Mathematical Perspectives on the Federal Thrift Savings Plan (TSP)

Military Operations Research Society Symposium
Working Group 20 (Manpower & Personnel)
June10, 2008

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Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **01 JUN 2008**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**
4. TITLE AND SUBTITLE **Mathematical Perspectives on the Federal Thrift Savings Plan (TSP)**		5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)		5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Operations Research Center U.S. Military Academy at West Point**		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR'S ACRONYM(S) 11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
See also ADM202527. Military Operations Research Society Symposium (76th) Held in New London, Connecticut on June 10-12, 2008, The original document contains color images.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**UU**	**49**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

76th MORSS
10-11-12 JUNE 2008

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Disclaimer

- ***You will not receive any personal financial advice during this talk, as I am not officially qualified or certified to do so.***

- ***However, my presentation is intended to get you to think mathematically about one of the retirement savings options available to many of you.***

Questions for Consideration

- Why might I be more risk tolerant than I currently believe?
- What are the L (Lifecycle) funds? How are they constructed? Why might they be of interest (or not) to me?
- What if stock and index fund returns are not normally distributed, as is commonly assumed?
- How does the choice of reward and risk measures affect optimal TSP portfolios?

Big Picture on Saving $$$

"There is no scholarship for retirement!"

-Unknown

- Spouse's 401(k) with matching funds
- Roth IRA (for Soldier/civilian and spouse)
- Thrift Savings Plan (TSP)
- Spouse's 401(k) without matching funds
- Coverdale Educational Savings Accounts
- 529 Tuition Plans (prepaid or savings)

(ordering of these depends on tax considerations)

Thrift Savings Plan (TSP) Overview

- Largest defined contribution retirement savings and investment plan
  - 3.7 million participants
  - $210 billion in assets
- 401(k) equivalent for government employees and uniformed service members
- 5 non-traded core funds
- Can rebalance daily with no direct costs

Core TSP Funds

"RISKY" ASSETS: F, C, S, I

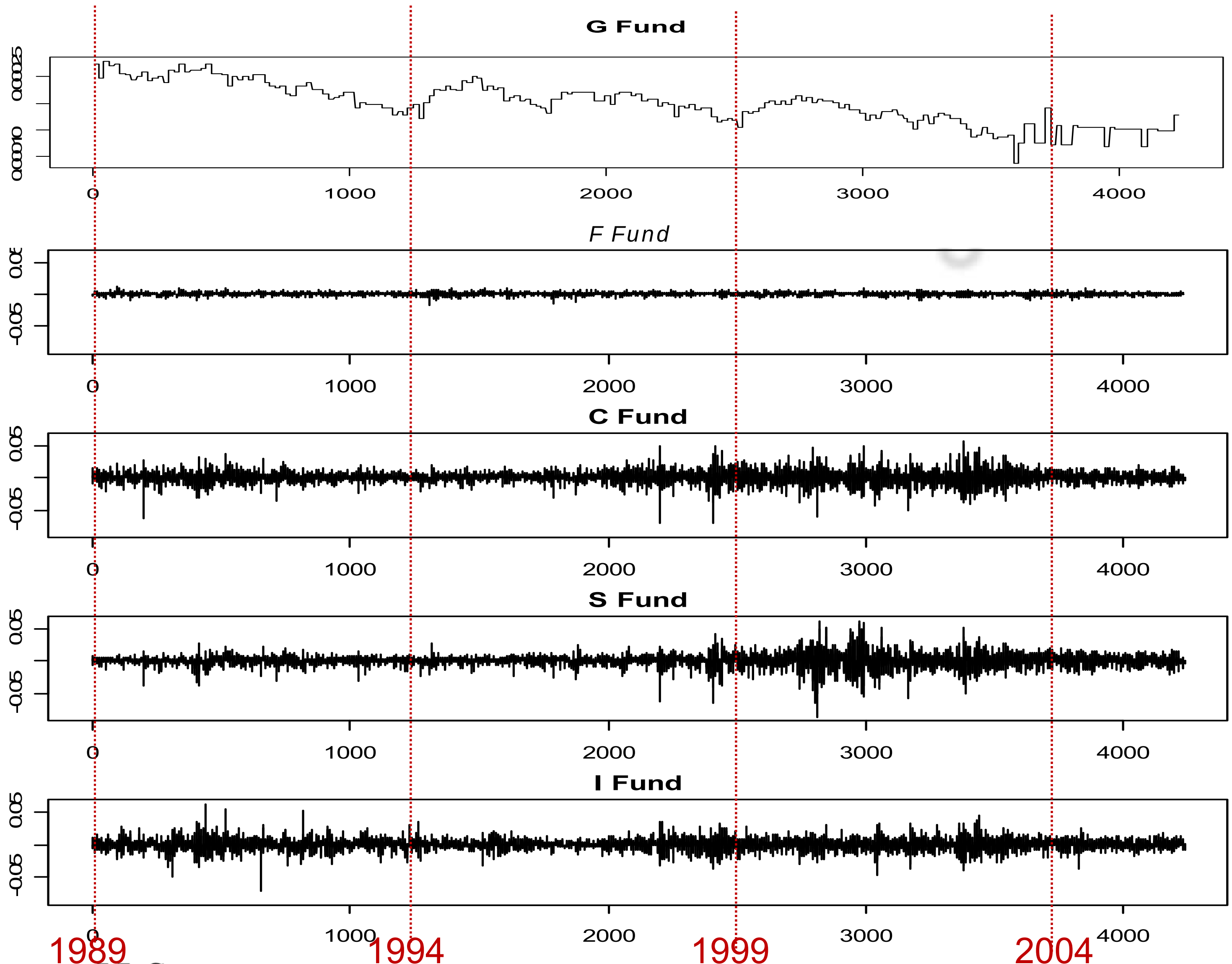
Fund	Description	Assets*	Mean Return#	Standard Deviation#
G	short-term, specially issued Treasury securities	$66.6B (39.2%)	6.4%	1.5%
F	tracks Lehman Brothers U.S. Aggregate (LBA) Index	$10.2B (6.0%)	7.3%	5.6%
C	tracks S&P 500 Index	$66.7B (39.3%)	13.0%	17.9%
S	tracks Dow Jones Wilshire 4500 Completion Index	$13.7B (8.1%)	13.3%	19.9%
I	tracks MSCI EAFE (Europe, Australia, Far East) Index	$12.6B (7.4%)	7.8%	18.7%

* As of Dec 31, 2005 # For the period 1988-2005

Returns / Investment Horizon

- Returns
  - Arithmetic: $$r_{i,t} = \frac{S_{i,t} - S_{i,t-1}}{S_{i,t-1}}$$
  - Log : $$r_{i,t} = \ln(S_{i,t}) - \ln(S_{i,t-1}) = \ln\left(\frac{S_{i,t}}{S_{i,t-1}}\right)$$
- Investment Horizon: 20 years
  - Point of military (not ultimate) retirement
  - System encourages 20 year careers
  - Employment options vary greatly
  - Can move TSP assets to other plans

Daily Returns Time Series



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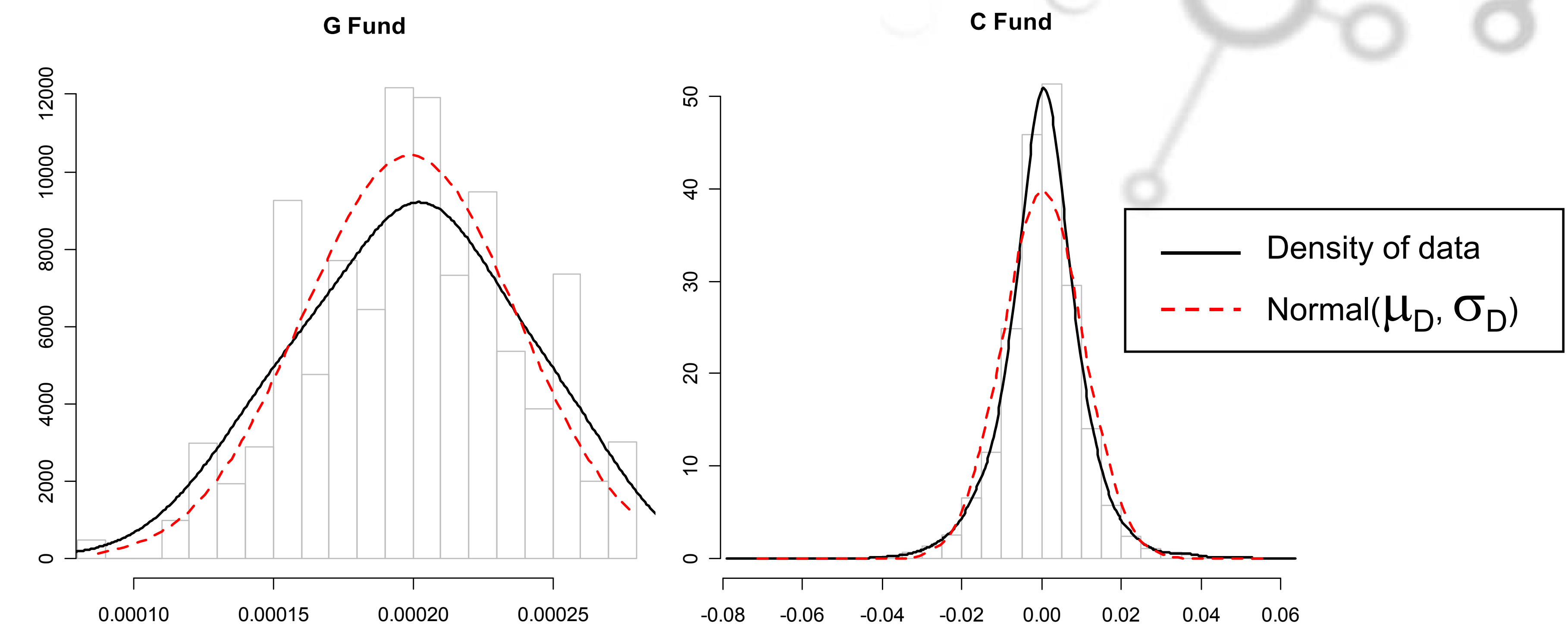
Daily Returns Distributions



- G fund appears approximately Gaussian
- C, S, I, and F funds are more peaked with heavy tails
- Goodness of Fit testing at common levels of significance rejects Normal for F, C, and S funds, even with batched means

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Mean-Variance Portfolio Optimization
(Markowitz, 1952)

Let X_i = fraction of funds invested in asset *i*

$\bar{R}_i$ = expected return of asset *i*

$\bar{R}_P$ = expected return of portfolio *p*

σ_i^2 = variance of return of asset *i*

σ_{jk} = covariance of return of asset *j* with asset *k*

Minimize
$$\sigma_P^2 = \sum_{j=1}^{N} X_j^2 \sigma_j^2 + \sum_{j=1}^{N} \sum_{\substack{k=1 \\ k \neq j}}^{N} X_j X_k \sigma_{jk}$$

