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# The Effects of Laser Phase Noise on Laser Radar Performance *

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## ABSTRACT

Laser phase noise is shown to degrade the sensitivity in target vibration measurements using a CW laser with an FM discriminator. Its effects on the performance of a radar employing a linear FM waveform is also discussed.

## 1. INTRODUCTION

In microwave Doppler radar, the phase noise of the microwave oscillator is a critical parameter that can limit the dynamic range of a radar system in general and the clutter visibility of an MTI radar in particular. Much effort has gone into the design of microwave oscillators having low phase noise; these include raising the quality factor Q of the microwave oscillator cavity by using low-loss microwave components, including superconducting components if necessary.

In laser radars, the phase noise of the transmitter or local oscillator is equally important. The phase noise in a laser oscillator, analogous to the phase noise in an RF or microwave oscillator, is a measure of the frequency instability of the laser source in the frequency domain. Similarly, in the time domain, the frequency instability is represented by the fractional frequency fluctuation in terms of the square root of the Allan variance. The Allan variance is used frequently in time standards applications, while the phase noise is most often employed in laser radar applications. In addition, the Allan variance is most appropriate for long-term frequency stability description, while the phase noise usually refers to short-term frequency fluctuations.

This paper discusses the parameters used in describing the laser frequency stability in both time domain and frequency domain. *It will be shown that the phase noise of a laser transmitter or local oscillator can* degrade the output signal-to-noise ratio of a CW FM laser radar used in vibration sensing will be shown to be degraded by the phase noise so that it is below that set by the shot noise of a laser local oscillator. As a result, shot-noise-limited operation in general will not be realizable. This is especially true in long range vibration sensing. For the case of dynamic target speckle (present for diffuse targets in oscillatory transverse motion), it will be shown that the required low phase noise in a laser transmitter may be relaxed depending on the level of the speckle noise relative to that of the phase noise.

## 2. LASER FREQUENCY STABILITY REPRESENTATION

The phase noise in an RF or microwave oscillator is usually represented in the frequency domain as $\mathcal{L}(f)$ in dBc/Hz vs. the offset frequency $f$ from the carrier. This is usually a log-log plot which can be approximated by line segments with different slopes. In the time domain, On the other hand, the laser

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frequency instability is represented by the root Allan variance as $\sigma_y(\tau)$ vs. the measurement time $\tau$ also by line segments with different slopes in a log-log plot, where $\sigma_y(\tau)$ is defined as the fractional frequency fluctuation for a given measurement time $\tau$. For each of the slopes, there is a one-to-one corresponding transformation between the time domain representation and the frequency domain representation of the oscillator noise. For each of these five exponential dependences, a transformation from the time domain to the frequency domain or vice versa can be done using one of a set of formulas developed by the National Institute of Science and Technologies (NIST) [1]. For small values of phase fluctuation, $\phi$, the single sideband phase noise is related to $S_\phi(f)$ based on the frequency modulation theory for small angle modulation.

$$\mathcal{L}(f) = \frac{1}{2rad^2} S_\phi(f) \quad . \tag{1}$$

The function $S_\phi(f)$ is defined as the spectral density of the phase fluctuation of the frequency source as a function of the offset frequency $f$ from the sinusoidal carrier $\nu_0$. $S_\phi(f)$ is measured in units of $rad^2/Hz$ Since $S_\phi(f)$ is measured in rad$^2$/Hz, $\mathcal{L}(f)$ is measured in Hz$^{-1}$ units. $\mathcal{L}(f)$ is defined as the ratio of the power density in one phase modulation sideband to the carrier power. When the small angle condition (meaning that the mean square phase fluctuations are small relative to one radian squared) is not met, Eq.(1) is not valid. A straight line with a Y-axis intercept at $-30$ dBc/Hz and a slope of $-10$ dB can be drawn in the plot as a criterion to show that the small angle is valid only if the phase noise plot is below this line. The phase noise above this line must be interpreted in radians squared per Hertz, not in dBc/Hz as $\mathcal{L}(f)$ is defined. In this case, the phase fluctuation is best represented by $S_\phi(f)$. In addition, the vertical scale must also be adjusted by 3 dB since $S_\phi(f)/2$ is actually graphed.

Next, we want to examine the frequency stability of some existing $CO_2$ lasers. Most of the measured laser frequency stability data have been obtained and presented in the time domain. In this paper, the frequency stability of frequency stabilized Hughes waveguide lasers and the effect of the laser frequency stability on a laser radar system using such a laser or lasers will be discussed. The frequency stability of the Hughes waveguide laser was measured in the time domain [2] by heterodyning together two Hughes $CO_2$ *waveguide lasers which were stabilized* to molecular absorption line centers; the frequency stability was therefore given in terms of the Allan variance . Figure 1 shows a plot of the observed data points (represented by stars).

