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THEORY OF HEAT PIPES

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Please make the following changes in LA-3246-MS.

Page

- 10 final term of Eq. (1) should be " $\rho_l g z \sin \varphi$ ".
- 12 line 16 should read "...evaporator section,  $0 < z < l_e$ ,  
 and removed in the condenser section,  $l_e < z < l$ . The  
 temperature and consequently the ...".
- 15 expression in third line from bottom should be  
 " $\langle \rho_l \vec{v} \cdot \nabla \vec{v} \rangle \approx \rho_l \langle \vec{v} \rangle^2 / \epsilon r_c$ ".
- 23 enclose each of the two derivatives in Eq. (26) in large  
 brackets.
- 27 first term on right hand side of Eq. (35) should be  
 " $\rho_l g l \sin \varphi$ ".
- 28 right hand side of line 7 should be " $\Delta T_v = - 0.7^\circ K$ ".
- 29 lower left expression in Eq. (37) should be  

$$\frac{(1-4/\pi^2)Q_e^2}{8\rho_v r_v^4 L^2}.$$
- 34 delete "and" in line 4.
- 34 the number "2/3" in line 12 should be " $\sqrt{2/3}$ ".
- 34 sentence beginning on line 15 should be "Under these  
 conditions for uniform heat addition and removal the  
 maximum heat transport is found to be".
- 34 in the upper expression of Eq. (43) the exponent of  
 $r_v$  should be 3 instead of 2.
- 35 before the first sentence insert "A wick of uniform  
 thickness is not optimal, but is not much inferior to  
 the best distribution of wick thickness."

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**OF THE UNIVERSITY OF CALIFORNIA LOS ALAMOS NEW MEXICO**

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**THEORY OF HEAT PIPES**

by

**T. P. Cotter**

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Abstract

A heat pipe is a self-contained structure which achieves very high thermal conductance by means of two-phase fluid flow with capillary circulation. A quantitative engineering theory for the design and performance analysis of heat pipes is given.

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## Contents

	<u>Page</u>
<b>Abstract</b>	3
1. Introduction	7
2. General heat pipe structure	8
3. Static condition	10
4. Start-up	12
5. Steady-state heat pipe regime	14
6. Solution of the steady-state equations	24
7. Maximum heat flux	28
8. Multi-component fluids	31
9. Optimal heat pipes	33
References	36

## Figures

1. Cylindrical Heat Pipe Structure	37
2. Distribution of Pressures in a Heat Pipe	37

## 1. Introduction

The "Grover Heat Pipe"<sup>1</sup> is a self-contained engineering structure which exhibits a thermal conductance greatly in excess of that which could be obtained by the use of a homogeneous piece of any known metal. This property is achieved within the containing envelope by the evaporation of a liquid, transport of the vapor to another part of the container, condensation of the vapor and return of the condensate to the evaporator through a wick of suitable capillary structure. The quantitative engineering theory for the design and performance analysis of heat pipes, alluded to but not elaborated in the original description of these devices, is supplied herein.

There are obviously many practical uses for a structure of extraordinarily large thermal conductance. The heat pipe principle is indeed applicable over a very wide range of sizes, shapes, temperatures and materials. Unlike solid heat conductors, however, heat pipes cannot be characterized by a single property (an "equivalent thermal conductivity", say), since the behavior and limitations of a heat pipe are largely integral properties of the device as a whole. Furthermore, even if the size, shape, temperature and materials of a heat pipe are specified, the mass, vapor volume fraction, thermal conductance and maximum heat flux are individually (through not independently) under the control of the designer. The particular application will determine which allowed combination of these properties is most desirable.

At the present time, certain quantitative features of heat pipe behavior have not in fact been verified experimentally, though they can be predicted with some confidence. A few properties cannot yet even be treated with any conviction.

It is impractical to furnish a sufficient number of specific calculations to be generally useful. This report is intended simply to offer some initial orientation to the quantitative principles of heat pipes, and to serve as a stimulus for further experimentation, applications, and improvement of the theory.

## 2. General heat pipe structure

The advantages of heat pipes are best realized when they are long and thin, that is, take the form of long cylinders or extended thin planar structures. For definiteness the discussion here will be confined to right circular cylinders of large length-to-diameter ratio. The course of the analysis for other shapes will be evident, though not always straightforward in detail. As shown in Fig. 1, such a heat pipe consists of a containing tube of length  $l$  with outer radius  $r_p$ , an annular capillary structure saturated with a wetting liquid, with outer radius  $r_w$ , and a vapor space of radius  $r_v$ .

Since heat is added to and removed from the heat pipe through the container wall by ordinary thermal conduction, this should be as thin as other considerations permit, in order to minimize radial temperature differences. The container wall must of course sustain

the difference between the internal and the ambient pressure. Heat pipes become effective at internal vapor pressures as low as a hundredth of an atmosphere, and improve with increasing pressure. It will ordinarily be possible nearly to match the ambient pressure by choice of a working fluid with an appropriate vapor pressure at the desired operating temperature. The question of the long-term compatibility of the container with the working fluid might determine its thickness, or the container might even be a structural element with other functions in the larger device of which the heat pipe is a part.