Subject to:
$$\sum_{i=1}^{N} X_i = 1$$

$$\bar{R}_P = \sum_{i=1}^{N} X_i \bar{R}_i$$

$$X_i \geq 0, \quad i = 1, \ldots, N$$

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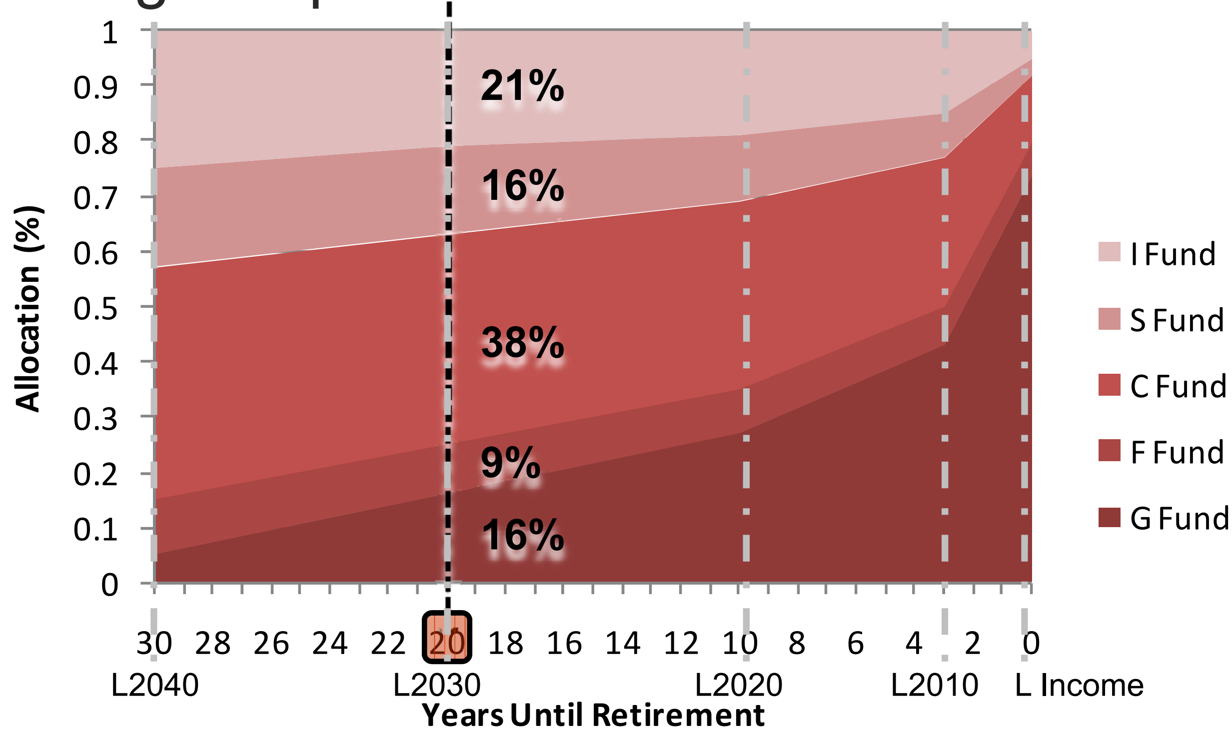
L (Lifecycle) Funds

- Invest in 5 core TSP funds based on time horizon to provide highest possible rate of return for risk taken.



- Over time, investments shift away from stocks and into bonds.
- L Funds are great, but …

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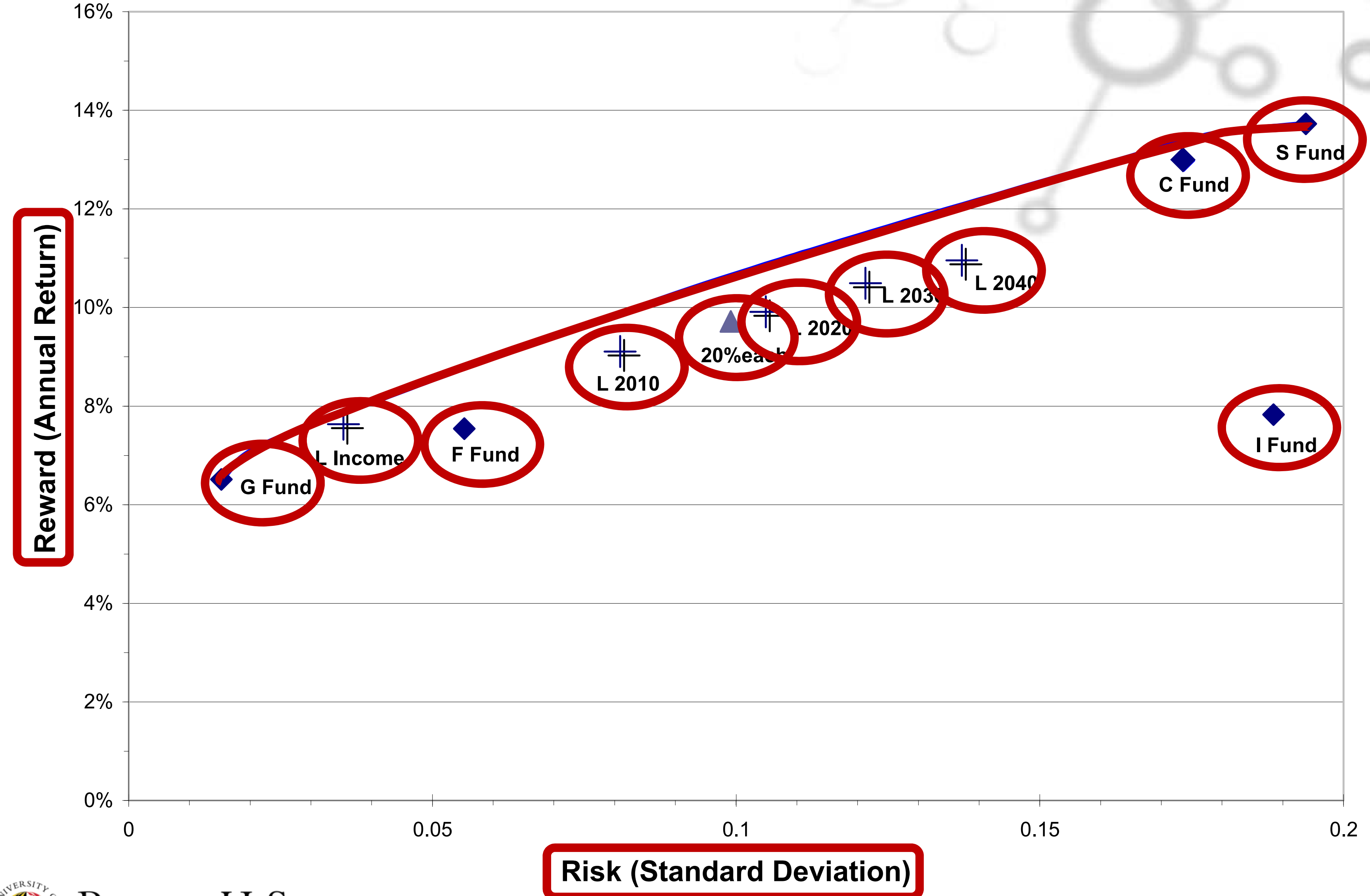
Reward-Risk Profile of TSP Funds



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VG-ICA Factor Model

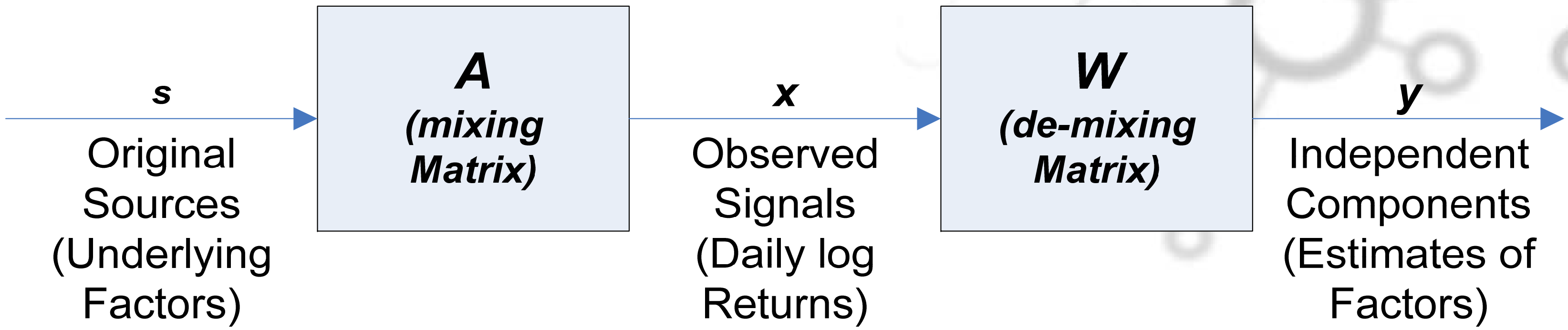
(Madan & Yen, 2004)

$$(R - \mu) = XB + \varepsilon$$

$$D = XB + \varepsilon$$

1. Use **Independent Component Analysis (ICA)** on asset returns D to identify underlying factors X
2. Fit the **Variance Gamma (VG)** distribution to each retained factor by MLE
3. Use Expected Utility to determine optimal portfolio of VG-ICA factors; convert back to optimal portfolio of assets (TSP funds)

Independent Component Analysis (ICA)



- Principal Component Analysis (PCA)
  - Focus on finding **uncorrelated** components in Gaussian data
  - Maximizes explained variance
  - Uses second-order statistics
- Factor Analysis
  - Essentially PCA with extra terms to model noise
- ICA
  - Focus on **independent** and non-Gaussian components
  - Maximizes non-Gaussianity (to maximize information)
  - Uses higher-order statistics

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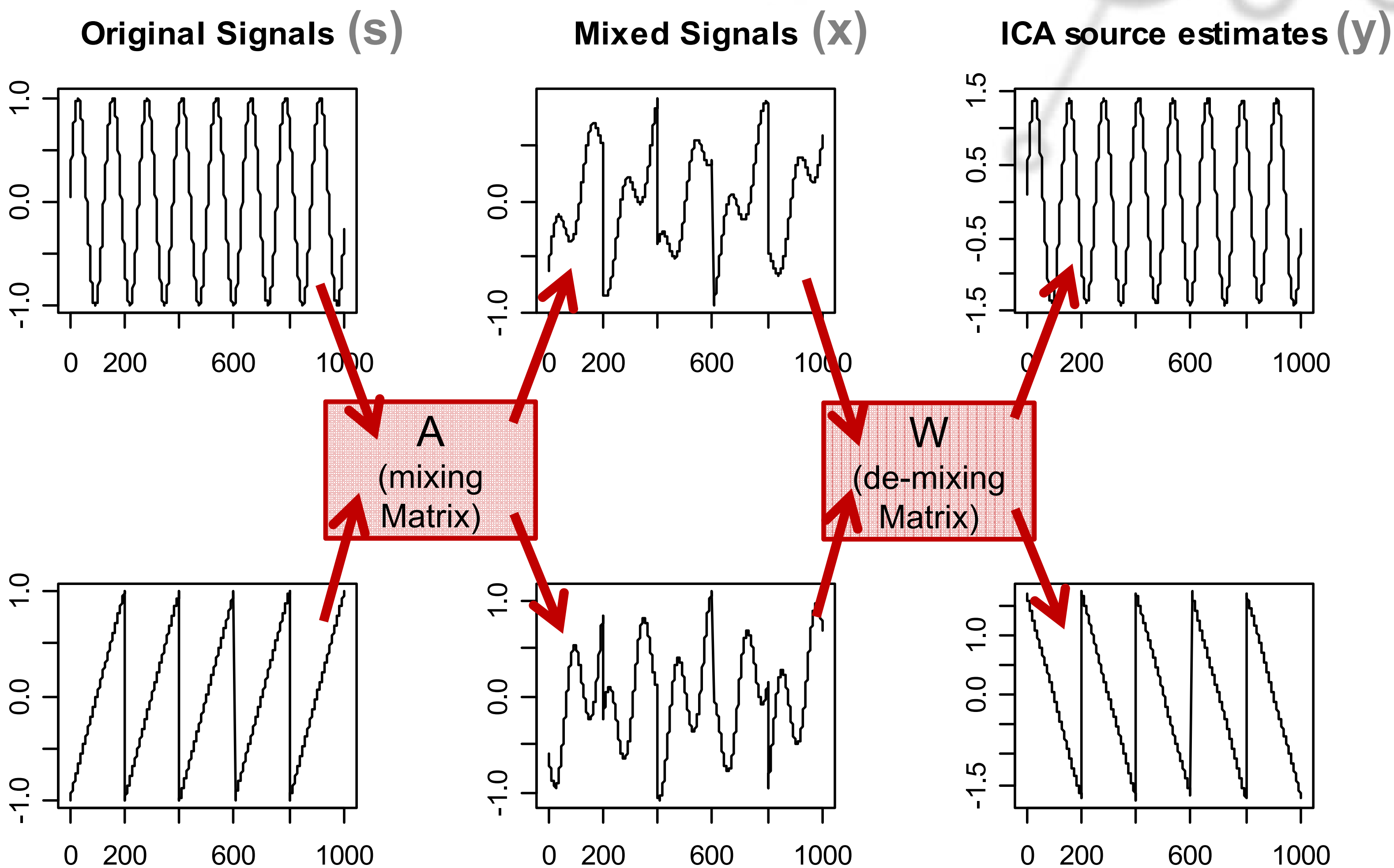
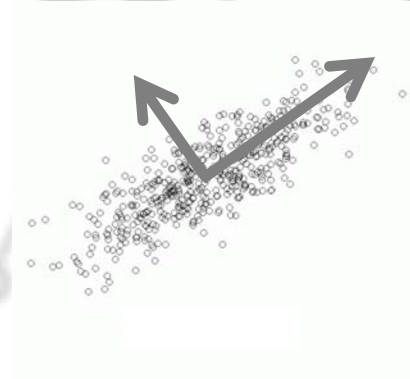
Another ICA Example

