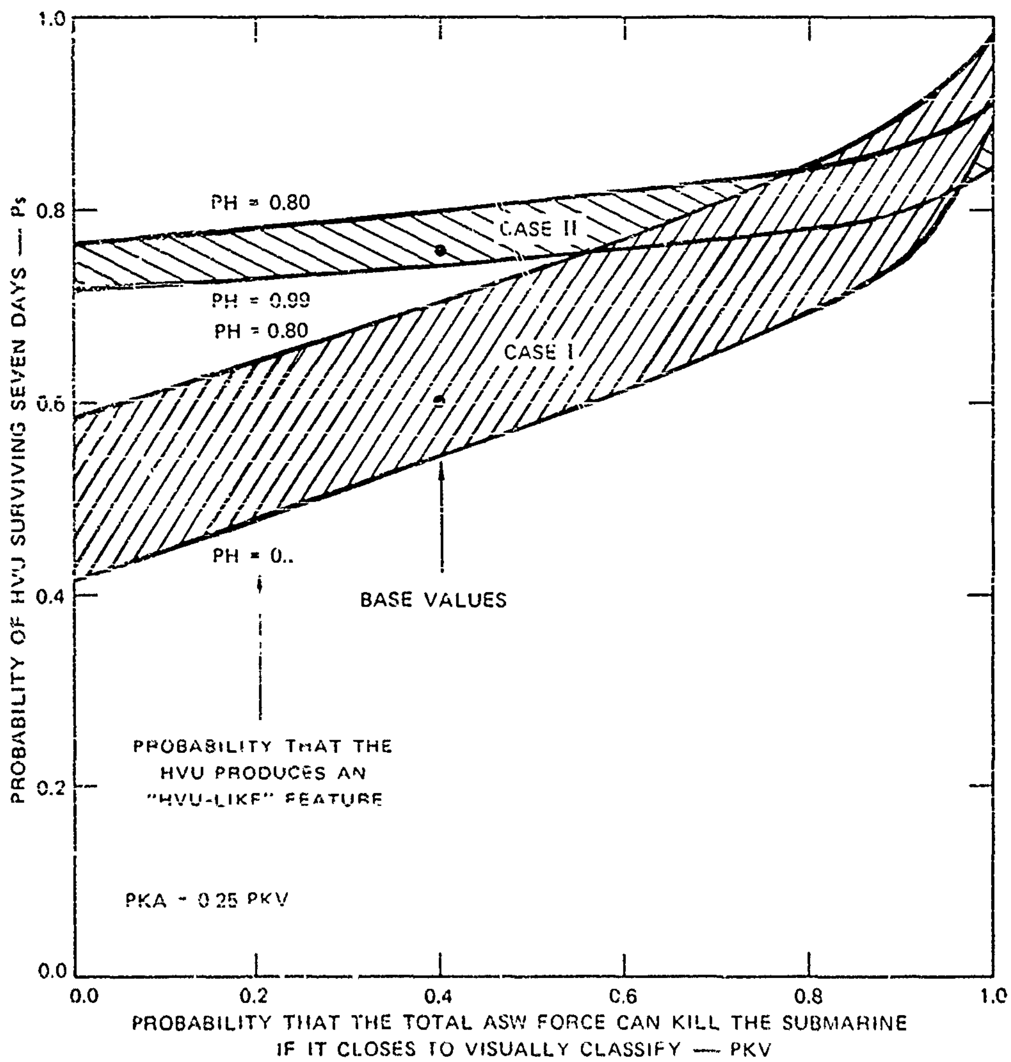


FIGURE 34 EFFECT OF ASW EFFECTIVENESS AND HVU FIDELITY ON SURVIVAL

Case II is insensitive to ASW, while Case I shows improved HVU survivability with increased ASW. The effects of ACM are also constant, except for very high levels of ASW effectiveness .