The details of the wick admit of wide variation. It need not be disposed against the inside surface of the container as shown, though this will ordinarily be the best place for it for several reasons. Since evaporation and condensation take place at the vapor-liquid interface, this disposition of the wick allows the necessary radial heat transfer to occur through the medium of highest thermal conductance and thus minimizes radial temperature differences. This also makes the hydraulic diameter of the vapor space as large as possible which minimizes axial pressure gradients in the flowing vapor. The wick may be a woven cloth, roving, felt, sinter, etc., or even simply slots or grooves in the container wall.<sup>2</sup> The capillary structure will be characterized by its mean pore radius, permeability and liquid volume fraction. It is desirable but possibly not essential that the heat pipe be self-priming; that is, if the requisite amount

of liquid is placed anywhere in the container it spontaneously saturates the entire wick. This is accomplished by having sufficiently small capillary pore size throughout the wick.

The working fluid must wet the wick material; that is, the contact angle (the angle formed by a wedge of liquid in equilibrium contact with the solid substrate) must be less than  $\pi/2$ . While not essential, it is desirable that the fluid also wet the container wall as this improves the heat transfer. There is little penalty for a modest excess of liquid over the amount required to saturate the wick. A deficiency on the other hand can be expected to reduce the maximum heat transport by reducing the effective wick volume in the evaporator section of the heat pipe.

### 3. Static condition

Suppose, first, that there is no heat addition or removal and that the pipe is at equilibrium with its length,  $z$ , at an angle  $\phi$  to a gravitational field of acceleration,  $g$ . The pressure distribution in the liquid phase,  $p_l(z)$ , obeys the usual hydrostatic law for an incompressible fluid:

$$p_l(z) = p_l(0) + \rho_l g \sin \phi \quad \begin{array}{l} \text{See Attached} \\ \text{Errata Sheet} \end{array} \quad (1)$$

where  $\rho_l$  is the density of the liquid. The pressure in the vapor phase  $p_v(z)$ , assumed an ideal gas, has a Boltzmann distribution in the gravitational field, but the variation of pressure is entirely negligible and we may take the pressure to be constant. The inter-

face between the liquid in the capillary structure and the adjacent vapor must assume a local radius of curvature,  $r(z)$ , so that surface tension,  $\gamma$ , supports the difference in pressure between the liquid and the vapor. Thus

$$p_v(z) - p_l(z) = \frac{2\gamma}{r(z)} \quad (2)$$

Now the vapor pressure of the liquid depends not only on the temperature  $T$ , but also somewhat on the radius of curvature of the liquid-vapor interface,  $r$ . This dependence of the vapor pressure  $p = p(T, r)$ , is given by

$$p(T, r) = p(T, \infty) e^{-\frac{2\gamma M}{\rho_l R T r}}$$

where  $M$  is the molecular weight of the vapor and  $R$  is the universal gas constant. The quantity  $2\gamma M / \rho_l R T$  typically has order of magnitude  $10^{-6}$  cm or less, and since the capillary pore sizes of practical interest exceed  $10^{-4}$  cm, we may neglect this dependence and take  $p = p(T)$  only. In the present equilibrium case the vapor pressure of the liquid must be equal to the pressure in the adjacent vapor,  $p(T) = p_v$ .

In a capillary structure of minimum pore radius  $r_c$ , containing a liquid for which the contact angle is  $\theta$ , the smallest radius of curvature that the meniscus can achieve is  $r_c \sec \theta$ . The liquid-vapor interface under some circumstances may be at the surface of the capillary structure or even outside it, so the maximum radius of curvature may be at least as large as the radius of the vapor space.



evaporation the vapor pressure of the liquid in the evaporator must exceed the pressure in the adjacent vapor. Similarly, in order to continue condensing, the pressure in the condenser vapor must exceed the vapor pressure of the adjacent liquid. As a result of evaporation the liquid-vapor interface in the evaporator recedes somewhat into the capillary structure and the radius of curvature of the meniscus consequently decreases there. Condensation of vapor increases the radius of curvature of the meniscus in the condenser, if it is not already essentially infinite. Thus, according to (2), the pressure distribution in the liquid changes in the direction which drives liquid from the condenser to the evaporator. The resulting distribution of pressures is shown in Fig. 2.

The starting transient is somewhat more complex when the material which will become the working fluid is below its melting point initially. As heat is added to the evaporator section, the material there is brought to its melting temperature and above, and vapor is formed which moves down the vapor duct to the condenser section. Material adjacent to the evaporator section is heated to the melting point partly by the condensation of vapor on its surface and partly by axial conduction of heat along the container wall and the wick. The melt zone thus moves out into the condenser section. In order for the startup to fail, liquid must continually be depleted by evaporation faster than it becomes available by melting, until all liquid formed

is deposited as solid in the condenser section. In a large number of tests of heat pipes of various kinds, successful startup appears to be a fairly general rule as long as there are no unexpected malformations of the wick.

#### 5. Steady-state heat pipe regime

We consider now the quantitative dynamics of the long cylindrical heat pipe in steady-state operation. We shall derive equations which determine the distribution of energy flow, material flow, temperature and pressure within a heat pipe when it is placed in a specified external thermal environment. The basic working relations are obtained from the general equations of conservation of mass, momentum and energy by taking averages over the radial cross section of the pipe and making simplifying assumptions.