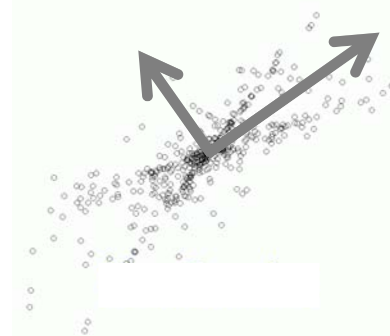


ICA versus PCA

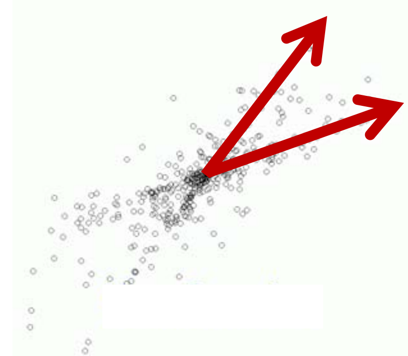
- **Principal Component Analysis** (PCA) finds:
  - directions of maximal variance in Gaussian data (second-order statistics).



  - directions of maximal variance in non-Gaussian data (second-order statistics).



- **Independent Component Analysis** (ICA) finds directions of maximal independence in non-Gaussian data (higher-order statistics).



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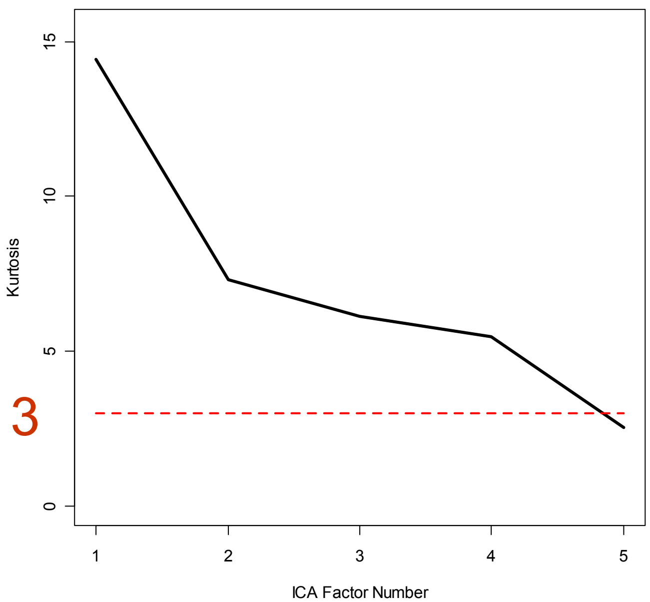
How Many ICs to Keep?

"Scree" Plot



R^2 Values from Regression

# ICs Kept	F Fund	C Fund	S Fund	I Fund
5	1	1	1	1
4	.9980	.9994	.9998	.9949
3	.0611	.8983	.9652	.9869
2	.0611	.8962	.9629	.1276

- Dropping more than one IC reduces fit on at least one fund
- The first four have excess kurtosis

→ Keep 4 Independent Components (ICs)

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VG Process and Distribution

- **Pure jump process** with two representations
  - Time-changed Brownian motion (Madan & Seneta, 1990)

$$X_{VG}(t;\nu,\theta,\sigma) = b(\gamma(t;1,\nu),\theta,\sigma)$$

  - Difference of 2 Gamma processes (Madan, Carr & Chang, 1998)

$$X_{VG}(t) = G_p(t) - G_n(t)$$

- **Parameters:** σ controls spread
  ν affects kurtosis
  θ impacts skewness

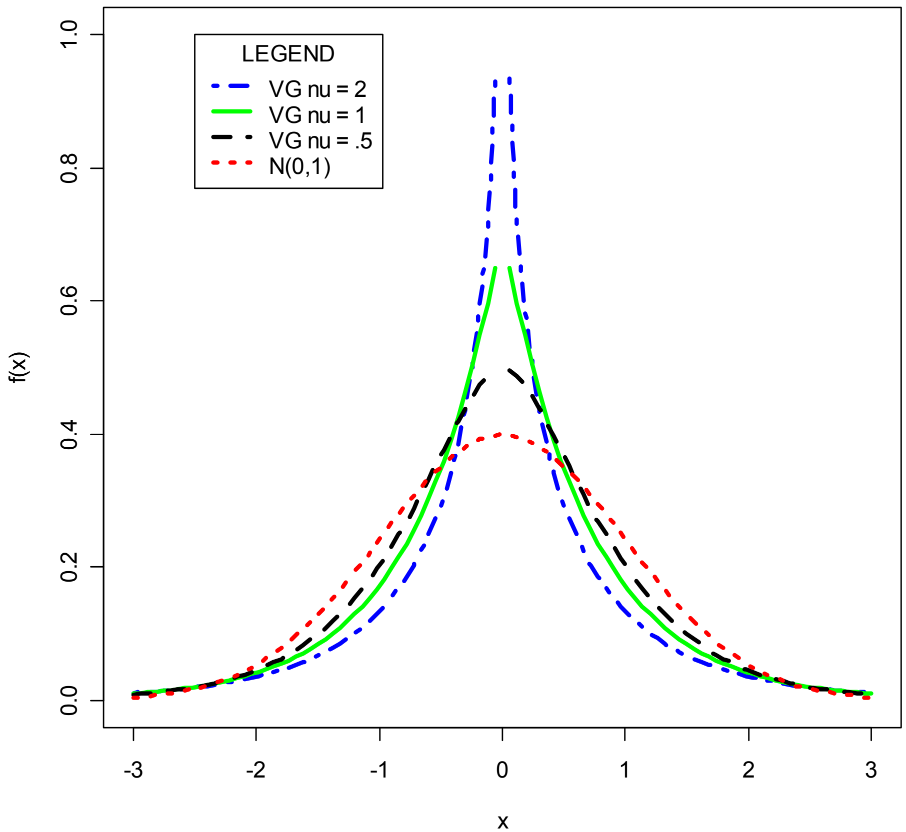
- **Density Function** (Madan, Carr & Chang, 1998)

$$h(z) = \frac{2\exp(\theta x/\sigma^2)}{\nu^{1/\nu}\sqrt{2\pi}\sigma\Gamma(\frac{1}{\nu})}\left(\frac{x^2}{2\sigma^2/\nu+\theta^2}\right)^{\frac{1}{2\nu}-\frac{1}{4}} K_{\frac{1}{\nu}-\frac{1}{2}}\left(\frac{1}{\sigma^2}\sqrt{x^2(2\sigma^2/\nu+\theta^2)}\right)$$

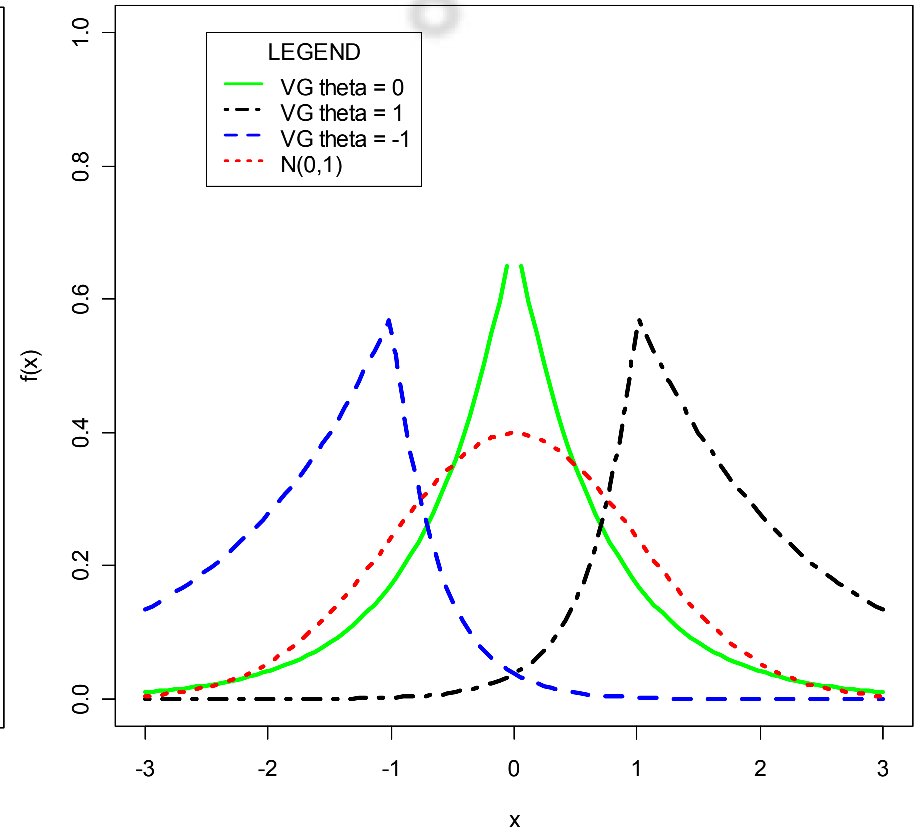
with $x = z - \theta$ where $z = \ln(S(t)/S(t-1))$

Examples of VG Distributions

Effect of ν



Effect of θ



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Fitting VG by MLE

- Given observed IID data $X_1, X_2, \ldots, X_n$, define the **likelihood function** as:

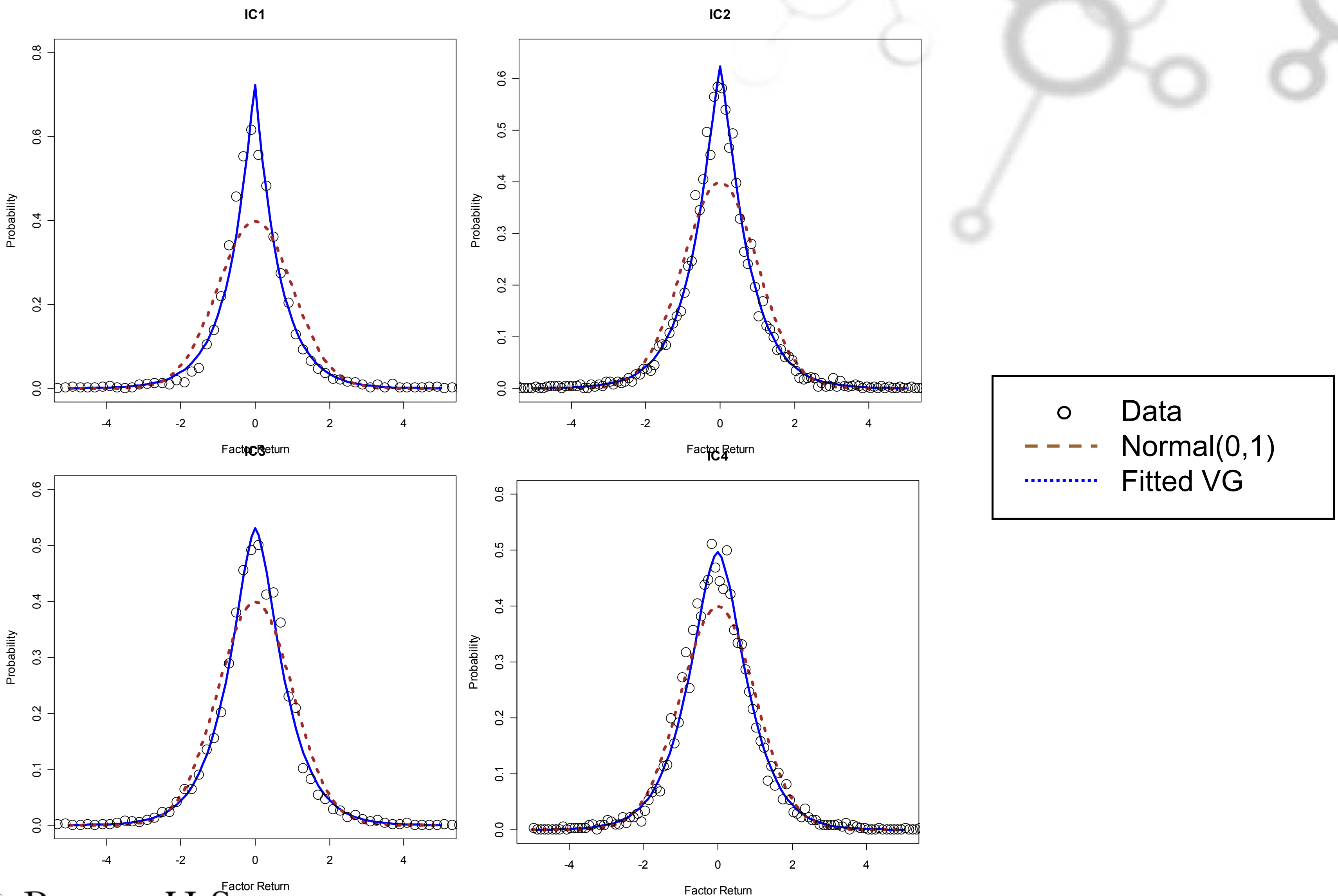
$$L(\theta) = f_\theta(X_1) f_\theta(X_2) \cdots f_\theta(X_n)$$

- The **MLE (maximum likelihood estimator)** $\hat{\theta}$ maximizes L(θ) over all permissible values of θ.
- Actually, maximizing the **log likelihood function** *ln*(L(θ)) is easier
- For the VG distribution with three parameters, this becomes:

$$l(\sigma, \nu, \theta) = \ln L(\sigma, \nu, \theta) = \sum_i f_{(\sigma, \nu, \theta)}(X_i)$$

(using pdf given before from Madan, Carr, and Chang, 1998)

Comparison of Fitted VG and Normal(0,1)



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Fitted VG Parameters / Chi-Square Statistics

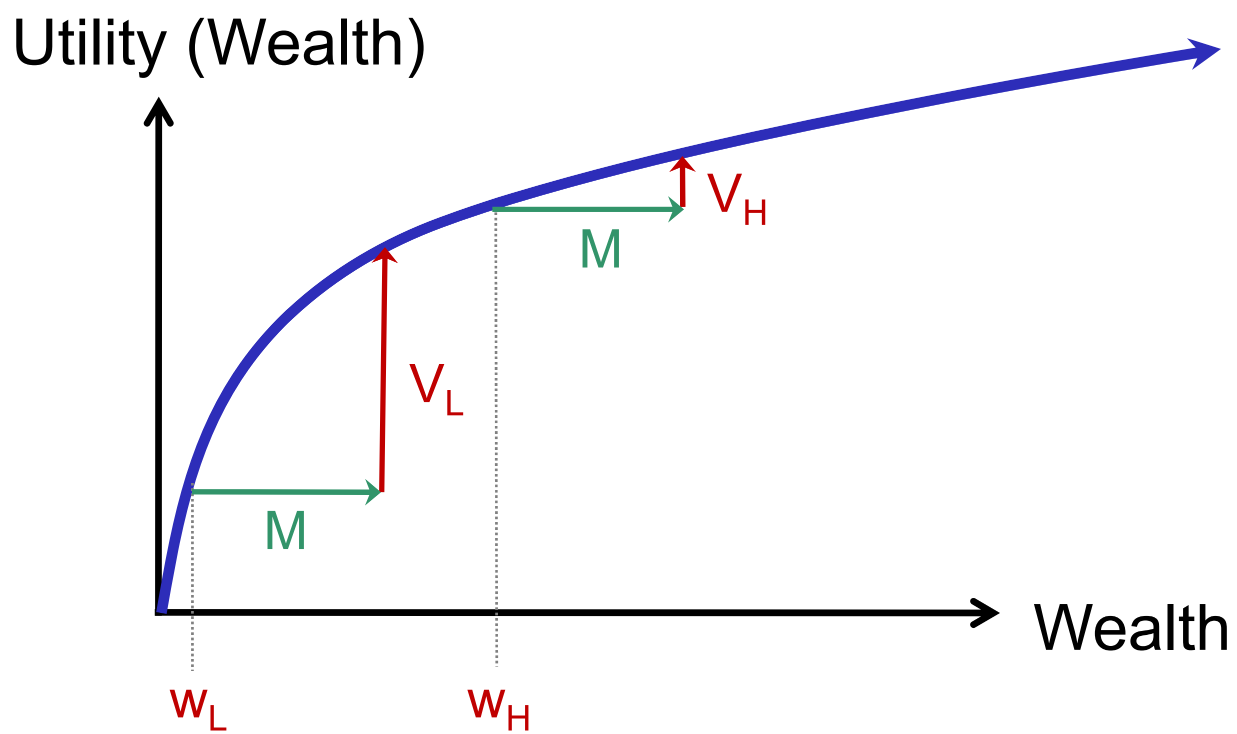
IC#	Fitted VG Parameters - Daily (Annualized)			X^2 Test Statistic (p-values) ($X^2_{.01,17} = 33.41$)	
	σ	*v*	θ	VG(σ,v,θ)	N(0,1)
IC1	0.933 (14.814)	0.969 (.00385)	-0.015 (-3.773)	99.01 (8.08E-13)	546.89 (7.83E-104)
IC2	0.980 (15.558)	0.820 (.00326)	-0.001 (-0.222)	13.37 (0.82)	295.73 (1.50E-51)
IC3	0.989 (15.703)	0.586 (.00232)	-0.004 (-1.019)	34.94 (0.014)	171.70 (1.32E-26)
IC4	0.991 (15.739)	0.468 (.00186)	-0.005 (-1.149)	25.29 (0.151)	118.07 (2.54E-16)

- Some excess kurtosis and slight negative skewness in each IC
- VG fits much better than Normal distribution

Utility Theory & Risk Aversion

Utility- a measure of relative satisfaction obtained

Risk Aversion- concave utility function, as shown below



Aside on Risk Aversion/Tolerance
(Jennings & Reichenstein, 2001)

- Pensions considered when planning retirement income….. but NOT when calculating asset allocation
- Pensions and investment portfolio generate retirement funds; why not consider both in total portfolio?
- Many similarities between inflation-indexed Treasury bonds (TIPS) and military retirement
  - Linked to Consumer Price Index (CPI)
  - Backed by federal government
- Suggest treating after-tax present value as a "pseudo-bond" in total portfolio
- Discounting can be at recent TIPS rates (3%-5%) or higher personal discount rate (18+%)
- Results in more aggressive (risk tolerant) portfolio in active investments than would otherwise result

NPV of Military Retirement

(Jennings & Reichenstein, 2001)

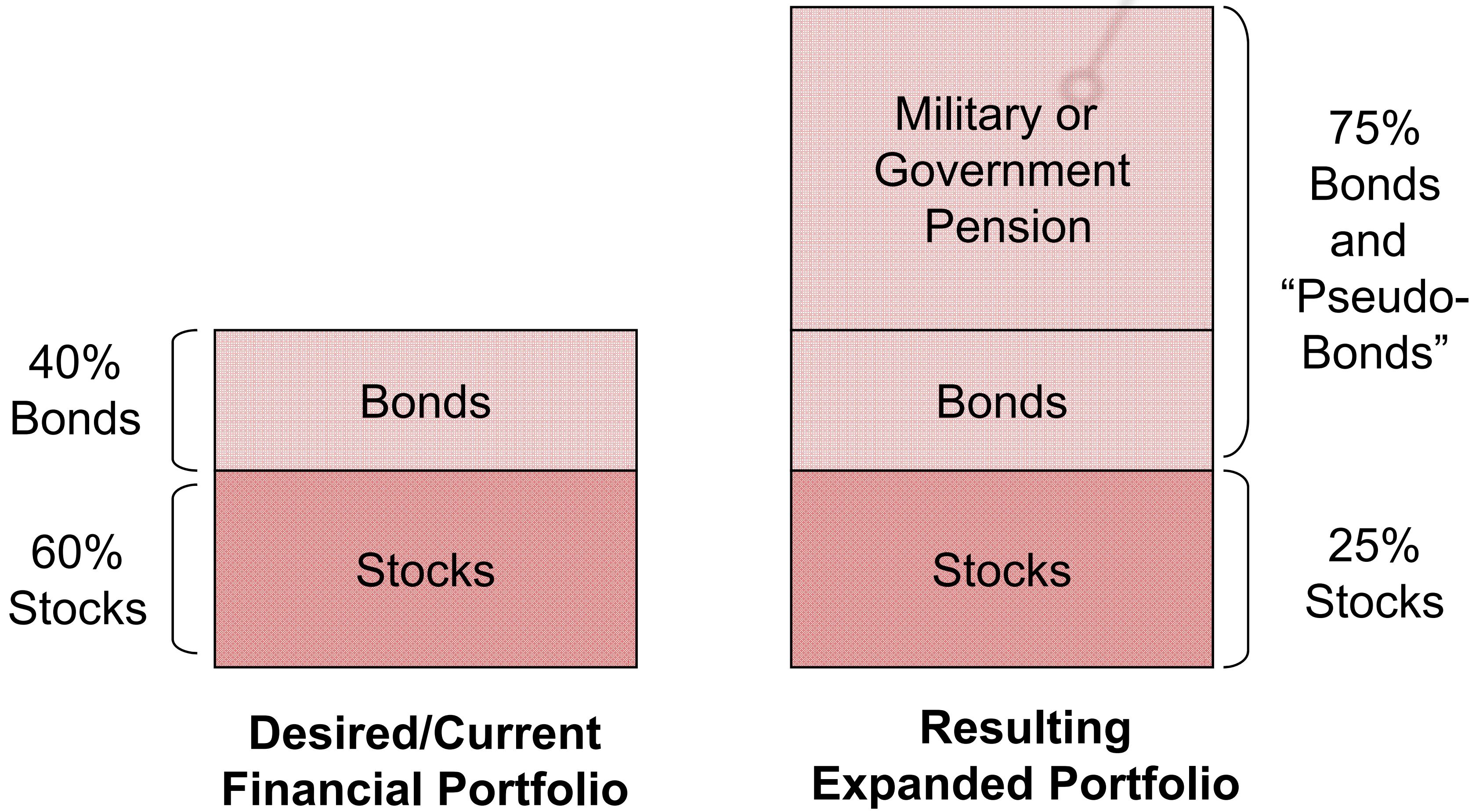
Rank at Retirement	Years of Service	After-Tax NPV
LTC	20	$726,674
LTC	22	$802,690
COL	24	$994,468
COL	26	$1,096,490
COL	28	$1,166,125
COL	30	$1,205,255