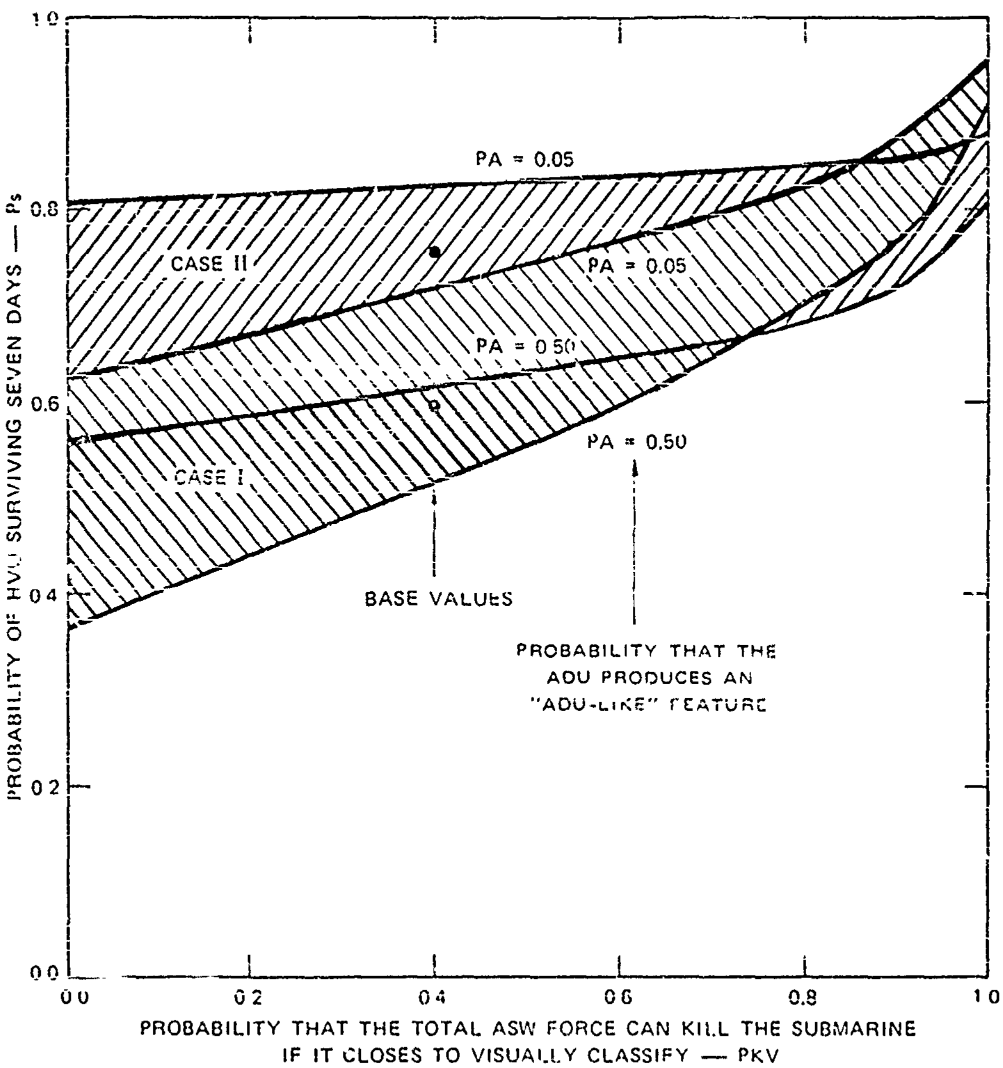


FIGURE 35 EFFECT OF ASW EFFECTIVENESS AND ADU FIDELITY ON SURVIVAL

IV EVENT STEP SIMULATION MODEL

A. General Description

An event step simulation model was developed to examine the sensitivity of a major assumption of the Acaso model: the HVU is assumed to survive if an ADU is attacked. The scenario behind the assumption is that, for a given attack, the submarine would launch all his weapons and then retire from the objective area. Recognizing the somewhat unrealistic nature of this assumption, the Acaso model was expanded into an event step simulation model.

The expanded model retains the basic structure of the Acaso model. However, as shown in Figure 36, to allow the submarine a reattack capability several "outcomes" from the "Attack ADU" state have been added:

1) Weapon depletion--this outcome is analogous to the former "Attack ADU" state. However, additional parameters in the model (salvo size, weapon load) determine how many attacks the submarine can complete before all of the weapons are expended.

2) ASW attrition--there is a specified number of ASW units in an ADU. When an ADU is attacked, it is assumed that the acoustic deception device itself is not destroyed; however, the weapons may acquire and destroy the ASW units. If at a later time the same ADU is contacted by the submarine, there will be a lower ASW threat to the submarine during the contact prosecution.

3) Submarine attrition--when the submarine attacks an ADU it is assumed that its position can be localized by the surviving ASW units. Therefore, the submarine must successfully break contact before he can resume search for the HVU.

The event step simulation model uses an event scheduling methodology that utilizes the probability and delay time functions derived for the Acaso model. Due to the similarity of the two models, a summary of the model differences will be presented in lieu of a complete model description.