The conservation of mass of a fluid of density  $\rho(z,r)$  in steady flow with velocity  $\vec{v}(z,r)$  is expressed by

$$\nabla \cdot \rho \vec{v} = 0 \quad (4)$$

Since there is no flow normal to the outer boundaries of the liquid region, the velocity components  $v_z$  and  $v_r$  satisfy the boundary conditions

$$v_z(0,r) = v_z(l,r) = v_r(z,r_w) = 0 \quad (5)$$

The total axial flow of vapor,  $\dot{m}_v$ , and of liquid,  $\dot{m}_l$ , at axial position  $z$  are respectively

$$\dot{m}_v(z) = \int_0^{r_v} \rho(z,r) v_z(z,r) 2\pi r dr; \dot{m}_\ell(z) = \int_{r_v}^{r_w} \rho(z,r) v_z(z,r) 2\pi r dr \quad (6)$$

Applying Gauss' theorem to (4) in a cylindrical region between 0 and z and inside  $r_w$ , and using (5) and (6), yields

$$\dot{m}_v(z) + \dot{m}_\ell(z) = 0 \quad (7)$$

The momentum equation for steady incompressible flow is

$$\nabla p = \rho \vec{g} + \eta \nabla \cdot \nabla \vec{v} - \rho \vec{v} \cdot \nabla \vec{v} \quad (8)$$

where p is the pressure and  $\eta$  the coefficient of viscosity. The implications of this equation for the liquid and vapor flows are quite different.

As an approximation valid for the flow of liquid through the porous structure of the wick we now obtain a version of Darcy's law. Consider the average of (8) over a small area with dimensions small compared to the thickness of the wick but large compared to the average radius of a capillary pore,  $r_c$ . Since  $\langle \vec{v} \rangle$ , the area average  $\vec{v}$ , includes regions occupied by solid wick structure, the average flow velocity within the pores is  $\langle \vec{v} \rangle / e$ , where e is the fraction of wick volume occupied by liquid. Since the fluid velocity is of order  $\langle \vec{v} \rangle / e$  within a capillary passage and vanishes on the pore surface, then in order of magnitude,  $\langle \rho_\ell \vec{v} \cdot \nabla \vec{v} \rangle \approx \rho_\ell \langle \vec{v} \rangle^2 / e r_c$  and  $\langle \eta \nabla \cdot \nabla \vec{v} \rangle \approx -\eta_\ell \langle \vec{v} \rangle / e r_c^2$ . The ratio of magnitudes of these two terms is just the Reynold's number for the average flow in a pore,

See Attached  
Errata Sheet

$\rho_{\ell} vr / \eta_{\ell}$ , which will be small compared to unity in all cases of present interest. The final inertial term in (8) is therefore neglected and we have

$$\nabla p_{\ell} = \rho \cdot \vec{g} - b \eta_{\ell} \langle \vec{v} \rangle / \epsilon r_c^2 \quad (9)$$

where  $b$  is a dimensionless constant depending on the detailed geometry of the capillary structure. For non-connected parallel cylindrical pores  $b \sim 8$ . For realistic capillary structures, with tortuous and interconnected pores,  $b \sim 10-20$ .

The average radial and axial pressure gradients will be inversely proportional to the flow areas in the radial and axial directions respectively. For long thin pipes, that is as long as  $r_v \ell \gg r_w^2$ , the radial pressure gradient will be negligible, and we may assume that both the flow velocity and pressure in the liquid depend only on  $z$ . Thus specializing (9) to the axial direction and using the definition of the total liquid mass flow, (6), we have

$$\frac{d p_{\ell}}{dz} = \rho_{\ell} g \sin \phi - \frac{b \eta_{\ell} \dot{m}_{\ell}(z)}{\pi(r_w^2 - r_v^2) \rho_{\ell} \epsilon r_c^2} \quad (10)$$

The dynamics of the vapor flow is decidedly more complex, partly because in general an equation like (10) relating the local pressure gradient with the local mass flow does not even exist, and partly because of the inertial term in (8) is often not negligible in cases

of interest. The vapor flow in the evaporator and condenser of a heat pipe is dynamically identical to pipe flow with injection or suction through a porous wall. This problem has been studied by Yuan and Finkelstein<sup>3</sup> for cylindrical pipes, and by Knight and McInteer<sup>4</sup> for flow between plane parallel walls. We summarize and quote the partial results of these authors.

Both analyses assume incompressible laminar flow and uniform injection or suction. Several regimes must be distinguished, depending on the magnitude of a Reynolds number,  $R_r$ , based on the radial flow velocity at the channel wall,  $v_r = v_r(z, r_v)$ , the channel radius,  $r_v$ , the vapor density,  $\rho_v$ , and viscosity,  $\eta_v$ :

$$R_r = - \frac{\rho_v r_v v_r}{\eta_v} = \frac{1}{2\pi\eta_v} \frac{dm_v}{dz} \quad (11)$$

Note that  $R_r$  is positive for evaporation and negative for condensation. For all values of  $R_r$  solutions are found for which the axial velocity profiles are symmetric about the channel axis, with the profiles at different axial stations differing only by a velocity scale factor proportional to the distance from the axial origin of the flow. For  $|R_r| \ll 1$ , viscous effects dominate and the axial velocity profile is close to the usual parabolic shape for Poiseuille flow. The pressure decreases in the direction of flow, with a gradient larger than that of Poiseuille flow in the case of evaporation, and smaller if vapor is condensing. In this regime the flow properties can

be calculated by a straightforward perturbation expansion in powers of the Reynolds number. The resulting pressure gradient is given approximately by<sup>3</sup>

$$\frac{dp_v}{dz} = - \frac{8\eta_v \dot{m}_v}{\pi \rho_v r_v^4} \left( 1 + \frac{3}{4} R_r - \frac{11}{27} R_r^2 + \dots \right) \quad (12)$$

This expression is derived on the assumption that  $R_r$  as defined in (11) is a constant, independent of  $z$ . This will often be the case in practical applications of heat pipes.