Assumptions:

- Officer currently at 18 years of service
- 28% tax bracket
- 4% TIPS rate / inflation

“Pseudo-Bond” Example

(Nestler, 2007)

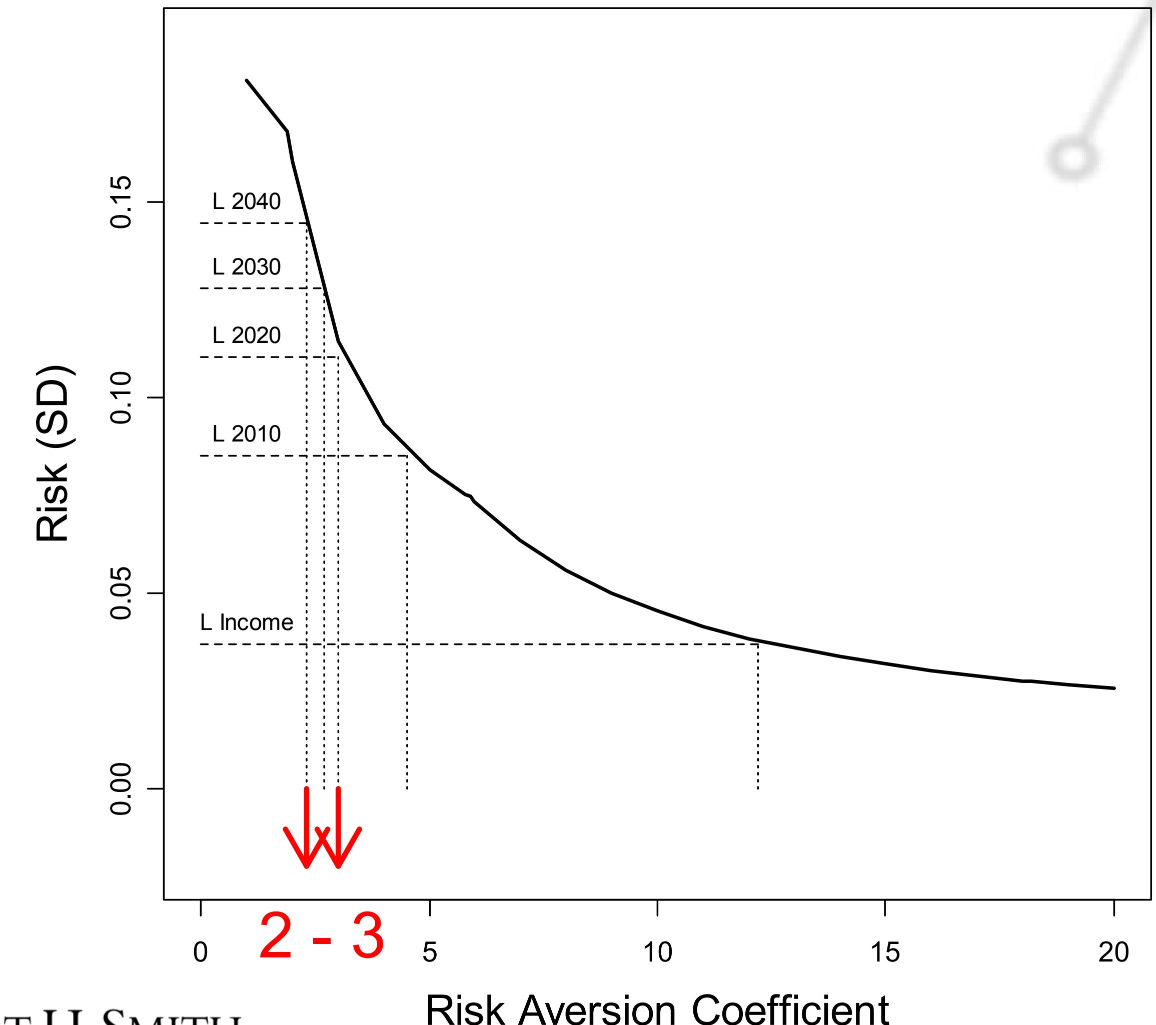


Negative Exponential Utility

$$U(w) = -e^{-cw}, \quad c > 0$$

- Constant Absolute Risk Aversion (CARA) -- no "wealth effect"
- Computational tractability advantage over other (log, power) utility functions
- Analytical solution to maximization problem is available using Certainty Equivalent (CE)
- CE is well-known for Normal and given for VG-ICA (Madan and Yen, 2004)

Implied Risk Aversion Coefficient



Portfolios for Comparison

Model	G Fund	F Fund	C Fund	S Fund	I Fund
VG-ICA (Daily)	0%	**1%**	**43%**	**30%**	**26%**
Riskless	**100%**	0%	0%	0%	0%
TSP "Market Portfolio"	**39%**	**6%**	**39%**	**8%**	**8%**
L 2030	**16%**	**9%**	**38%**	**16%**	**21%**
L 2040	**5%**	**10%**	**42%**	**18%**	**25%**

NOTE: These portfolios are created with returns assumed to be Normally distributed.

Stochastic Dominance

- Generalizes utility theory; don't need a specific utility function

- First-Order Stochastic Dominance (FOSD)
  - Assumes only monotonicity; strongest result
  - A FOSD B IFF $F_B(x) \geq F_A(x), \forall x$

- Second-Order Stochastic Dominance (SOSD)
  - Also assumes risk aversion
  - A SOSD B IFF $\int_{-\infty}^{x} [F_B(u) - F_A(u)]du \geq 0, \forall x$

- Easy to test with empirical data

Traditional Risk Measures

- Dispersion Measures
  - Variance (or Standard Deviation)
    - Treats gains and losses equally
  - Semi-Variance
    - Only considers observations below mean
  - Mean Absolute Deviation (MAD)
    - Average absolute deviation from the mean
- "Safety Risk" Measures
  - Value-at-Risk (VaR)
  - Expected Tail Loss (ETL)

Value-at-Risk (VaR)

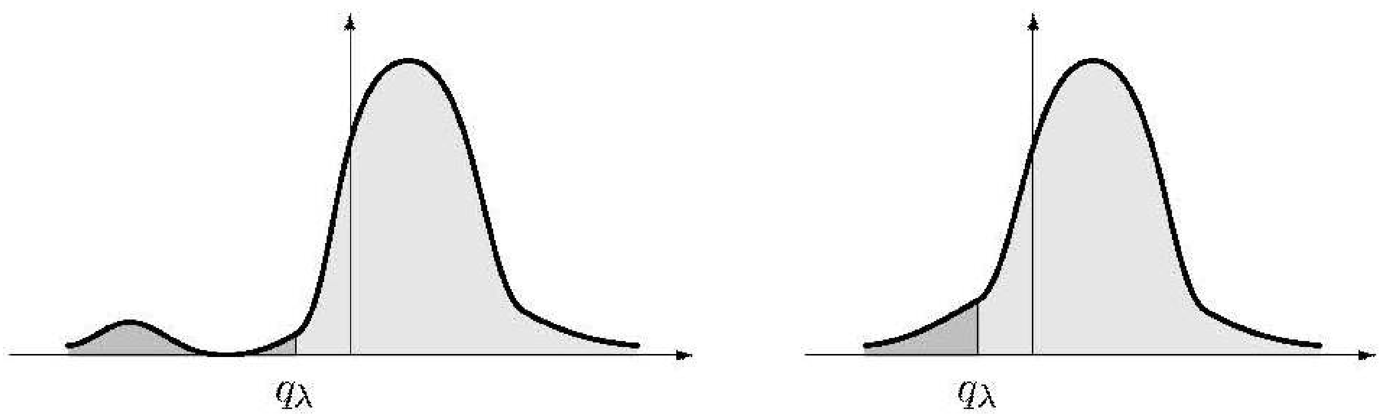
– "Expected maximum loss over a fixed horizon for a given confidence level"

$$P(X \geq VaR_\lambda(X)) = \lambda$$

– Standard risk measure for past 12 years

– Does not reward diversification

– Addresses size but not shape of tail



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Coherent Measures of Risk
(Artzner, Delbaen, Eber, & Heath, 1999)

- Axioms for coherency:
  - Translation invariance $\rho(X-\alpha)=\rho(X)-\alpha$
  - Monotonicity $X>Y \Rightarrow \rho(X)>\rho(Y)$
  - Sub-additivity $\rho(X+Y)\leq\rho(X)+\rho(Y)$
  - Positive homogeneity $\rho(\lambda X)=\lambda\rho(X)$
- Variance: not monotonic or translation invariant
- VaR: not sub-additive in non-Gaussian world
- Other measures that are coherent exist.

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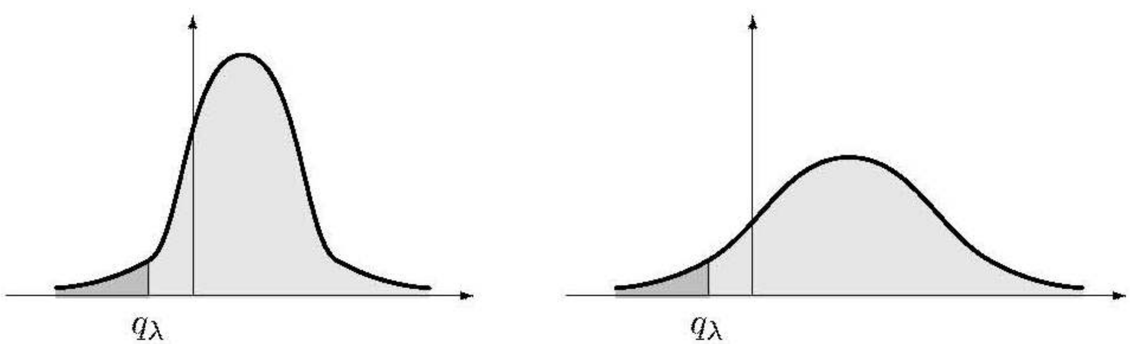
Conditional VaR

- "Expected value of all losses greater than VaR for a specified λ."

$$CVaR_{\lambda}(X) = E[X \mid X > VaR_{\lambda}(X)]$$

- Also known as Expected Shortfall (Rockafellar & Uryasev, 2001) and Tail VaR (Acerbi, Nordio, et Al., 2001)
- Accounts for size and shape of left tail but ignores rest of distribution