PRECEDING PAGE BLANK-NOT FILMED

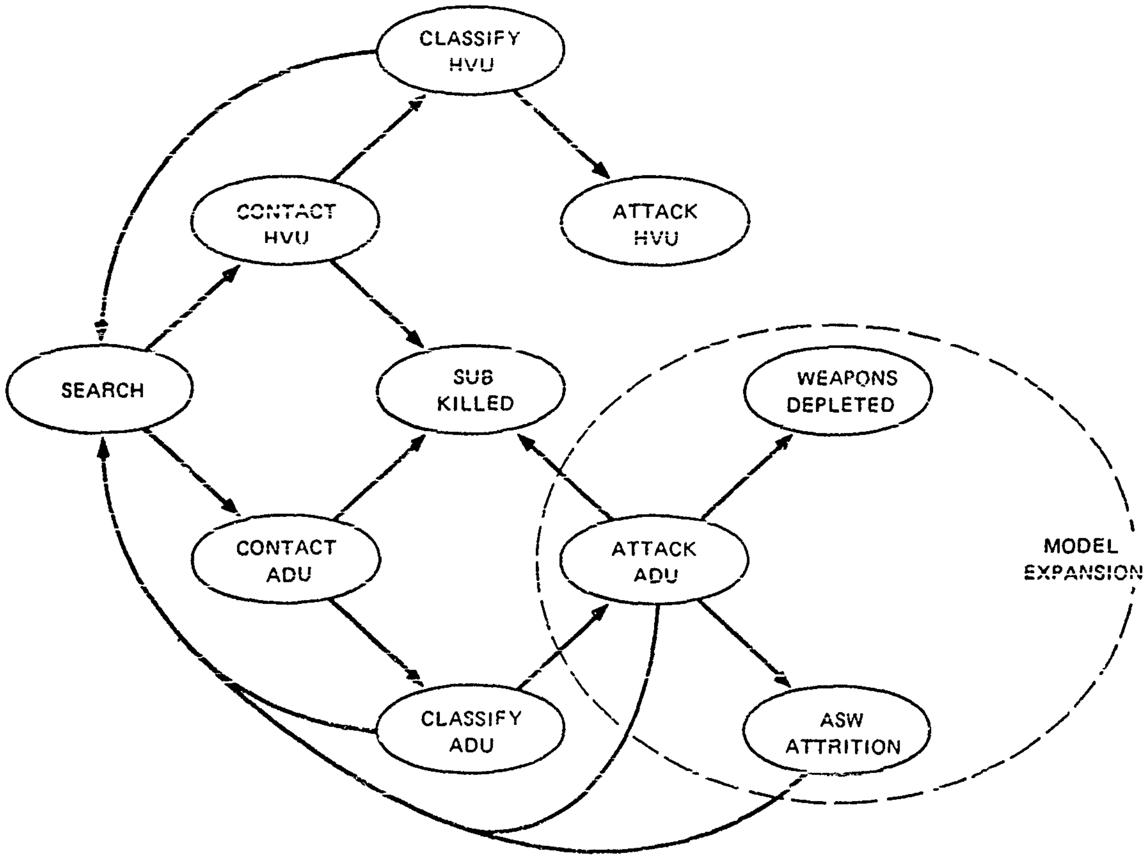


FIGURE 36 EVENT STEP SIMULATION MODEL FLOW

1. <u>ASW Parameters</u>

In the expanded model a specific number of ASW units are assigned to the HVU and each of the ADUs. All ASW units are assumed to be identical, but unlike the Acaso model, the numerical distribution of the ASW units between the HVU and each ADU is at the discretion of the model user. Three input parameters describe the ASW capability of an ASW unit.

PK_0 = probability of one ASW unit killing the submarine, given the submarine closes to acoustic classification range

PK_1 = probability of one ASW unit killing the submarine, given the submarine closes to visual classification range

PK_2 = probability of one ASW unit killing the submarine, after the submarine's position is revealed by attacking the ADU.

An additional parameter, ASW_i, describes how many ASW units are in company with the ith unit.

2. Submarine Parameters

Several additional parameters are required to describe the reattack potential of the submarine and the effectiveness of the weapons against the ASW units.

NMX = total number of missiles carried by the submarine

NML = number of missiles launched per attack

PK_m = probability of one missile destroying one ASW unit, given that the acoustic decoy was under attack.

ASW attrition is modeled by reducing the number of ASW units in company with the ADU under attack (ASW_i) in proportion to the number of missiles launched (NML) and the missiles probability of kill (PK_m).

B. Analysis

In order to compare the results of the expanded model to the Acaso model, identical base cases were used. The general scenario is the same--one submarine in search of an HVU in an operating area of 200-nmi radius. Also in the area are three ADUs. To approximate the ASW levels of the Acaso model, a distribution of three ASW units with the HVU and one ASW unit each of the ADUs was used.

The Case I results are summarized in Table 10. This is the case where the submarine expects to encounter ADUs ($\hat{P}_0$ = 0.3). As can be seen, the measures of effectiveness for the base case of the expanded model closely approximate those obtained from the Acaso model. For Run 2 the submarine is allowed one reattack of four missiles. Run 3 is

TABLE 10

EVENT STEP SIMULATION MODEL RESULTS SUMMARY (CASE I)

RUN	INPUT			MEASURES OF EFFECTIVENESS				
	NML/NMX	PK_m	PK_2	P_s	P_1	P_2	P_3	P_4
ACASO BASE CASE	–	–		0.60	0.15	0.24	0.34	0.28
1. BASE CASE	8/8	0.0	0.0	0.62	0.20	0.27	0.30	0.27
2. MULTI-ATTACK WITHOUT ASW ATTRITION	4/8	0.0	0.6	0.57	0.18	0.37	0.04	0.41
3. MULTI-ATTACK WITH ASW ATTRITION	4/8	1.0	0.6	0.55	0.19	0.30	0.06	0.45
4. MULTI-ATTACK WITH ASW ATTRITION	2/8	1.0	0.6	0.55	0.17	0.31	0.00	0.51

identical to Run 2, except when an ADU is attacked all of the ADU's ASW protection is destroyed. For Run 4 the submarine is allowed a maximum of four attacks of two missiles apiece, but this tactic does not change the probability of HVU survival. In general, when the submarine is careful in his classification, the reattack capability does not significantly help the submarine.

The primary sensitivity to the submarine's reattack capability of Case I can be seen in the changes of P_3 (out of missiles) and P_4 (out of time). As more reattacks are allowed, the primary reason for HVU survival changes from weapon depletion (P_3) to the submarine's being unable to find and correctly classify the HVU (P_1).

The ASW capability of the ADUs maintains its importance even when ASW attrition is assumed. P_2 does not drop below 30% because for Case I the submarine tends to close for visual classification and face the stronger ASW threat prior to attack. Since the submarine has good classification capability (does not attack ADUs), and the chances are small that the submarine will encounter an ADU it has already attacked, the allowance for ASW attrition for Case I does not significantly reduce the importance of ASW.

The results for runs under Case II assumptions (submarine does not expect ADUs, $\hat{P}_0 = 0.9$) are presented in Table 11. As with Case I, the Case II base case closely agrees with the Acaso base case.

TABLE 11
EVENT STEP SIMULATION MODEL RESULTS SUMMARY (CASE II)

RUN	INPUT			MEASURES OF EFFECTIVENESS				
	NML/NMX	PK_m	PK_2	P_s	P_1	P_2	P_3	P_4
ACASO BASE CASE	–	–	–	0.76	0.03	0.07	0.90	0.01
1. BASE CASE	8/8	0.0	0.0	0.77	0.03	0.06	0.91	0.01
2. MULTI-ATTACK WITHOUT ASW ATTRITION	4/8	0.0	0.6	0.65	0.06	0.34	0.53	0.07
3. MULTI-ATTACK WITH ASW ATTRITION	4/8	1.0	0.6	0.63	0.07	0.11	0.75	0.08
4. MULTI-ATTACK WITH ASW ATTRITION	2/8	1.0	0.6	0.46	0.12	0.16	0.37	0.35

Since the submarine does not expect ADUs for Case II, the submarine tends to attack each contact from the acoustic classification zone. Thus the submarine faces the minimum ASW threat, but tends to expend his weapons rapidly. For Case II the reattack capability was very important for the submarine, especially for Run 4, where four attacks were possible.