When  $|R_r|$  is large the evaporation and condensation cases becomes qualitatively different. Knight and McInteer<sup>4</sup> show this in theory for flow between plane parallel walls. Wageman and Guevara<sup>5</sup> have verified the following description experimentally for cylindrical pipe flow. For high evaporation rates,  $R_r \gg 1$ . The radial dependence of the velocity is not parabolic but is proportional to  $\cos \frac{\pi}{2} \left( \frac{r}{r_v} \right)^2$ . The pressure decreases in the direction of flow. The flow properties can be calculated by a perturbation expansion in powers of  $1/R_r$ . With high condensation rates on the other hand, the flow is of boundary layer type. The axial velocity is constant across most of the channel, with the transition to zero velocity occurring in a thin layer at the wall. The pressure increases in the direction of fluid motion as a consequence of partial dynamic recovery in the decelerating flow. In this regime only the limiting behavior can at present be described analytically, as perturbation expansions cannot be made

self-consistently. In either limit,  $|R_r| \rightarrow \infty$ , the pressure gradient is given by<sup>4</sup>

$$\frac{dp_v}{dz} = - \frac{\dot{m}_v}{4\rho_v r_v^4} \frac{dm_v}{dz} \quad (13)$$

where the difference in the flows enters only in the numerical coefficients: for evaporation,  $s = 1$ ; for condensation,  $s = 4/\pi^2$ . As before, (13) is strictly correct only for constant  $R_r$ .

Nothing appears to be known about the stability of these flows, so that nothing definite can be said about the onset of turbulence. The transition criterion as well as the properties of the fully developed turbulent flow will depend also on a Reynolds number,  $R_z$ , based on the mean axial velocity  $\bar{v}_z$ ,

$$R_z = \frac{\rho_v r_v \bar{v}_z}{\eta_v} = \frac{\dot{m}_v}{\pi r_v \eta_v} \quad (14)$$

We might, however, use (12) or (13), as appropriate, without regard to the problem of turbulence, for lack of better information, except for one case of practical interest where we may proceed on a sounder basis. If the evaporator and condenser of a heat pipe are connected by a long insulated section, then, since the returning condensate will be heated by the outgoing vapor, there will be a small but ordinarily negligible net condensation along the insulated part so that  $R_r \approx 0$ . If  $R_z < 1000$ , then the expression (12) for laminar flow is appropriate. If, however,  $R_z > 1000$ , and the length exceeds, say

50  $r_v$ , then we should expect fully developed turbulent flow. In this case we should use instead of (12), the empirical Blasius law

$$\frac{dp_v}{dz} = - \frac{.0655 \eta_v^2}{\rho_v r_v^3} R_z^{7/4} \quad (15)$$

To complete the discussion of the flow dynamics we must state the connections between the vapor and liquid pressures and the vapor and liquid mass flows. As in the equilibrium case the interface meniscus assumes a radius of curvature satisfying (2), except that in the steady state  $p_v$  also depends on  $z$ . The two mass flows are coupled with the liquid temperature at the interface,  $T(z, r_v)$ , which in turn determines the vapor pressure of the liquid,  $p$ . The local condensation rate  $\frac{dm_l}{dz}$  is given by the gas kinetic formula

$$\frac{dm_l}{dz} = - \frac{dm_v}{dz} = \frac{\alpha r_v (p_v - p)}{\sqrt{RT/2\pi M}} \quad (16)$$

The numerical factor  $\alpha \approx 1$  includes both the probability of condensation of an impinging vapor molecule, and the "roughness" of the meniscus interface formed on the capillary structure. Equation (16) also applies for surface evaporation, but not for boiling evaporation, i.e., the formation of vapor bubbles within the capillary structure.

We now discuss the transport of energy. If  $\vec{q}$  is the energy flux, then in the steady state and in the absence of sources, conservation

of energy requires

$$\nabla \cdot \vec{q} = 0 \quad (17)$$

The convective and conductive contributions to the steady state heat flux are given by

$$\vec{q} = h\rho\vec{v} - k\nabla T \quad (18)$$

where  $h$  is the specific enthalpy of the fluid and  $k$  the thermal conductivity of the local medium. Net heat transport due to radiation ordinarily makes a negligible contribution in a heat pipe and it is therefore neglected. The total axial heat transport,  $Q(z)$ , is

$$Q(z) = \int_0^{r_p} q_z(z,r) 2\pi r dr \quad (19)$$

The desired approximation to (19) follows from a definition of the heat pipe regime. The device is operating in the heat pipe regime when, though the heat flow may be very large, the axial and radial temperature gradients throughout are very small, excepting only the radial temperature gradient in the container wall and wick. Using the axial component of (18) in (19), assuming the heat pipe regime prevails, so that axial conduction terms are small compared to convective terms, we have

$$Q(z) = \int_0^{r_v} h_v \rho_v v_z 2\pi r dr + \int_{r_v}^{r_w} h_\ell \rho_\ell v_z 2\pi r dr \quad (20)$$