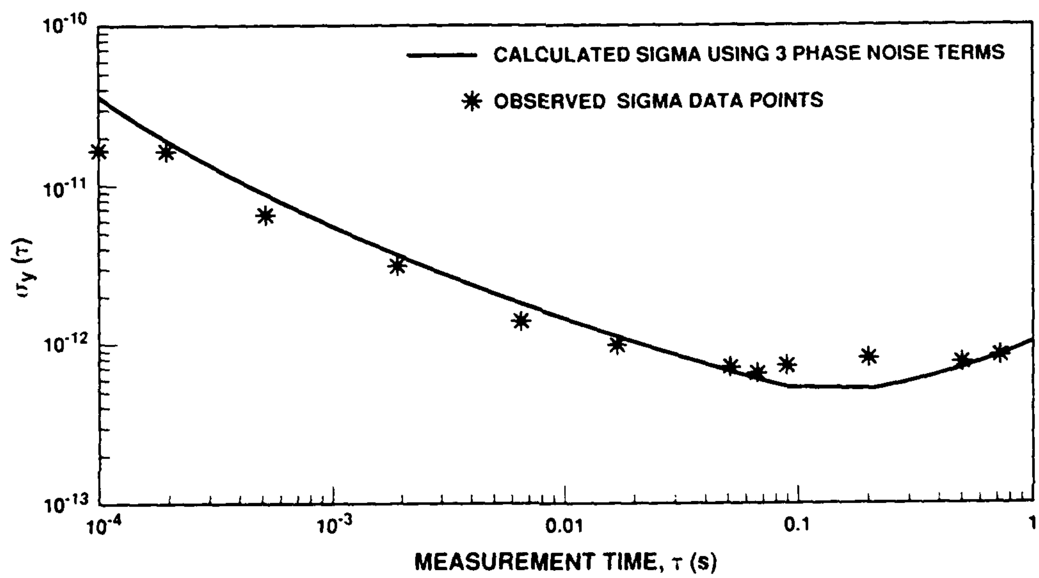


*Figure 1: Root Allan Variance of Stabilized Hughes $^{12}CO_2$ Lasers*

## 3. CONVERSION FROM ALLAN VARIANCE TO SSB PHASE NOISE

In order to use the data for laser radar applications, we need to convert the frequency stability representation from the time domain to the frequency domain. an NIST software program was used. The solid curve in Fig. 1 is the calculated Allan variance for the Hughes $CO_2$ laser fit to the original observed data points connected. Using the set of NIST conversion formulas, we first obtained the following equation for $S_y(f)$

$$S_y(f) = \frac{1.5 \times 10^{-25}}{f^2} + 4.0 \times 10^{-26} + 1.3 \times 10^{-35} f^2 \ . \tag{2}$$

By multiplying $S_y(f)$ represented by Eq.(2) by $\nu_0^2/(2f^2)$, where $\nu_0$ is the frequency of the laser in Hz, we next obtain the SSB phase noise $\mathcal{L}(f)$ as shown in Eq.(3) below.

$$\mathcal{L}(f) = 5 \times 10^{-9} + \frac{16}{f^2} + \frac{60}{f^4} \ . \tag{19}$$

Figure 2 shows the SSB phase noise, Eq.(3), plotted in dBc/Hz, solid curve. Note that the curve as it stands can go above the 0 dB point at a low frequency. According to the small phase angle condition, the small phase angle condition is not met and it is best to treat the plot as an $S_\phi(f)/2$ plot rather than a $\mathcal{L}(f)$ plot. The reason that we have retained the $\mathcal{L}(f)$ symbol is trying to bridge the gap between microwave and optics and that it is much easier to stay with the symbol $\mathcal{L}(f)$ rather than changing the symbol.