Classes of Weighted VaR
(Cherny, 2006; Cherny & Madan, 2007)

$$WVaR_{\mu}(X) = \int_{[0,1]} CVaR_{\lambda}(X)\mu(dx)$$

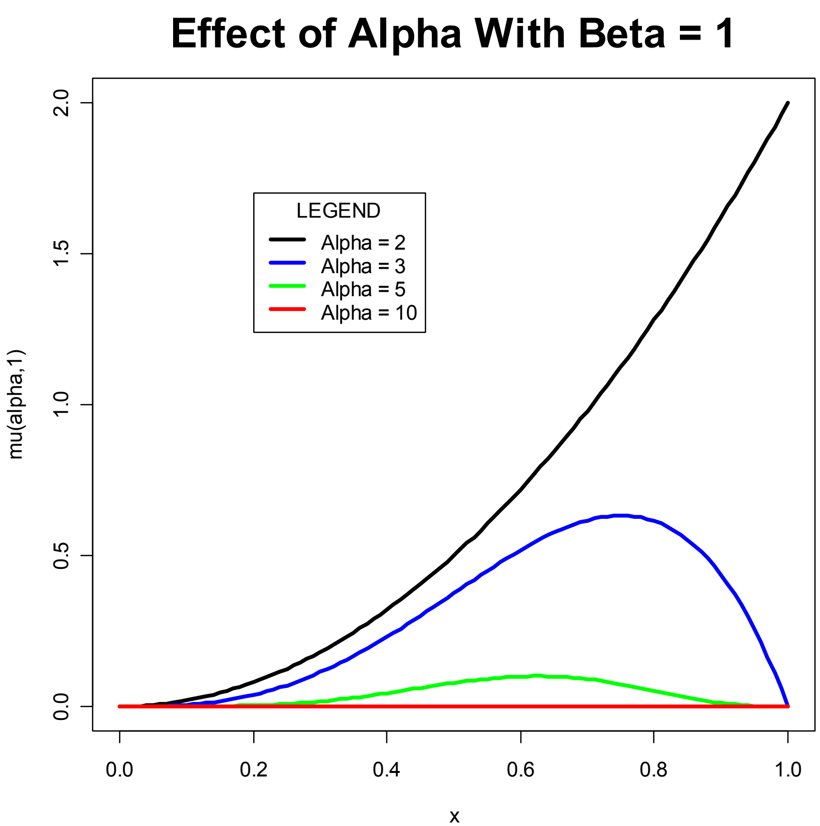
- Beta VaR(α,β)

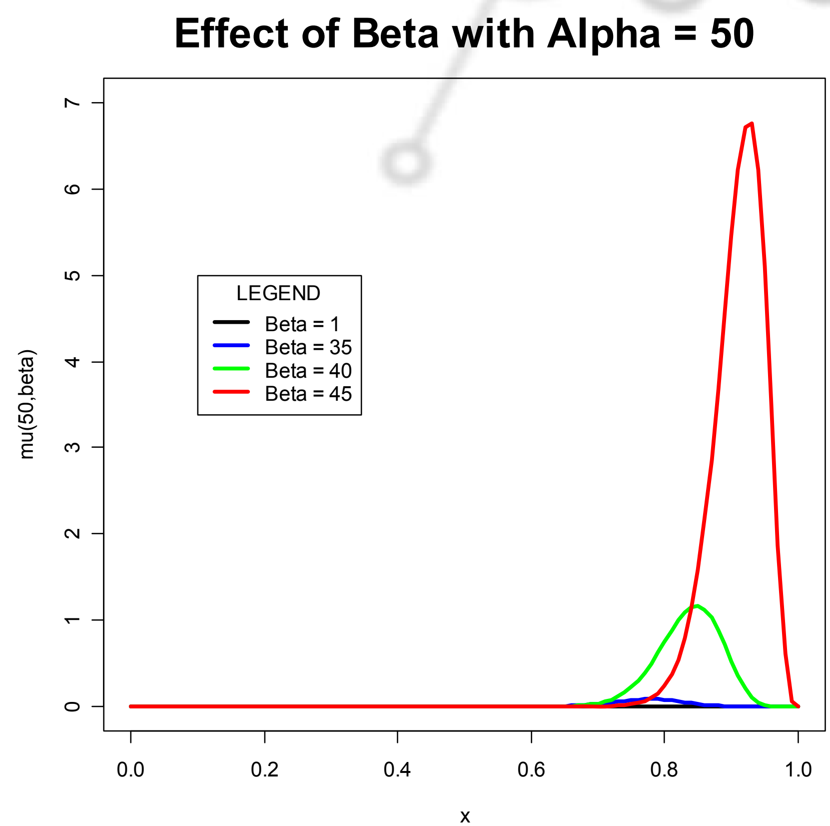
$$\mu_{\alpha,\beta}(dx) = B(\beta+1, \alpha-\beta)^{-1} x^{\beta}(1-x)^{\alpha-\beta-1}dx, \quad x \in [0,1]$$

  - Expectation of average of the β biggest of α independent copies of portfolio loss
  - Faster to estimate than CVaR

- Alpha VaR(α)
  - Essentially Beta VaR with β=1
  - Expectation of biggest of α copies of portfolio loss

Effect of Alpha and Beta





Can allow for more risk by decreasing α or increasing β

Performance (Reward-Risk) Measures

- Sharpe Ratio

$$SR = \frac{E(X)}{\sigma_X}$$

- STARR Ratio

$$STARR = \frac{E(X)}{CVaR_{\lambda}(X)}$$

- R-Ratio
  (Rachev)

$$R = \frac{CVaR_{\lambda_1}(-X)}{CVaR_{\lambda_2}(X)}$$

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New Portfolio Performance Measures *(Nestler, 2007b)*

- Similar to R-Ratio but use Alpha-VaR and Beta-VaR in place of CVaR
- AVaR-Ratio: $$AVR = \frac{AVaR_{\alpha_1}(-X)}{AVaR_{\alpha_2}(X)}$$
- BVaR-Ratio: $$BVR = \frac{BVaR_{\alpha_1,\beta_1}(-X)}{BVaR_{\alpha_2,\beta_2}(X)}$$

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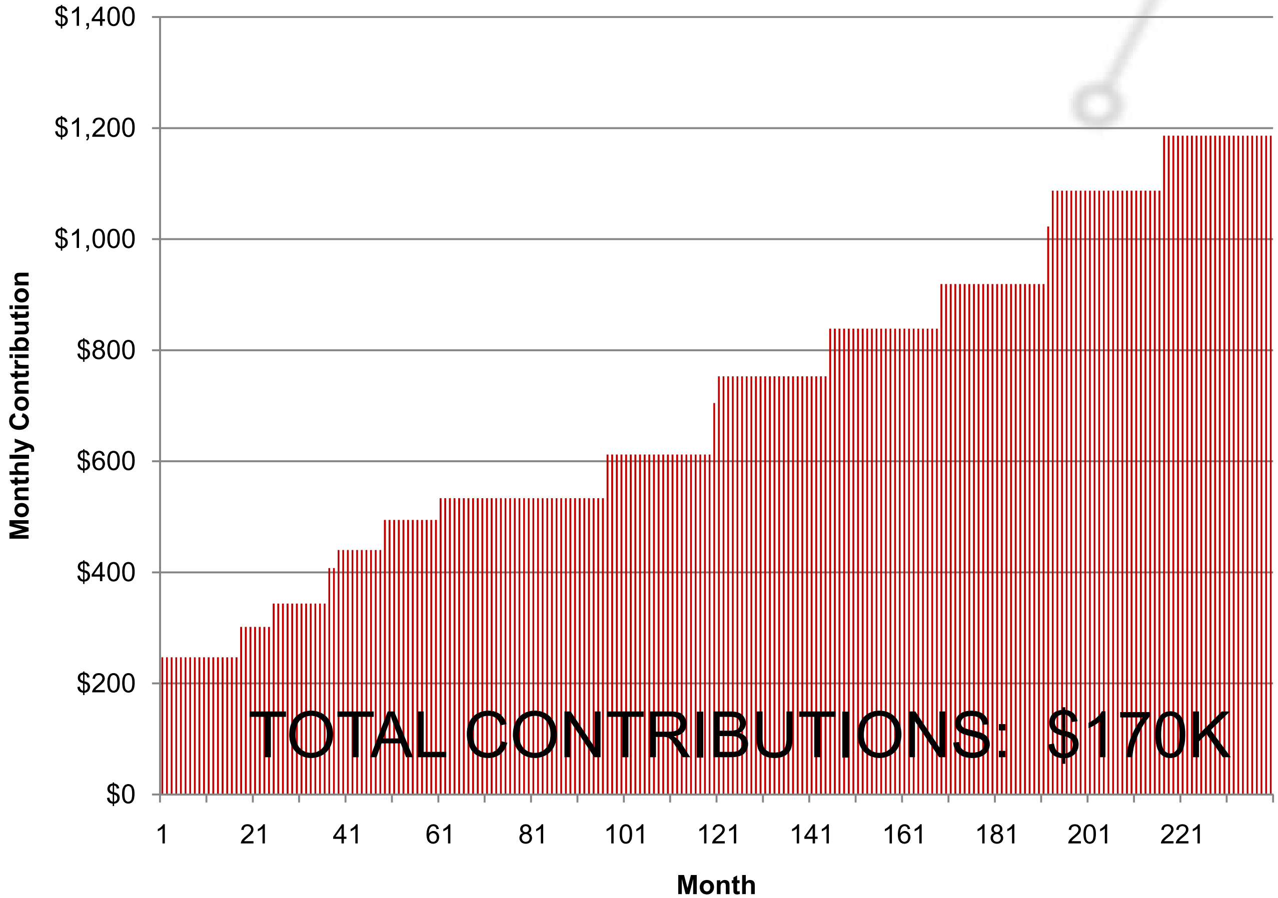
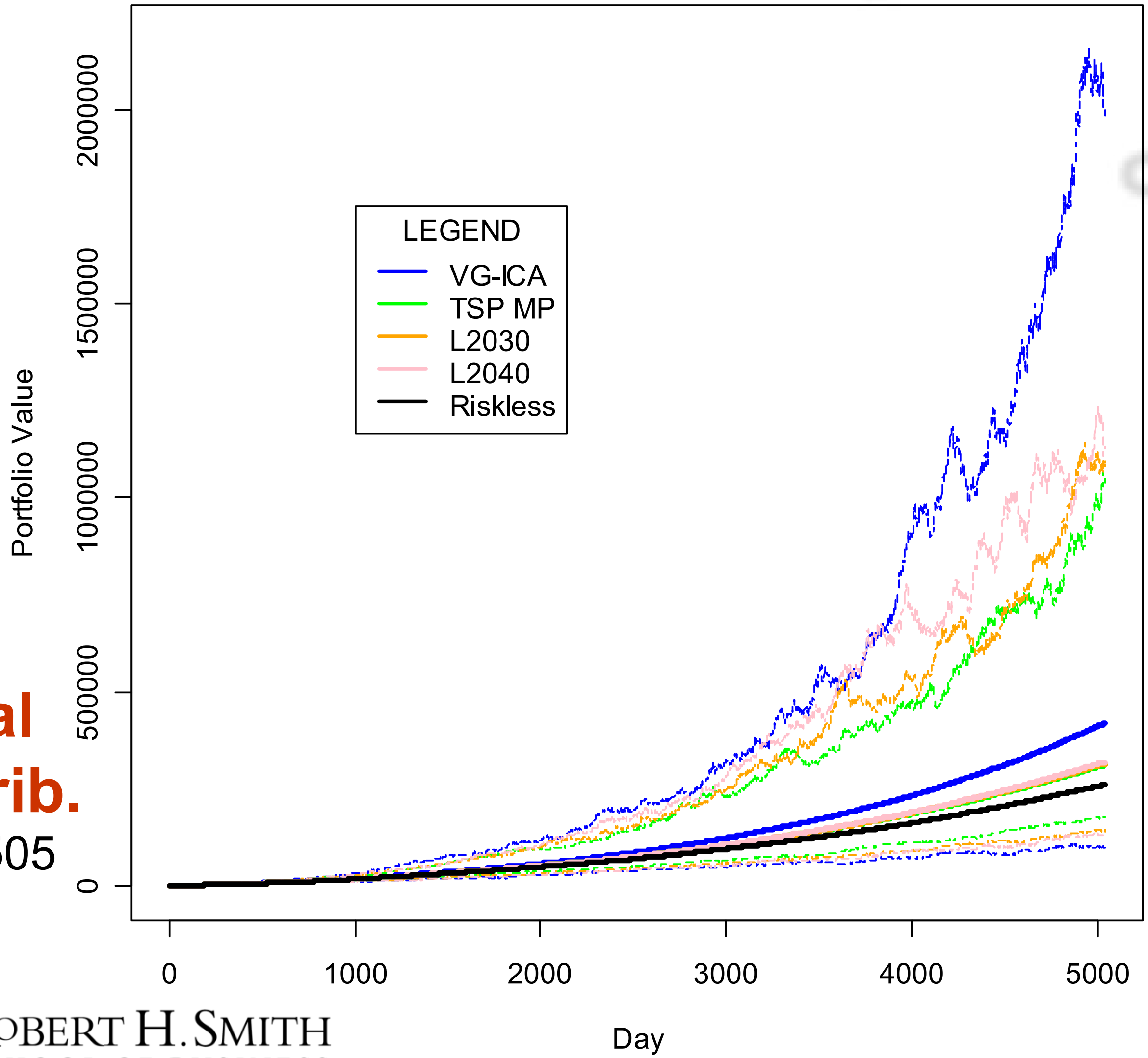
Monthly Contribution