Again, the allowance of ASW attrition was not too important for HVU survival. In Run 2 (without ASW attrition) the ASW was more important due to killing the submarine after ADU attacks, but there was a corresponding decrease in P_3. Comparing Run 2 with Run 3, where ASW attrition is allowed, P_s (HVU survival) is about the same, but the reasons for HVU survival change--the submarine runs out of missiles (P_3) instead of being killed by the ASW units (P_2).

In conclusion, the reattack capability is not too important under Case I because the submarine has good classification and tends to use his weapons judiciously. However, for Case II the reattack capability is important because by attacking each contact from the acoustic zone, the submarine faces the minimum ASW threat and can survive until the HVU is eventually found and attacked.

Appendix A

MARKOV REPRESENTATION OF THE HVT/LVT MODEL

Appendix A

MARKOV REPRESENTATION OF THE HVT/LVT MODEL

This appendix develops a Markov representation of ORI's HVT/LVT* model. The purpose of representing this model as a Markov process was to make an independent check of the closed form equations developed by ORI[2]. The two different ways of calculating the MOE** yield very similar, but not identical results.

Figure A-1 shows the state diagram for the Markov representation. The symbols near the arrows are the transition probabilities from state to state. The definition of some of the symbols are quoted from the ORI report:

σ: "Probability that a submarine, on encountering an LVT, will not be destroyed by its local ASW defense before the target is classified."

δ: "Probability that a submarine will correctly classify an LVT on encounter, given that it survives the local ASW defense at the LVT. With probability $1 - \delta$, the submarine mistakenly attacks the LVT and is removed as an HVT threat."

h: "Mean time (days) needed by a submarine that encounters and survives the local ASW defense at the LVT, to make a target classification decision (which will be correct with probability δ)."

(Note: the symbols with subscript zeros are the parameters for the case of contacting the HVT.)

T_0: "The time required for one submarine to completely explore the deployment area at the search rate consistent with detecting the quiet HVT."

---

*Operations Research Incorporated. High value target/low value target.

**Measure of effectiveness.

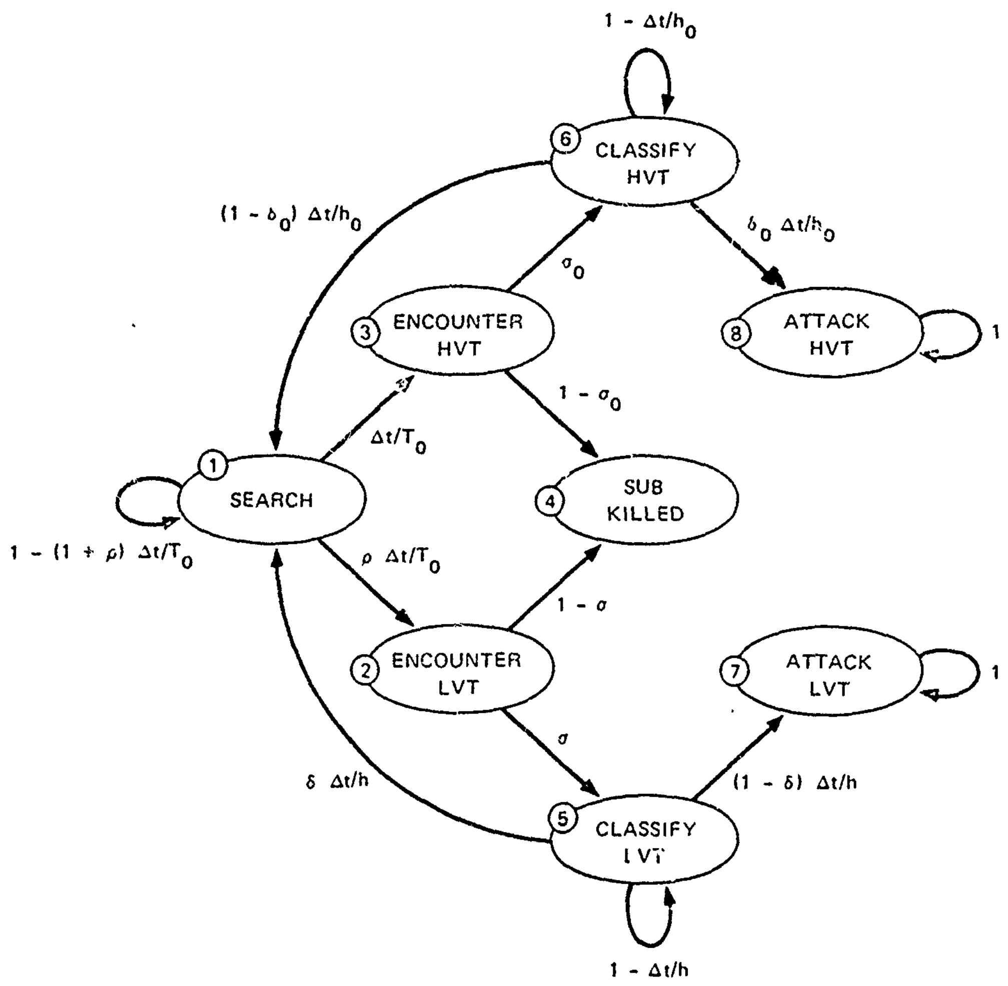


FIGURE A-1 MARKOV REPRESENTATION OF ORI'S HVT/LVT MODEL

ρ: Ratio of LVT-to-HVT search rates (HVT search rate is $1/T_0$).