The specific enthalpies of vapor and liquid depend on temperature

and are related by

$$h_v(T) = h_\ell(T) + L(T) \quad (21)$$

where  $L(T)$  is the heat of vaporization at temperature  $T$ . The vapor region is at nearly uniform temperature so  $h_v$  can be taken out of the first integral of (20). If we define a mean specific enthalpy of liquid,  $\bar{h}_\ell$ , by

$$\bar{h}_\ell = \frac{\int_{r_v}^{r_w} h_\ell \rho_\ell v_z 2\pi r dr}{\int_{r_v}^{r_w} \rho_\ell v_e 2\pi r dr} \quad (22)$$

then using (6), (7), (21) and (22), the expression for  $Q(z)$  becomes

$$Q(z) = \dot{m}_v \left[ 1 + (h_\ell - \bar{h}_\ell)/L \right] \quad (23)$$

Here  $h_\ell$  is the liquid specific enthalpy at the temperature of the vapor-liquid interface. Since  $|(h_\ell - \bar{h}_\ell)/L| < \Delta T/(L/c_\ell)$ , where  $\Delta T$  is the radial temperature difference across the wick and  $c_\ell$  is the specific heat of the liquid, and for liquids  $L/c_\ell \sim 10^3 \text{K}$ , the bracketed quantity will differ negligibly from unity in any reasonable case. Thus finally we have the somewhat obvious conclusion that the axial transport of energy is essentially entirely accomplished by the vapor convection of latent heat of condensation:

$$Q(z) = L \dot{m}_v(z) \quad (24)$$

The heat pipe is coupled to the external environment through the

net rate of heat addition per unit length of pipe,  $H = H(z, T_p, Q)$ . As indicated,  $H$  may depend on:  $z$  explicitly, when heat is added with a known distribution, as might be the case with electron bombardment or induction heating of the pipe surface;  $T_p = T(z, r_p)$ , the temperature of the external surface of the pipe, as in the case of radiation or conduction to a reservoir of specified temperature; and  $Q = Q(z)$ , as in the case of heating or cooling using parallel forced convection by an external fluid. In any case a prescription of the environment determines  $H$  as a known function of its arguments. Applying Gauss' theorem to (17) in a cylinder of radius  $r_p$  and length  $dz$ , and using (19), gives

$$\frac{dQ(z)}{dz} = -2\pi r_p q_r(z, r_p) = H(z, T_p, Q) \quad (25)$$

The radial heat flux through the container wall and wick to the vapor-liquid interface in the heat pipe regime is found by applying Gauss' theorem to (17) in an annulus,  $r_p \gg r \gg r_v$ , with thickness  $dz$ . Using (18) and (25), one may obtain the following relation:

$$2\pi r_v k_w \frac{\partial T(z, r)}{\partial r} \Big|_{r_v} = 2\pi r_p k_p \frac{\partial T(z, r)}{\partial r} \Big|_{r_p} \left[ 1 + (\bar{h}_\ell - h_\ell)/L \right] \quad (26)$$

The bracketed quantity again differs negligibly from unity, implying that convection contributes little to the radial transport of energy through the wick. The temperature at the outside of the container and the vapor-liquid interface are thus related by the standard result

for radial thermal conduction in a composite cylinder:

$$T_p = T_v + H/K \quad (27)$$

where

$$T_p = T(z, r_p) \quad ; \quad T_v = T(z, r_v)$$

$$\frac{1}{K} = \frac{1}{2\pi} \left( \frac{1}{k_p} \ln \frac{r_p}{r_w} + \frac{1}{k_w} \ln \frac{r_w}{r_v} \right)$$

The equations (2), (10), (16), (24), (25), (27) and one of (12), (13) or (15) as appropriate, provide a basis for the quantitative calculation of heat pipe properties.

## 6. Solution of the steady-state equations

Rather accurate approximate solutions of the equations of the preceding section can be obtained fairly simply. In the heat pipe regime the temperature is nearly uniform throughout the whole vapor space and the distribution of axial heat and mass flows differ little from what they would be if the vapor temperature was exactly a constant,  $T_0$ . If (27) is solved for  $T_p$ , we may express  $H$  in (25) as a function of  $z$ ,  $T_v$  and  $Q$ .

$$\frac{dQ}{dz} = H(z, T_v, Q) \quad (28)$$

The heat flows through the two ends of the pipe either are

negligible or at worst may be known functions of the local vapor temperature, which we denote by  $F_0(T_v)$  and  $F_\ell(T_v)$  respectively. The effective average temperature,  $T_0$ , and its associated axial heat flux distribution,  $Q_0(z)$ , are then obtained as the solution of

$$\frac{dQ_0}{dz} = H(z, T_0, Q_0) \quad (29)$$

$$Q_0(0) = F_0(T_0) \quad ; \quad Q_0(\ell) = F_\ell(T_0)$$

Since this is a two-point boundary value problem on a first order differential equation it can in general only be satisfied for particular values of  $T_0$ . In a physically well defined problem  $H(z, T_0, Q_0)$  will depend explicitly on  $T_0$ , and the value of  $T_0$  satisfying (29) will be unique.

With this good approximation to the heat flux we may obtain the vapor and liquid mass flows from (24) and (7)

$$\dot{m}_v(z) = -\dot{m}_\ell(z) = Q_0(z)/L(T_0) \quad (30)$$

The vapor mass flow in turn determines the distribution of pressure in the vapor, to within a constant, by integration of (12), (13) or (15) as appropriate. Using  $\dot{m}_v(z)$  and  $p_v(z)$  in (16) then determines the vapor pressure of the liquid to within an additive constant. Consistent with the accuracy of the calculation, this constant may be taken as  $p(T_0)$ . Since the vapor pressure is a known function

of the liquid surface temperature, this determines  $T_v(z)$ . The self-consistency of the approximate calculation is verified if the total variation of  $T_v(z)$ , found in this way, is small compared to  $T_0$ . Finally, the liquid mass flow determines the axial distribution of pressure in the liquid by integration of (10). Throughout the foregoing all the temperature dependent properties, with the exception of the vapor pressure, are sufficiently slowly varying that they may be taken as constants evaluated at  $T_0$ .