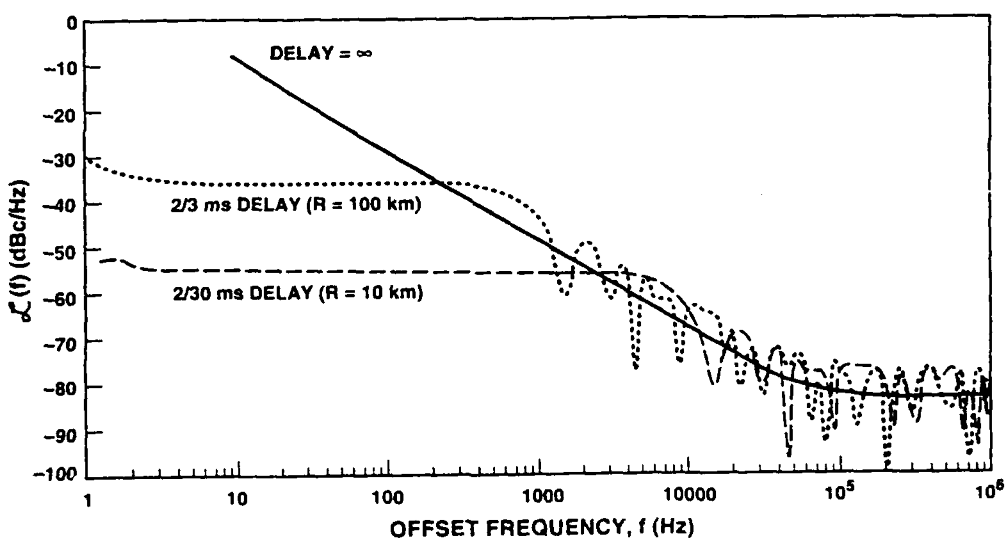


*Figure 2: Effective SSB Phase Noise for Two Time Delays*

## 4. EFFECTIVE PHASE NOISE IN HETERODYNE DETECTION

Having determined the SSB phase noise such as that shown by the solid curve in Fig. 5, we want to show that in heterodyne operation, the effective phase noise is range or time delay dependent. The reason is that when the return signal is time delayed so that by the time it gets back to the transmitter/receiver, the local oscillator frequency which is assumed to be derived from the laser transmitter by a beamsplitter has shifted only slightly so that the laser phase of the local oscillator is still correlated to that of the return laser beam. The amount of correlation depends on the laser frequency stability

The effect of time delay on the SSB phase noise can be accounted for by multiplying the infinitely long delay SSB phase noise such as that shown by the solid curve in Fig. 2 by a time delay function $K^2(f)$ [4]:

$$\mathcal{L}(f)_d = \mathcal{L}(f)K^2(f) \ , \tag{4}$$

where

$$K^2(f) = 2[1 - cos(2\pi f \tau_d)] \ , \tag{5}$$

and where $\tau_d$ is the time delay and $f$ is the offset frequency.

At small offset frequencies and/or at small time delays, the phase noise is reduced because the delay function is very small so that the effective phase noise is reduced. Because of this large phase noise reduction by the delay function for small delays, a mediocre homodyne $CO_2$ laser radar system can often perform well without any special measure to ensure low phase noise in the laser transmitter/receiver. The often heard laser radar jargon used to describe the above adequacy in the laser spectral purity for short range homodyne operation might go like this: the target is well within the coherence length of the laser.

The effective phase noises for two delays, 2/30 ms and 2/3 ms, are obtained as shown using Eq.(4). While the short delay reduces the phase noise out to an offset frequency of about 2 kHz, the long delay of 2/3 ms is effective in reducing the phase noise only for frequencies below about 300 Hz. Because of the large reduction in phase noise, the small phase angle requirement is relaxed; this is especially true for short laser radar ranges. It is noted that the phase noise tends to have the highest density near the transition frequency around 1 kHz. Since the phase noise is frequency distributed, for a laser radar system with a detection bandwidth $B$, it is convenient to represent the overall effect of the phase noise on system performance by integrating the noise density over the bandwidth of operation.

Table 1 lists the integrated phase noise for different time delays and bandwidths for the above laser calculated using a software program on phase noise analysis in radar systems [4].

*Table 1: Integrated phase noise for four bandwidths*

| Time delay $\tau_d$ (s) | Integrated phase noise (dBc) for bandwidth B | | | |
|---|---|---|---|---|
| | (B=100 kHz) | (B=10 kHz) | (B=1 kHz) | (B=500 Hz) |
| $1.0 \times 10^{-5}$ | −26 | −33 | −42 | −45 |
| $3.162 \times 10^{-5}$ | −22 | −25 | −33 | −36 |
| $1.0 \times 10^{-4}$ | −17 | −18 | −23 | −26 |
| $3.162 \times 10^{-4}$ | −12 | −13 | −15 | −17 |
| $6.67 \times 10^{-4}$ | −9 | −10 | −11 | −13.5 |
| $1.0 \times 10^{-3}$ | −7 | −7 | −8 | −9 |

It can be seen from Table 1 that the integrated phase noise is quite high. This means that the resultant dynamic range of the laser radar receiver will be reduced to about 10 dB for a delay of 2/3 ms (corresponding to a range of 100 km) and about 20 dB for a delay of 2/30 ms (corresponding to a range of 10 km). In terms of detection, weak target returns will be buried by the phase noise associated with the laser radar return from a strong target.