Δt: Time increment of the Markov process ($\Delta t \ll h, h_0, T_0$).

V: Probability that the HVT has not been encountered and correctly classified (and thus attacked) by one submarine at the end of "t" days.

The MOE V is computed by multiplying the transition matrix P by itself $n = t/\Delta t$ times. At the beginning of the process (t=0) the probability that the submarine is in the search state is assumed to be 1.0. Therefore, the MOE is just:

$$V = 1 - (P^n)_{18}$$

where $(P^n)_{18}$ is the probability that the submarine has transitioned from state #1 to state #8 by the time t days.

The discrete-step Markov process of Figure A-1 is an approximation to the continuous time process. For small Δt, the probabilities vary only the third decimal place when Δt is varied from 1/32 day to 1/64 day. This is accurate enough for use as an MOE and comparison with ORI's closed form equation.

ORI's closed form equation for the one-submarine case is:

$$V = \frac{\rho K + K_0}{\rho K + E_0} + \frac{\delta_0 \sigma_0}{\rho K + E_0} \exp\left(\frac{-t(\rho K + E_0)}{T_0 + h\rho(1-K) + h_0(1-E_0)}\right) ,$$

where

$$K = 1 - \delta\sigma$$

$$E_0 = 1 - (1-\delta_0)\sigma_0$$

$$K_0 = 1 - \sigma_0 .$$

The equation can be rewritten in a somewhat simpler form by substituting:

$$A = \frac{\delta_0 \sigma_0}{\rho K + E_0} ,$$

and noting that

$$1 - A = \frac{\rho K + K_0}{\rho K + E_0} .$$

The equation then becomes:

$$V = 1 - A(1 - e^{-t/T}) \quad ,$$

where

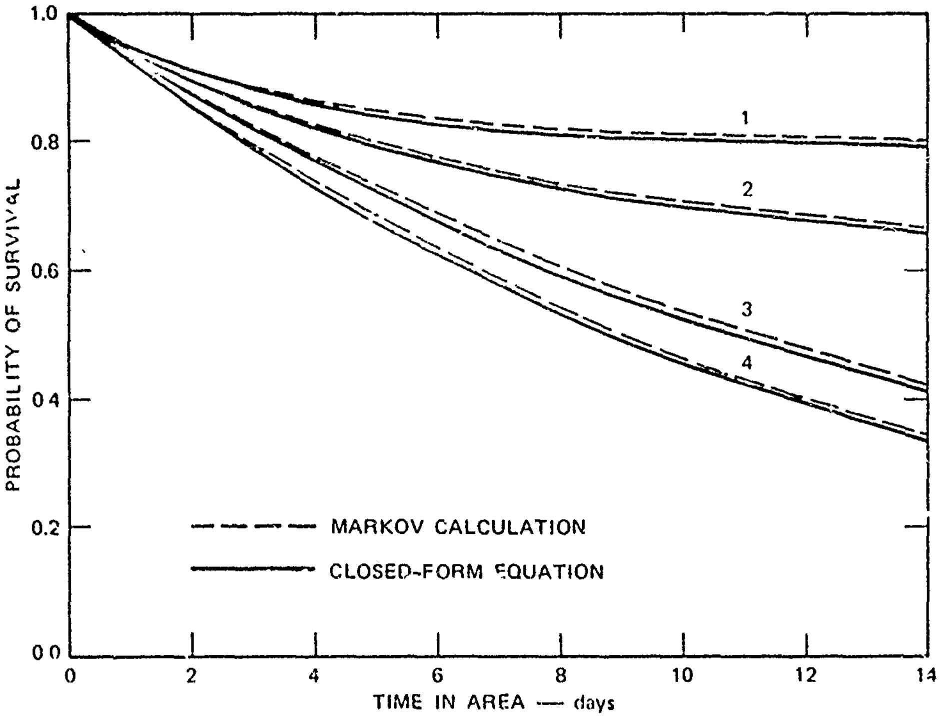
$$A = \delta_0 \sigma_0 / B$$

$$T = (T_0 + \rho\ \delta\ \sigma\ h + (1-\delta_0)\sigma_0 h_0)/B$$

$$B = \rho(1-\delta\sigma) + (1-\sigma_0) + \delta_0 \sigma_0 \quad .$$

The probability V approaches the probability 1 - A in an average time T.

Figure A-2 shows the comparison between the Markov and closed form calculations for various parameter combinations. The Markov calculation is higher than the equation by a very small amount (usually less than 0.01).



PARAMETER	CURVE NUMBER			
	1	2	3	4
T_0	10.00	10.00	12.80	12.80
ρ	5.00	5.00	10.30	0.00
h_c	0.10	0.10	0.21	0.21
h	0.10	0.10	0.21	0.21
σ_0	0.80	0.80	1.00	1.00
σ	0.90	0.90	1.00	1.00
δ_0	0.75	0.75	1.00	1.00
δ	0.65	0.95	1.00	1.00

FIGURE A-2 COMPARISON OF CLOSED-FORM EQUATION AND MARKOV CALCULATION

Appendix B

ACASO MODEL COMPUTEk PROGRAM

Appendix B

ACASO MODEL COMPUTER PROGRAM

The Acaso model was programmed in FORTRAN for a CDC 6000 series time snaring system. Table B-1 is a comparison listing of the model parameters and their corresponding program variable names. Table B-2 is the program listing of the Acaso model.