We now obtain the total pressure and temperature variations along a heat pipe for a particular, but rather commonly met case: constant heat addition along the evaporator, and constant heat removal along the condenser. Thus

$$Q_0(z) = L \dot{m}_v(z) = \begin{cases} \frac{z}{l_e} Q_e & ; 0 \leq z \leq l_e \\ \frac{l-z}{l-l_e} Q_e & ; l_e \leq z \leq l \end{cases} \quad (31)$$

where  $l_e$  is the length of the evaporator and  $Q_e$  is the total heat input to the evaporator. Integrating (12) and (13), neglecting the term in  $R_r^2$  in the former, and assuming  $\rho_v$  is constant in both, gives

$$\Delta p_v = p_v(l) - p_v(0) = \begin{cases} -\frac{4\eta_v l Q_e}{\pi \rho_v r_v^4 L} & ; R_r \ll 1 \\ -\frac{(1-4/\pi^2) Q_e^2}{8 \rho_v r_v^4 L} & ; R_r \gg 1 \end{cases} \quad (32)$$

From (16) one obtains

$$\Delta p = p(T_v(l)) - p(T_v(0)) = \Delta p_v - \frac{l Q_e \sqrt{RT_0/2\pi M}}{l_e(l-l_e)L \alpha r_v} \quad (33)$$

It is a requirement for the heat pipe regime that both  $\Delta p_v$  and  $\Delta p$  be significantly smaller in magnitude than  $p(T_0)$ . For the small pressure differences occurring in the heat pipe regime the Clapeyron-Clausius equation may be used to calculate the temperature difference:

$$\Delta T_v = T_v(l) - T_v(0) = \frac{RT_0^2}{ML p(T_0)} \Delta p \quad (34)$$

The liquid pressure differences, found by integrating (10), is

$$\Delta p_l = p_l(l) - p_l(0) = \rho_l g \sin \phi + \frac{b \eta_l Q_c l}{2\pi(r_w^2 - r_v^2) \rho_l e r_c^2 L} \quad (35)$$

As an illustration of the magnitudes of pressure and temperature drops typically encountered, we cite an experimental horizontal liquid sodium heat pipe which was reported in the original description of these devices.<sup>1</sup> The relevant specifications are given in the left

column and various derived properties in the right:

$Q_e = 500$ watts	$m_v(\ell_e) = 0.1$ gm/sec
$T_o = 920^\circ\text{K}$	$p(T_o) = 50$ mm Hg
$\ell = 90$ cm	$\Delta p_v = -0.2$ mm Hg
$\ell_e = 13$ cm	$\Delta p = -0.5$ mm Hg
$r_v = .64$ cm	$\Delta p_\ell \approx 2$ mm Hg
$r_w = .80$ cm	$\Delta T_v = -0.7$ K <small>See Attached Errata Sheet</small>
$r_c = .012$ cm	

The main features of a working heat pipe are evident here: The transport of considerable heat is accomplished by the circulation of a small amount of working fluid; this circulation requires but small pressure differences; and the accompanying temperature difference is so small that its precise magnitude is not of importance in practical applications.

#### 7. Maximum heat flux

While the thermal conductance of a heat pipe is very large there are, however, limitations on the magnitudes of both the total and local energy fluxes.

The total axial heat transport may increase only if the force of capillary origin can sustain the required circulation of fluid. As previously noted, the maximum difference in pressure between vapor and adjacent liquid that can be supported by surface tension in the

capillary structure is  $(2\gamma \cos \theta)/r_c$ . It is therefore necessary that

$$p_v(z) - p_l(z) \leq \frac{2\gamma \cos \theta}{r_c} \quad ; \quad 0 \leq z \leq l \quad (36)$$

In the general the largest pressure difference will occur at the beginning of the evaporator section,  $z = 0$ . If the heat pipe has been prepared with a fully saturated wick, the vapor-liquid interface meniscus will have large radius of curvature at the terminal end of the condenser and, therefore,  $p_v(l) \approx p_l(l)$ . Thus if (36) is satisfied at  $z = 0$ , it will be satisfied for all  $z$ .

For the particular case of uniform heat addition and removal we may use (32) and (35) together with the preceding remarks to write down explicitly the limiting condition on the total axial heat flux:

$$\left. \begin{array}{l} \frac{4\eta_v l Q_e}{\pi \rho_v r_v^4 L} \\ (1 - 4/\pi^2) Q_e^2 \\ \frac{8\eta_v r_v^4 L}{\pi \rho_v r_v^4 L} \end{array} \right\} + \rho_l g l \sin \phi + \frac{b\eta_l Q_e l}{2\pi(r_w^2 - r_v^2)\rho_l e r_c^2 L} \leq \frac{2\gamma \cos \theta}{r_c} ; \quad \left\{ \begin{array}{l} R_r \ll 1 \\ R_r \gg 1 \end{array} \right. \quad (37)$$

There is a further limitation on the local radial heat flux in the evaporator section of the heat pipe. The liquid in the interior

of the wick here is necessarily superheated. We should therefore expect that the limitation will be closely connected with the conditions for the onset of boiling in the capillary structure and the quantitative properties of the subsequent evaporation and heat transfer. The problems here are more complex even than those encountered in pool boiling of liquids, for in addition to the local limitations of boiling heat transfer, the undoubtedly deleterious interaction of boiling with the overall circulation of liquid throughout the capillary structure will be important. There is no generally useful experimental information available yet on this problem.