•Assumes saving 10% of base pay each month (median for TSP)



Realistic Scenario: Portfolio Value
(5000 sample paths)



Upside Potential:

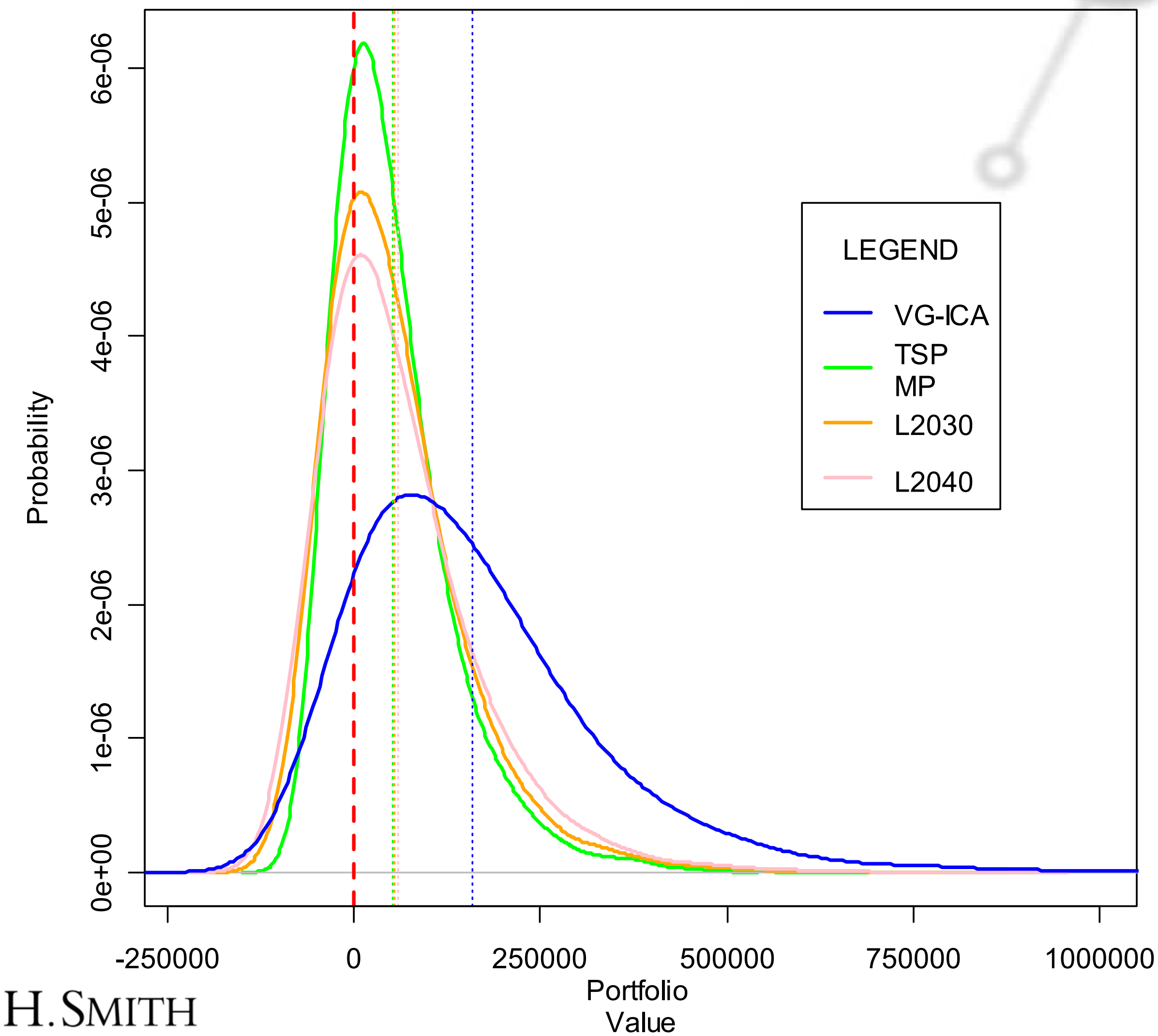
VG-ICA: $ 1,992,133
L2040: $ 1,127,069
L2030: $ 1,093,643
TSP MP:$ 1,064,902

Expected Value:

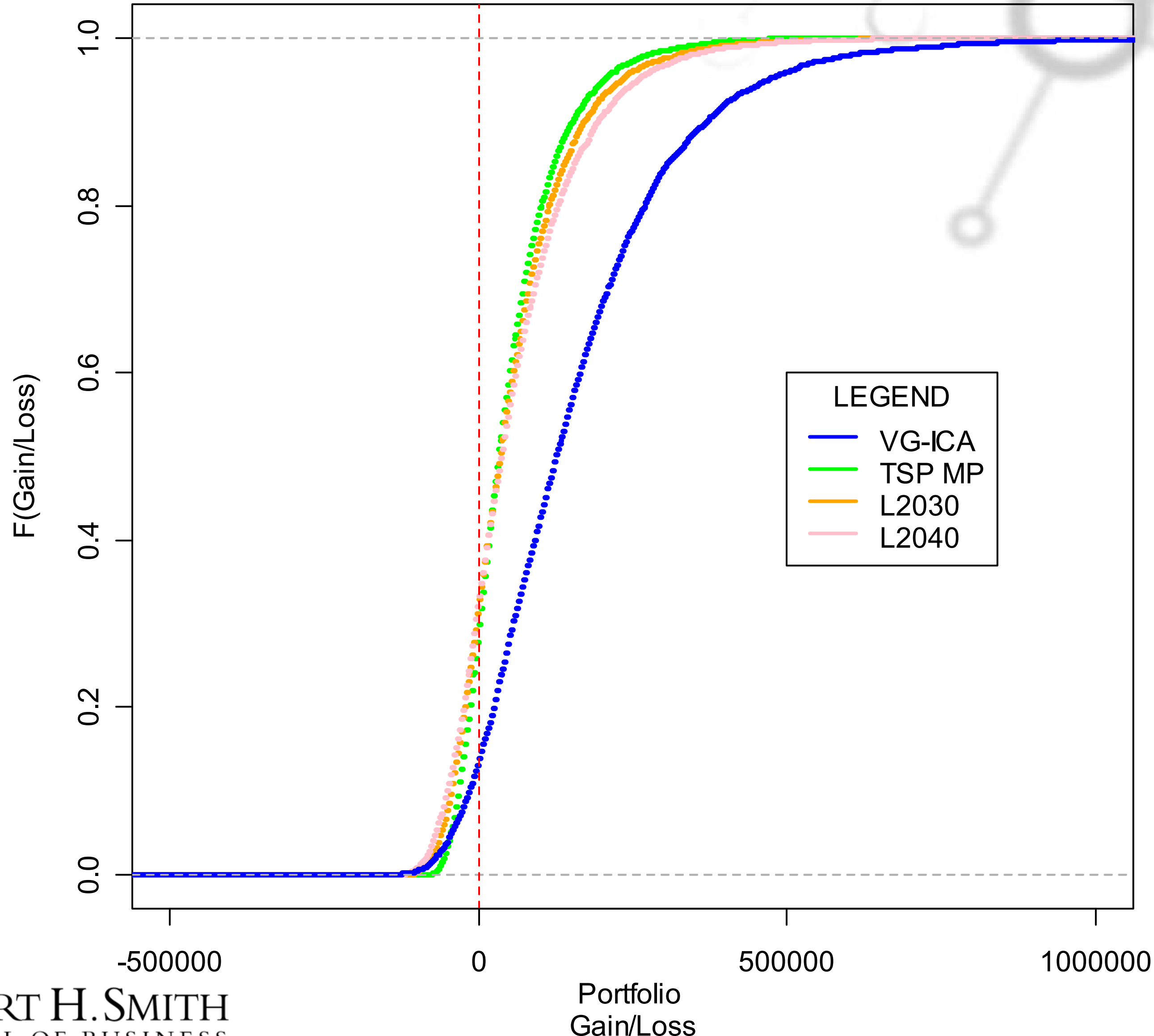
VG-ICA: $ 418,381
L2040: $ 318,840
L2030: $ 313,936
TSP MP:$ 310,247
Riskless:$ 259,642

Total Contrib.
$ 170,505

PDF of Discounted Portfolio Value



Realistic Scenario: CDF Comparison



NOTE:
No SD.