Table B-1

PROGRAM VARIABLES

Model Parameter	Program Variable	Model Parameter	Program Variable
Input:		Intermediate:	
		s_o	WO
		s	W
r_o	RO	PM	PM
r	R	PF	PF
v_o	VO	$\hat{X}_o$	XO
v	V	X_m	X1
u	U	X_f	X2
t	T(t/2 1 T)	X_o	[illegible]O
A	AR(AR · – A)	Y	Y
$\hat{P}_o$	PO	Y_1	Y1
$\hat{PH}$	PHE	Y_2	Y2
$\hat{PA}$	PLE	PV_o	PVO
Pt	PE	PV	PV
N	N	PHA_o	PHAO
PH	PH	PHA	PHA
PA	PL	F	F
F_o	FO	PPV_o	PPVO
PKA	PKA	PPV	PPV
PKV	PKV	PPA_o	PPAO
		PPA	PPA
		W_o	WAO
		W	WA

Table B-1 (Concluded)

Model Parameter	Program Variable	Model Parameter	Program Variable
ORI:		Output:	
T_o	TO	T	TT
ρ	RHO	A_o	AH
σ_o	SO	A	AL
σ	S	K_o	SKO
δ_o	DO	K	SK
δ	D	P_s	PS
h_o	HO	P_1	PKO
h	H	P_2	PK
t	T	P_3	PA
		P_4	PT

Table B-2

ACASO COMPUTER PROGRAM LISTING

```
PROGRAM HUUADU(OUTPUT)
DIMENSION POC(2)
DIMENSION PS(2),PK0(2),PK(2),PA(2),PT(2)
DIMENSION C1(2),C2(2),C3(2)
DATA R0 / 30.0 /
DATA R  / 60.0 /
DATA V0 / 10.0 /
DATA V  /  5.0 /
DATA U  / 10.0 /
DATA T  /  7.0 /
DATA AR /200.0 /
DATA N  /  3   /
DATA PHE/  0.95/
DATA PLE/  0.20/
DATA PH /  0.95/
DATA PL /  0.20/
DATA F0 /  0.5 /
DATA PKA/  0.1 /
DATA PKU/  0.4 /
DATA PE /  0.1 /
DATA POC/  0.3 ,  0.9 /
PI = 3.1415927
SPK = 4.0/PI-1.0
IH1 = 10H    PARAM1
IH2 = 10H         PS
IH3 = 10H      PK0
IH4 = 10H  PK     PA
IH5 = 10H     PT
IH6 = 10HC1    C2
IH7 = 10H  C3
C**** INSERT PARAM2 LOOP ****
P2 = K
XX = P2
PRINT 400,P2
400 FORMAT(//*    PARAM2*/F10.3)
PRINT 500,POC(1),POC(2)
500 FORMAT(34X,*CASE 1  P0 =*,F4.2,38X,*CASE 2  P0 =*,F4.2,/)
PRINT 600,IH1,(IH2,IH3,IH4,IH5,IH6,IH7,I=1,2)
600 FORMAT(6A10,A4,5A10,A4)
C**** INSERT PARAM1 LOOP ****
P1 = J
XX = P1
A = AR*AR*PI
SM = AMIN1(V0,U)
SX = AMAX1(V0,U)
```

Table B-2 (Continued)

```
W0 = SX + SPK*SM*SM/SX
SM = AMIN1(V,U)
SX = AMAX1(V,U)
W = SX + SPK*SM*SM/SX
T0 = (A-PI*P0*P0-N*PI*R*R)/(48.0*R0*W0)
IF(T0.LE.0.0) GO TO 200
RHO = (N*R*W)/(R0*W0)
DO 100 I = 1,2
P0 = P0C(I)
IF(P0.GT.1.0-PE/2.0) GO TO 10
PF = 0.5*PE/(1.0-P0)
IF(P0.LT.PE/2.0) GO TO 20
PM = 0.5*PE/P0
GO TO 30
10 PM = 0.5*PE/P0
PF = 1.0
GO TO 30
20 PM = 1.0
30 X0 = AG(PLE)-AG(1.0-PHE)
X1 = X0 + AG(PM)
X2 = AG(1.0-PF)
IF(X1.LT.X2) GO TO 40
IF(P0.LE.0.5) X1 = X2
X2 = X1
40 X0 = AG(PL) - AG(1.0-PH)
Y = AG(PL)-AG(PLE)
X1 = X1 + Y
X2 = X2 + Y
F = 0.0
IF(N.NE.0) F = (1.0-F0)/N
PPV0 = (1.0-PKV)**F0
PPA0 = (1.0-PKA)**F0
IF(N.NE.0) GO TO 50
PPV0 = 1.0-PKV
PPA0 = 1.0-PKA
50 PPV  = (1.0-PKV)**F
PPA  = (1.0-PKA)**F
PV0 = PROB(X2-X0) - PROB(X1-X0)
PV  = PROB(X2) - PROB(X1)
S = PV*PPV + (1.0-PV)*PPA
S0 = PV0*PPV0 + (1.0-PV0)*PPA0
WA0 = (1.0-PV0)*PPA0/S0
WA  = (1.0-PV) *PPA /S
PHA0 = 0.0
IF(PV0.LT.1.0) PHA0 = (1.0-PROB(X2-X0))/(1.0-PV0)
```