We can, however, give a conservative criterion for the nonoccurrence of boiling. The onset of boiling may be characterized by a critical bubble radius of curvature,  $r_b$ , which depends on the nature and geometry of the interface where bubbles nucleate. If the difference between the pressure of the vapor in the bubble and the pressure in the surrounding liquid is less than  $2\gamma/r_b$ , then the bubble will collapse. In a nucleating bubble the pressure in the vapor cannot exceed  $p(T(z,r))$ , the equilibrium vapor pressure of the liquid at the local temperature. Furthermore, in the capillary structure  $r_b$  cannot exceed  $r_c \sec \theta$ . Thus as long as

$$p(T(z,r)) - p_l(z,r) \leq \frac{2\gamma \cos \theta}{r_c} \quad (38)$$

bubbles cannot grow beyond the critical size, and true boiling will not

occur. Of course, there may indeed be no boiling under more severe conditions than given by this criterion.

### 8. Multi-component fluids

We consider the steady-state behavior of a heat pipe in which a mixture of liquids rather than a single pure compound is used as the working fluid. Conservation of mass again requires that

$$\dot{m}_v(z) + \dot{m}_\ell(z) = 0 \quad (7)$$

The mass of each component individually must also be conserved.

Letting  $f_v(z)$  and  $f_\ell(z)$  be the mass fractions of some designated component in vapor and liquid phases respectively, then

$$f_v(z)\dot{m}_v(z) + f_\ell(z)\dot{m}_\ell(z) = 0 \quad (39)$$

Using (7) to eliminate  $\dot{m}_\ell(z)$

$$\left[ f_v(z) - f_\ell(z) \right] \dot{m}_v(z) = 0 \quad (40)$$

This can be satisfied only if  $\dot{m}_v(z) = 0$  or  $f_v(z) = f_\ell(z)$ . The first alternative implies no local refluxing. If we assume that the steady state is close to thermodynamic equilibrium then the second alternative can only be met in one of three special ways: (a)  $f_v = f_e = 1$ , that is, only the pure component is present locally in both phases; (b)  $f_v = f_e = 0$ , that is, the designated component is locally totally absent; (c) we are dealing with the very special case of a constant

boiling mixture, which may in fact be regarded as constituting a new pure component. If none of the foregoing cases holds, then gross local thermodynamic non-equilibrium is implied.

If the heat pipe is originally charged with homogeneous liquid mixture, then in the early transient heat transport, the vapor leaving the evaporator will be richer in the more volatile components than the returning liquid, which thus tends to concentrate the less volatile components in the evaporator and the more volatile components in the condenser. Now, there is an essentially completely fractionated steady-state distribution of components which is consistent with this trend, with the previously enumerated near-equilibrium alternatives, and with the heat pipe dynamics which require very small pressure gradients throughout. This steady state consists of a series of segments each containing a pure component which is refluxing as an independent heat pipe. The components are arranged in order of increasing volatility with the most volatile at the terminal end of the condenser. The temperature distribution forms a series of plateaus, with the plateau temperatures decreasing in order of increasing volatility of the local component, in such a way that the pressure within the vapor is nearly constant throughout the entire pipe. Between the segments there are short transition zones of rapidly varying temperature within which there is no refluxing, the entire axial heat transport occurring by ordinary thermal conduction,

mainly through the container wall and wick. Along each temperature transition zone the stagnant two-component liquid and vapor phases vary in equilibrium concentrations in a way consistent with constancy of total vapor pressure. Since the distribution of density is not uniform, gravitationally induced convection, particularly of the vapor phase, may modify this distribution considerably.

If the heat pipe contains some non-condensable gas of low solubility in the working fluid, the foregoing applies equally well. It has been demonstrated<sup>1</sup> that in the steady state the gas is driven to the terminal end of the condenser where it forms a stagnant zone. The length of this zone is proportional to the mass of gas and to the mean temperature in the zone, and is inversely proportional to the pressure of the vapor in the refluxing section of the heat pipe. This feature might be useful. In such a heat pipe the working length of the condenser increases as the heat input to the evaporator is increased.

#### 9. Optimal heat pipes

A heat pipe will commonly be required to transport the largest possible amount of heat, subject to whatever subsidiary constraints arise in the particular application. In this case the maximum heat flux criterion (36) applies, in the form

$$\Delta p_l - \Delta p_v - \frac{2\gamma \cos \theta}{r_c} = 0 \quad (41)$$

If the subsidiary constraints do not involve the capillary pore size,  $r_c$ , then its optimum value may be deduced from (41) alone. According to (10) the viscous contribution to  $\Delta p_\ell$  is inversely proportional to  $r_c^2$ . The standard extremizing procedure then yields and the result that  $r_c$  should be so chosen that the viscous contribution to  $\Delta p_\ell$  is one-half the magnitude of the capillary pressure term,  $(2\gamma \cos \theta)/r_c$ . For example, in the case of uniform heat addition and removal,  $\Delta p_\ell$  is given by (35), and the optimum choice of  $r_c$  is

$$r_c = \frac{b\eta_\ell Q_e \ell}{4\pi(r_w^2 - r_v^2)\rho_\ell eL \gamma \cos \theta} \quad (42)$$

If the hydrostatic contribution to  $\Delta p_\ell$  is absent, and if the ratio  $r_v/r_w$  is not constrained by the subsidiary conditions, then the optimum value of  $r_v/r_w$  is  $2/3$ . This follows by noting that if  $\Delta p_v$  is obtained either from (12) or (13) as appropriate, and the optimal value of  $r_c$  is used for  $\Delta p_\ell$  in (41) then the greatest heat transport corresponds to the maximum value of  $r_v^4(r_w^2 - r_v^2)$ . Under these transport is found to be

$$Q_e = \begin{cases} \frac{\pi r_w^2 L \gamma \cos \theta}{3\ell} \left( \frac{e \rho_v \rho_\ell}{3b\eta_v \eta_\ell} \right)^{1/2} & ; R_r \ll 1 \\ \frac{4\pi r_w^2 L}{3} \left( \frac{2\rho_v \rho_\ell e \gamma^2 \cos^2 \theta}{(\pi^2 - 4)b\ell \eta_\ell} \right)^{1/3} & ; R_r \gg 1 \end{cases} \quad (43)$$

If, for example, a fixed total volume of wick is distributed in a uniformly heated evaporator, so as to minimize the liquid viscous pressure drop in this section of the pipe, the cross-sectional area of wick should be proportional to  $\sqrt{z}$ , and the pressure drop is 8/9 of that of the wick of constant thickness.