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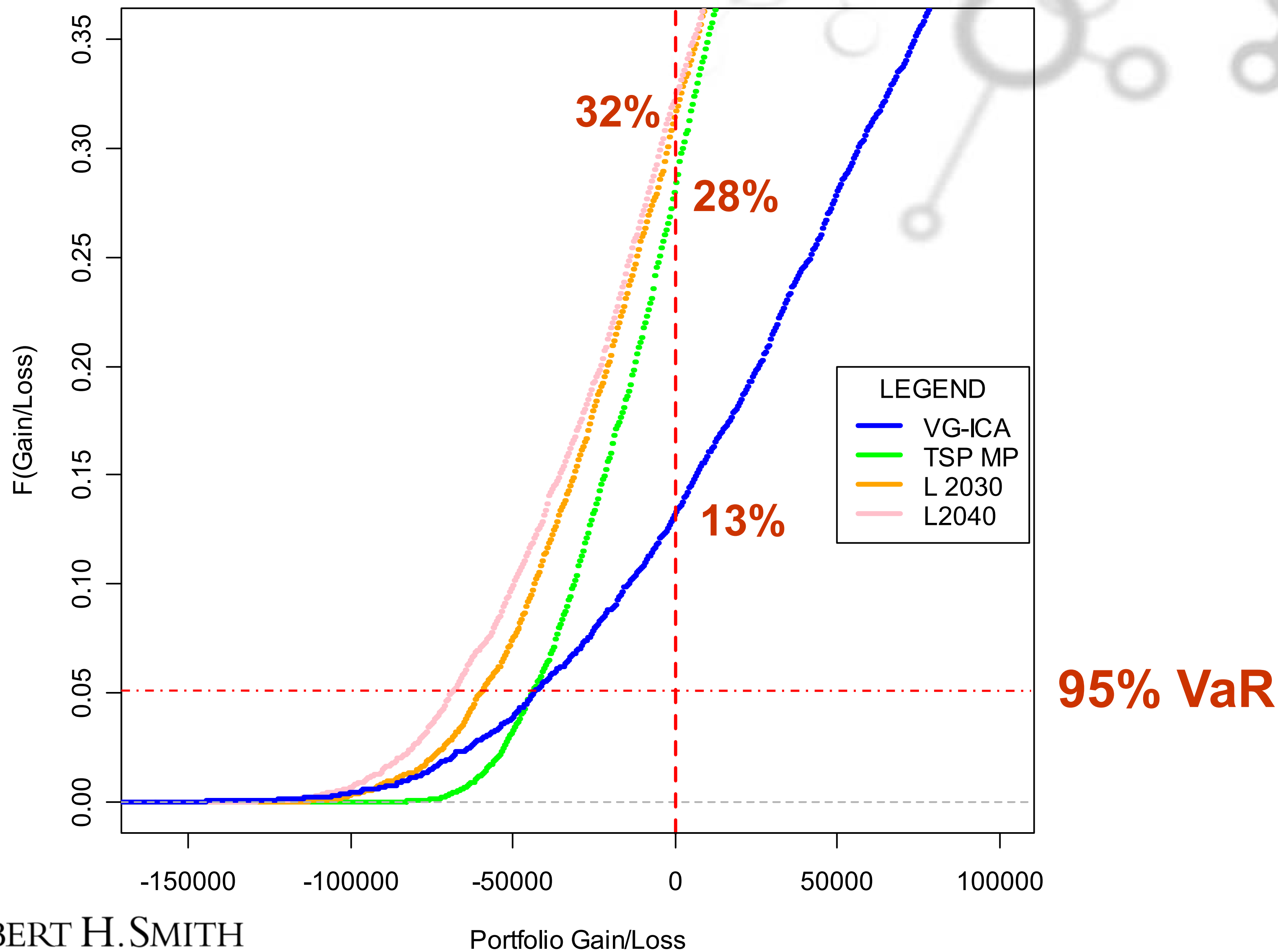
Realistic Scenario: Zoomed CDF Comparison



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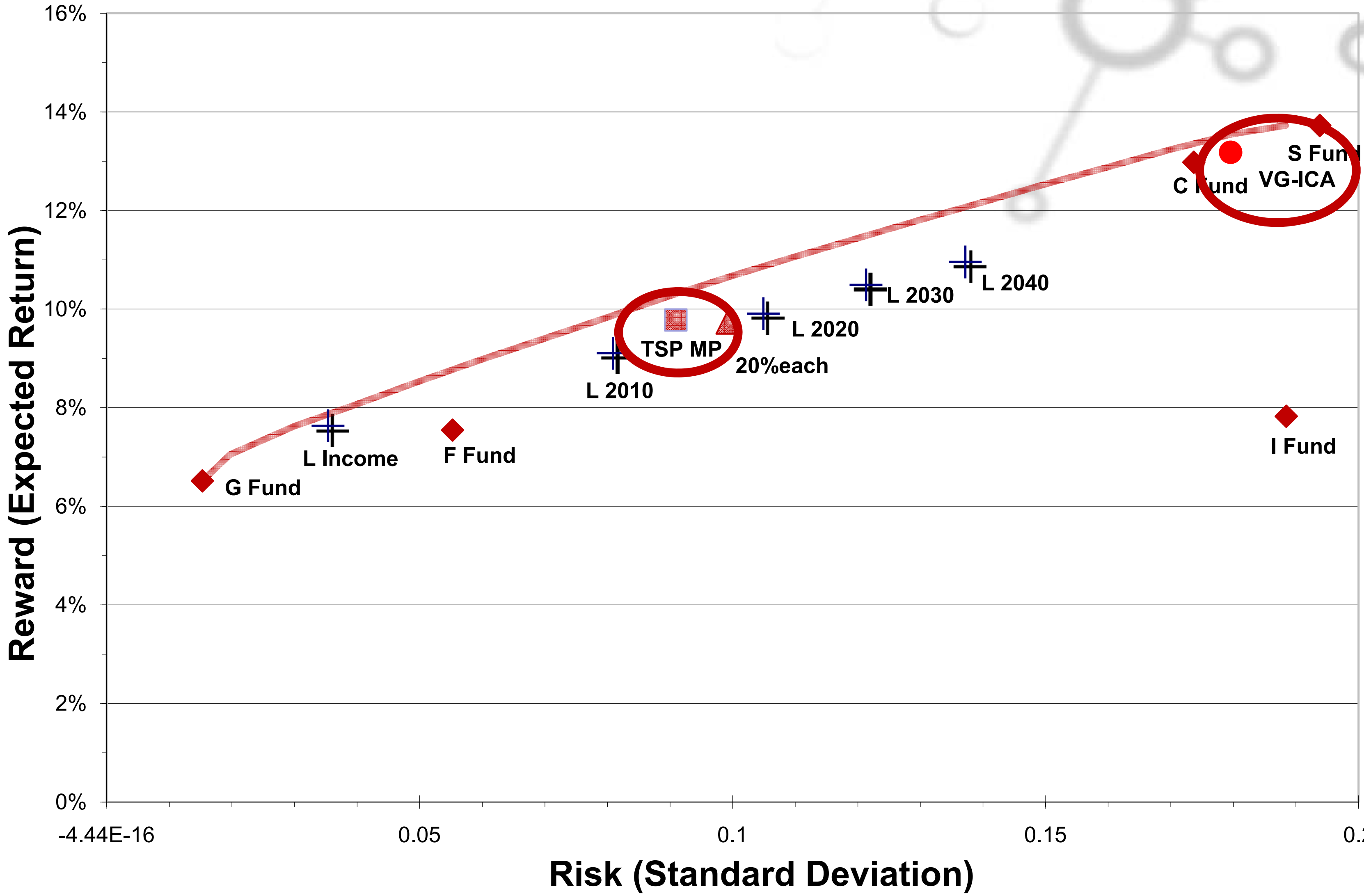
Realistic Scenario: Risk & Performance Measures

Risk Measure (↓ better)	VG-ICA	TSP MP	L 2030	L 2040
Std Dev	$ 168,885	$ 80,890	$ 94,515	$ 105,525
95% VaR	$ 43,382	$ 44,146	$ 60,378	$ 66,910
95% CVaR	$ 68,056	$ 54,789	$ 74,684	$ 82,783
Alpha VaR(50)	$ 77,575	$ 59,250	$ 81,352	$ 87,754
Beta VaR(50,5)	$ 43,938	$ 44,010	$ 60,203	$ 67,757

Performance Measure (↑ better)	VG-ICA	TSP MP	L 2030	L 2040
Sharpe Ratio	0.94	0.63	0.57	0.54
STARR Ratio	2.33	2.90	0.72	0.69
R-Ratio(.05,.05)	8.87	2.82	4.10	4.07
AVR	9.35	5.41	4.62	4.67
BVR	11.60	5.05	4.24	4.12

Traditional Reward-Risk Profile



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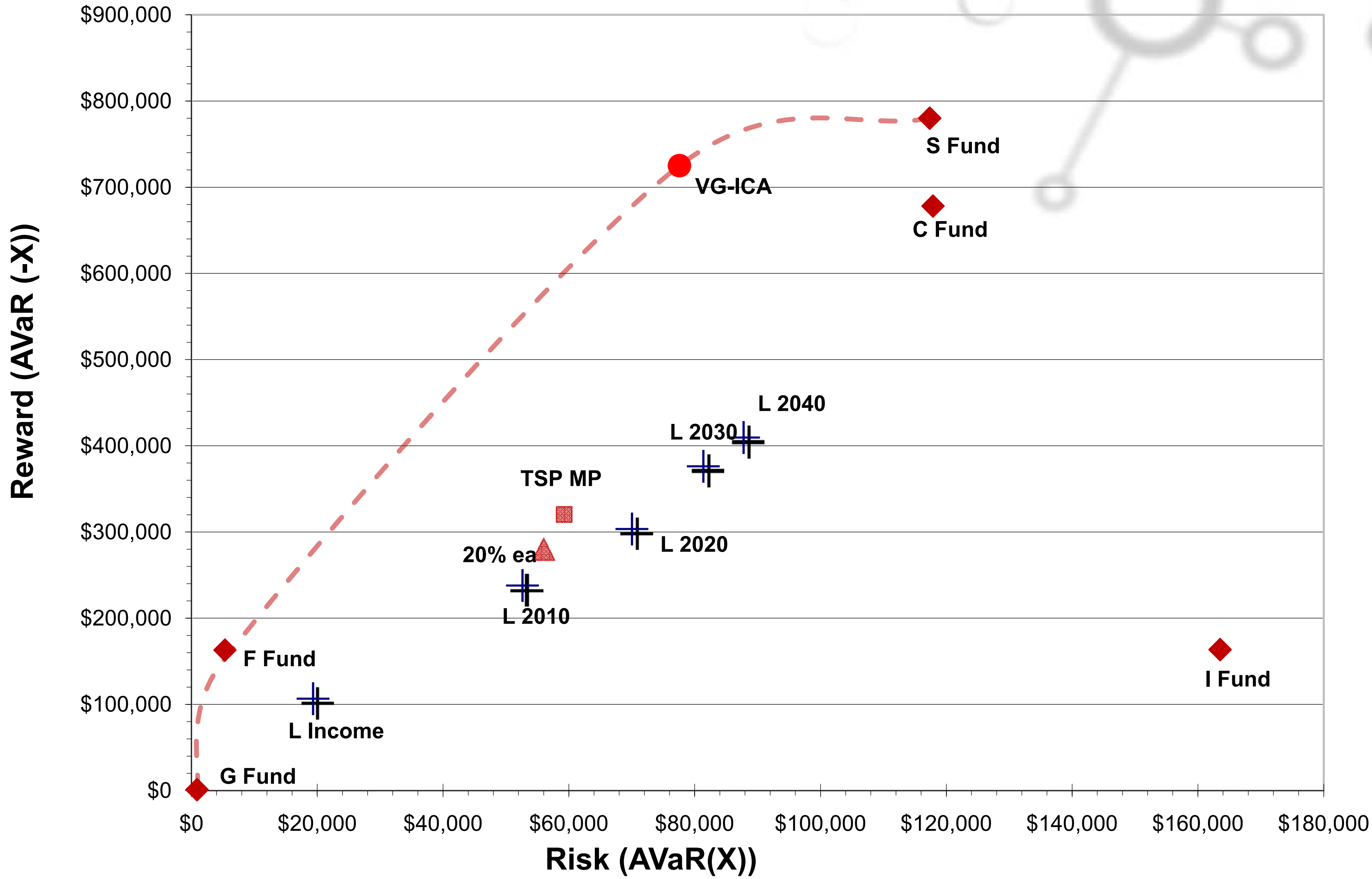
New Reward-Risk Profile



Some Possible Answers

- Why might I be more risk tolerant than I currently believe?
  - Counting military or government pension as "pseudo-bonds" could change the target stock-bond asset mix.
- What are the L (Lifecycle) funds? How are they constructed? Why might they be of interest (or not) to me?
  - "Set it and forget it" funds built using mean-variance optimization with returns assumed to be distributed Normally.
  - Depends on an individual's level of interest and involvement.
- What if stock and index fund returns are not normally distributed, as is commonly assumed?
  - Possible to take advantage of information contained in higher moments.
- How does choosing reward-risk measures affect optimal TSP portfolios?
  - Ability to capture information from entire distribution is useful.
  - Need to do further work on optimizing performance measures instead of using expected utility.

Q U E S T I O N S ?