Table B-2 (Continued)

```
PHA = 0.0
IF(PV.LT.1.0) PHA = (1.0-PROB(X2))/(1.0-PV)
D0 = WA0*PHA0 + (1.0-WA0)
D  = 1.0 - WA*PHA
H0 = (1.0-WA0/2.0)*R0/(W0*24.0)
H  = (1.0-WA /2.0)*R /(W *24.0)
BB = RHO*(1.0-D*S) + (1.0-S0) + D0*S0
TT = (T0 + RHO*D*S*H + (1.0-D0)*S0*H0)/BB
AH = D0*S0/BB
AL = RHO*S*(1.0-D)/BB
SK0 = (1.0-S0)/BB
SK  = RHO*(1.0-S)/BB
EX = 1.0 - EXP(-T/TT)
PS(I) = 1.0 - AH*EX
PA(I) = AL*EX/PS(I)
PK0(I) = SK0*EX/PS(I)
PK(I) = SK*EX/PS(I)
PT(I) = (1.0-EX)/PS(I)
C1(I) = PK0(I)
C2(I) = C1(I) + PK(I)
C3(I) = C2(I) + PA(I)
100 CONTINUE
PRINT 1000,P1,(PS(I),PK0(I),PK(I),PA(I),PT(I),
+                C1(I),C2(I),C3(I),I=1,2)
1000 FORMAT(F10.3,2(F10.3,F8.3,6F6.3))
200 CONTINUE
END
FUNCTION AG(P)
C
C   CALCULATES X SUCH THAT PROB(-INF,X) = P
C   NATL BUREAU OF STANDARDS, HANDBOOK OF MATH FUNCTIONS
C   P. 933 EA. 26.2.23
C
PD = P
C0 = 2.515517
C1 = 0.802853
C2 = 0.010328
D1 = 1.432788
D2 = 0.189269
D3 = 0.001308
ISWITCH = 0
IF(PD.EQ.1.0) GO TO 30
IF(PD.EQ.0.0) GO TO 20
IF(PD.LE.0.5) GO TO 10
PD = 1.0 - PD
```

Table B-2 (Concluded)

```
ISWITCH = 1
10 CONTINUE
T = SQRT(ALOG(1.0/(PD*FD)))
T2 = T*T
AG = (C0 + C1*T + C2*T2)/(1.0 + D1*T + D2*T2 + D3*T2*T)
AG = AG - T
IF(ISWITCH.EQ.1) AG = -AG
RETURN
20 AG = -999.
RETURN
30 AG =  999.
RETURN
END
FUNCTION PROB(Y)
C
C   CALCULATES PROBABILITY OF Y BEING BETWEEN -INFINITY AND Y
C   NATL BUREAU OF STANDARDS, HANDBOOK OF MATH FUNCTIONS
C   P. 932 EA. 26.2.19
C
X  = Y
D1 = 0.0498673470
D2 = 0.0211410061
D3 = 0.0032776263
D4 = 0.0000380036
D5 = 0.0000488906
D6 = 0.0000053830
ISWITCH = 0
IF(X.GE. 999.) GO TO 30
IF(X.LE.-999.) GO TO 20
IF(X.GE.0.0)   GO TO 10
ISWITCH = 1
X = -X
10 X2 = X*X
X3 = X2*X
PROB = 1.0+D1*X+D2*X2+D3*X3+D4*X2*X2+D5*X3*X2+D6*X3*X3
PROB = 1.0 - 0.5/PROB**16
IF(ISWITCH.EQ.1) PROB = 1.0 - PROB
RETURN
20 PROB = 0.0
RETURN
30 PROB = 1.0
RETURN
END
```

REFERENCES

1. Naval Operations Analysis, U. S. Naval Institute, Annapolis, Maryland (1968)

2. A. Kooharian and H. Young, "A Methodology for Evaluation of Task Force Defense Concepts (U)," Interim Technical Report No. 688, Operations Research, Inc., (5 November 1971) CONFIDENTIAL

PRECEDING PAGE BLANK-NOT FILMED

A BIBLIOGRAPHY OF SRI REPORTS RELATED TO ACOUSTIC DECEPTION

I CONCEPT STUDIES

1. A. Bien, R. M. Wrona, "Acoustic Countermeasures Concepts and Techniques for the Protection of a Carrier Task Force in Transit (U)," Interim Report 1, Project 2167-600, (April 1967) SECRET AD 382 015L.
2. A. Bien, P. A. Banks, and P. J. McDonough, "Acoustic Countermeasures Concepts and Techniques for the Protection of Carrier Task Forces (U)," Interim Report 2, Project 7000-210, (April 1968) SECRET AD 391 934L.
3. A. Bien, P. J. McDonough, and J. R. Olmstead, "Acoustic Countermeasures Concepts and Techniques, Volume I: Evaluation (U)," Final Report, Project 7000-210, (August 1969) CONFIDENTIAL AD 506 293L.
4. J. R. Olmstead, "Acoustic Countermeasures Concepts and Techniques, Volume II: Acoustic Kinematic Computer Model (U)," Final Report, Project 7000-210, (August 1969) CONFIDENTIAL AD 506 294L.
5. A. Bien, P. J. McDonough, and J. R. Olmstead, "Acoustic Countermeasures Concepts and Techniques, Volume III: Supporting Analysis and Data (U)," Final Report, Project 7000-210, (August 1969) SECRET AD 506 235L.
6. G. W. Black, "A Computer Program for Determining Multiple Target Acoustic Detection and Separation Zones (U)," Research Memorandum NWRC RM-55, Project 7000-210, (August 1969) CONFIDENTIAL AD 506 304L.

II EVALUATION STUDIES

A. OBJECTIVE AREA I

1. J. M. Moore, "Semi-Markov Models of Search in the Presence of Decoys," Research Memorandum NWRC RM-64, Project 1016-245, (January 1972) UNCLASSIFIED AD 736 961.
2. E. L. Wong, "Simulation Models of Search in the Presence of Decoys," Technical Note NWRC TN-37, Project 1016-245, (April 1972) UNCLASSIFIED.
3. A. Bien, "Evaluation of Tactical Deception Techniques in Carrier Task Force Defense (U)," Final Report, Project 1016-245, (December 1972) SECRET/NOFORN AD 525 077.