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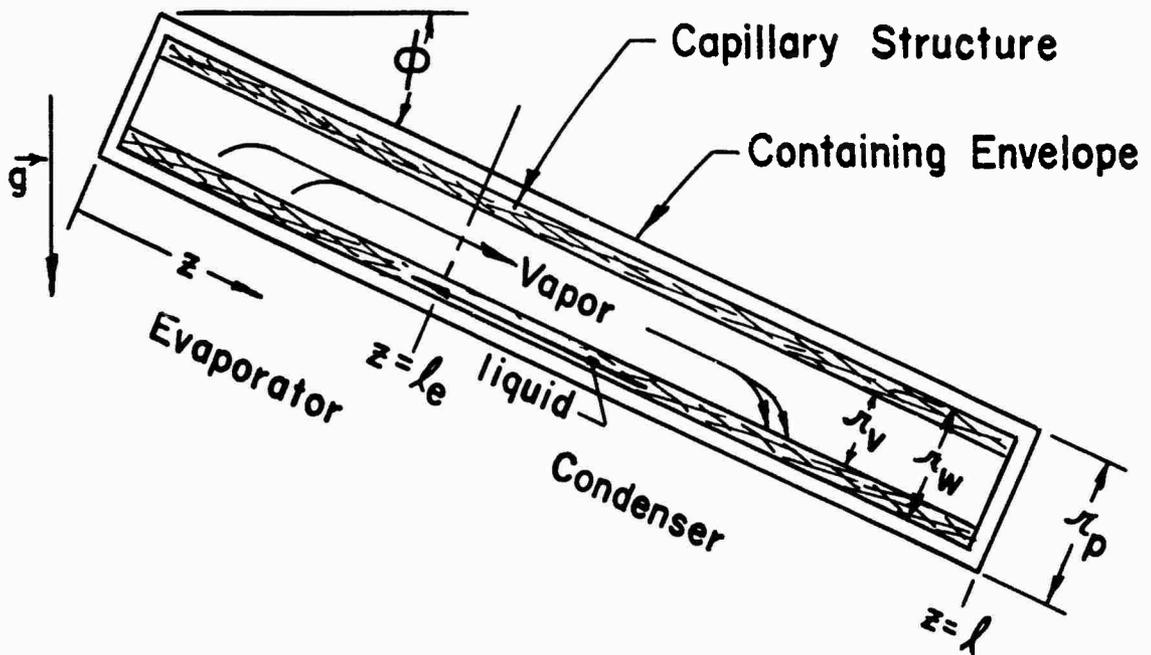


Fig. 1. Cylindrical Heat Pipe Structure

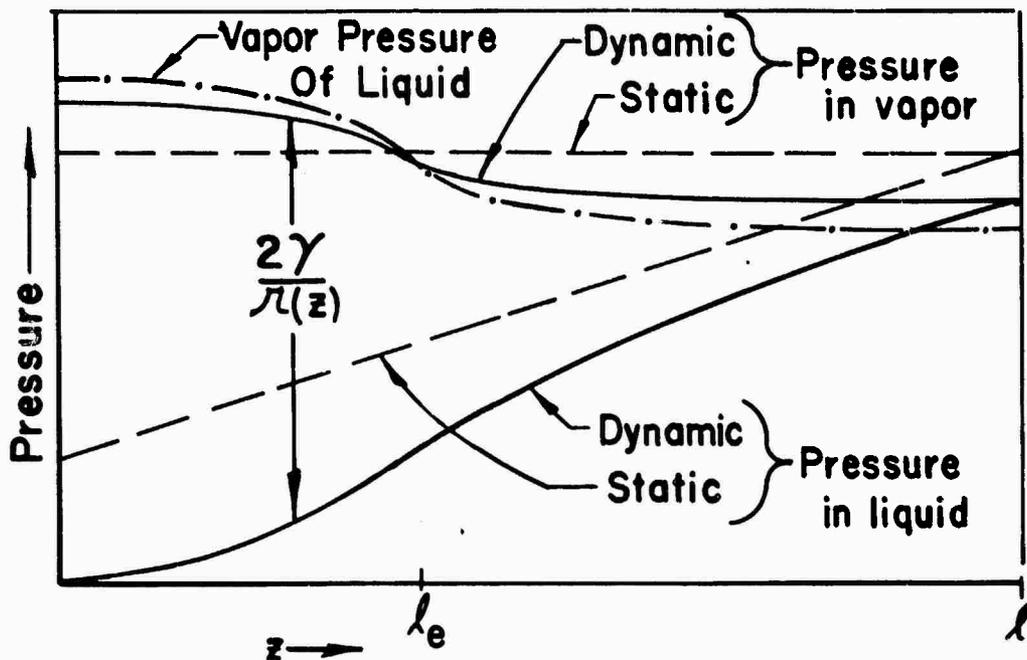


Fig. 2. Distribution of Pressures in a Heat Pipe