PRECEDING PAGE BLANK-NOT FILMED

B. OBJECTIVE AREA II

1. E. L. Wong, "Simulation Model for the Evaluation of Naval Tactical Deception Concepts (U)," Technical Note NWRC-TN-41, Project 1318-5, (January 1973) CONFIDENTIAL AD 527 991.

2. J. M. Moore, "Semi-Markov Model for Evaluating Naval Tactical Deception Concepts," Technical Note NWRC TN-42, Project 1318-5, (April 1972) UNCLASSIFIED AD 778 511.

3. J. M. Moore and A. Bien, "Naval Tactical Deception Concepts: Methodology Assessment and Concept Evaluation (U)," Final Report (Draft), Project 1318-15, (February 1974) SECRET.

C. TRANSIT

1. J. M. Moore, "Evaluation of Tactical Deception Techniques for Defense of a Carrier Task Group in Transit (U)," Final Report, Project 1318-15, (November 1973) CONFIDENTIAL.

III RELATED STUDIES

1. J. R. Olmstead and T. R. Elfers, "Combined Use of ACM and ECM Decoys to Defend Against the SLCM Threat (U)," Final Report, Project 2444-250, (February 1973) SECRET/NOFORN AD 529 048L.

2. J. R. Olmstead and T. R. Elfers, "Models of Acoustic Deception and ASW in a Task Group Operating Area (U)," Final Report, Project 1318-25, (December 1974) CONFIDENTIAL.

IV SPIN-OFF STUDIES

A. SOVIET SLCM

1. J. R. Olmstead, A. Bien, and R. R. Keenly, "Cruise-Missile Submarine Operations: An Information and Tactics Study (U)," Final Report, Project 1016-220, (December 1971) SECRET/NOFORN AD 519 402.

2. R. R. Newman, "Characteristics of a Carrier Task Force Objective Area Operations (U)," Technical Note NWRC TN-34, Project 1016-243, (January 1972) SECRET/NOFORN AD 519 209.

3. J. R. Olmstead and R. R. Keenly, "Cruise-Missile Submarine Supplemental Study: SS-N-7 Targeting Constraints (U)," Research Memorandum NWRC RM-68, Project 1873-220, (September 1972) SECRET/NOFORN AD 522 871.

B. CLASSIFICATION

1. G. W. Black, W. H. Frye, and J. R. Olmstead, "MUSAC--Multiple Source Acoustic Classification Model: A Representation of the Passive Sonar Classification Process," Final Report (Draft), Project 1318-240, (November 1973) UNCLASSIFIED.

DISTRIBUTION LIST

ORGANIZATION	NO. OF COPIES
Director, Defense Research and Engineering Washington, D. C. 20301	2
Chief of Naval Operations Department of the Navy Washington, D. C. 20350	
(Op-951)	1
(Op-961)	1
(Op-98)	1
(Op-0953)	1
(Op-987)	1
(Op-982)	1
(Op-03)	2
Office of Naval Research Department of the Navy Arlington, VA 22217	
(Code 431)	4
(Code 220)	1
(Code 230)	1
Antisubmarine Warfare Systems Project Office Naval Material Command Department of the Navy Washington, D. C. 20360	
(ASW-14)	1
(ASW-22)	1
Naval Air Systems Command Department of the Navy Washington, D. C. 20362	
(AIR-53362)	1
(AIR-370)	1
Naval Sea Systems Command Department of the Navy Washington, D. C. 20362	
(NAVSEA 06H1-3)	1
(NAVSEA 660E2)	1
(NAVSEA 0371)	1
Chief of Naval Material Department of the Navy Washington, D. C. 20360	
(MAT-03)	1
(MAT-03R1)	1
Commander Third Fleet FPO San Francisco 96601	1
Commander Second Fleet Norfolk, VA 23511	
(N7 Division)	2
Destroyer Development Group FPO New York 09501	2
Commander, Operational Test & Evaluation Force Norfolk, VA 23511	1
Naval Intelligence Support Center 4301 Suitland Road Washington, D. C. 20390	
(Code NISC-20)	1
(Code NISC-30)	1
Naval Research Laboratory Washington, D. C. 20375	
(Code 2620)	1
(Code 8100)	1
(Code 5700)	1
Naval Air Development Center Warminster, PA 18974	
(Code 20531)	1
Naval Ship Research and Development Center Bethesda, MD 20034	
(Code 186)	1
(Code 1926)	1
Naval Undersea Center San Diego, CA 92132	
(Code 14)	1
New London Laboratory Naval Underwater Systems Center New London, CT 06320	2
Naval Weapons Center (Code 12) China Lake, CA 93555	1
Naval Ordnance Laboratory Silver Spring, MD 20910	1
Naval Postgraduate School Monterey, CA 93940	
(Technical Library)	1
Naval War College Newport, RI 02840	
(Technical Library)	1
Defense Documentation Center Cameron Station Alexandria, VA 22314	2
Bolt, Beranek & Newman, Inc. 1701 N. Ft. Myer Dr. Arlington, VA 22209	1
Center for Naval Analyses 1400 Wilson Boulevard Arlington, VA 22209	1
Institute for Defense Analyses 400 Army-Navy Drive Arlington, VA 22202	1
Presearch, Inc. 2361 Jefferson Davis Hwy Arlington, VA 22209	1
Pacific Analysis Corporation 841 Bishop Street Honolulu, Hawaii 96813	1
Operations Research Inc. 1400 Spring Street Silver Spring, MD 20910	1