## 5. DOPPLER RESOLUTION OF DOPPLER RADAR LIMITED BY PHASE NOISE

Because the frequency stability of a laser oscillator is not infinitely high, the laser linewidth or phase noise is not zero. This means that the radar returns from two targets with slightly separated Doppler frequencies may not be resolved. For example, for the 2/3 ms delay curve shown in Fig. 5, if the difference between the Doppler frequencies of two targets is less than 1 kHz, the two Doppler signals will be overlapped, because the pulse energies of both targets are distributed approximately $\pm 1$ kHz around their peak Doppler frequencies. Assuming that the cross sections of both targets are equal, the final result is that the return signal appears to be broadened by a factor of two and is centered around the average Doppler frequency.

If the laser radar cross section of one of the targets is 1000 times larger than that of the other target and the delay is still 2/3 ms, the weaker target will not be detected if the Doppler frequency difference is less than 1 kHz because the integrated pulse energy will fall below the integrated phase noise of the stronger target. However, should the delay be reduced to 2/30 ms, the weaker target could still be detected until the cross section of the weaker target decreases to about 1/100,000 that of the stronger target. In the case of an infinitesimal linewidth or approximately zero phase noise, the Doppler frequency difference can be very small and the weaker target can still be detected unless it is limited by other noises such as shot noise or by the frequency resolution determined by the dwell time of the transmitted beam on the target.

In the above examples, we have assumed that the two targets have non-zero Doppler frequencies in general. When one of the targets is a background clutter with zero Doppler frequency, the above examples imply that slowly moving small objects may not be detected by a laser radar using lasers with high phase noises.

## 6. AN ULTRASTABLE LASER WITH VERY LOW PHASE NOISE

Based on the data presented in the preceding section the phase noise of the stabilized Hughes waveguide laser appears to be high. For this reason, we want to examine the short-term frequency stability of ultrastable $CO_2$ lasers that have been developed by Freed at Lincoln Laboratory [5]. Figure 3 shows Allan variance plots of the above ultrastable $CO_2$ laser which has an open Fabry-Perot cavity [5]. The open and solid circles represents the case when the laser was frequency locked by the saturated fluorescence technique, and the two cross-filled circles represent the short-term frequency stability data points experimentally observed [5]. If we plot these two cross-filled data points on Fig. 1. They fall below the data points (for the same measurement time $\tau$) by more than 10 dB. Therefore, the root Allan variance for the Fabry-Perot cavity ultrastable $CO_2$ laser can be represented by a curve approximately a factor of 10 below the solid curve shown in Fig. 1 in the measurement time range corresponding to these two cross-filled data points. In transforming to the frequency domain, we note that the phase fluctuation $S_\phi(f)$ is proportional to the square of the root Allan variance so that the SSB phase noise for the Fabry-Perot cavity ultrastable $CO_2$ laser is about 20 dB (because of the squaring operation) below that of the frequency stabilized Hughes laser shown in Fig. 2 for frequencies close to the carrier. We can therefore assume that the following equation is suitable for representing the SSB phase noise of the Fabry-Perot cavity laser (see Eq.(3)):

$$\mathcal{L}(f) = 5 \times 10^{-9} + \frac{0.16}{f^2} + \frac{0.60}{f^4} \quad . \tag{6}$$

By multiplying Eq.(6) by the factor $2f^2/\nu_0^2$, we obtain the spectral density of fractional frequency deviation, $S_y(f)$ similar to Eq.(2)

$$S_y(f) = \frac{1.5 \times 10^{-25}}{f^2} + 4.0 \times 10^{-28} + 1.3 \times 10^{-37} f^2 \quad . \tag{7}$$

When Eq.(7) is used as input to the NIST program, we obtain the root Allan variance for short-term frequency stability. Similarly, using Eq. (6), SSB phase noise for the ultrastable $CO_2$ laser for an infinitely long delay was plotted in Fig. 4. In addition, the effective phase noise for a 2/3 ms delay is also shown(dashed curve). In comparison with the SSB phase noise of the Hughes waveguide laser shown in Fig. 2(replotted here as the dotted curve), the effective SSB phase noise is seen to be about a factor of 100 smaller. If the phase noise were to be integrated, it would be easy to prove that the integrated phase noise for the same bandwidths shown in Table 1 would be about 20 dB lower for the same time delays shown.

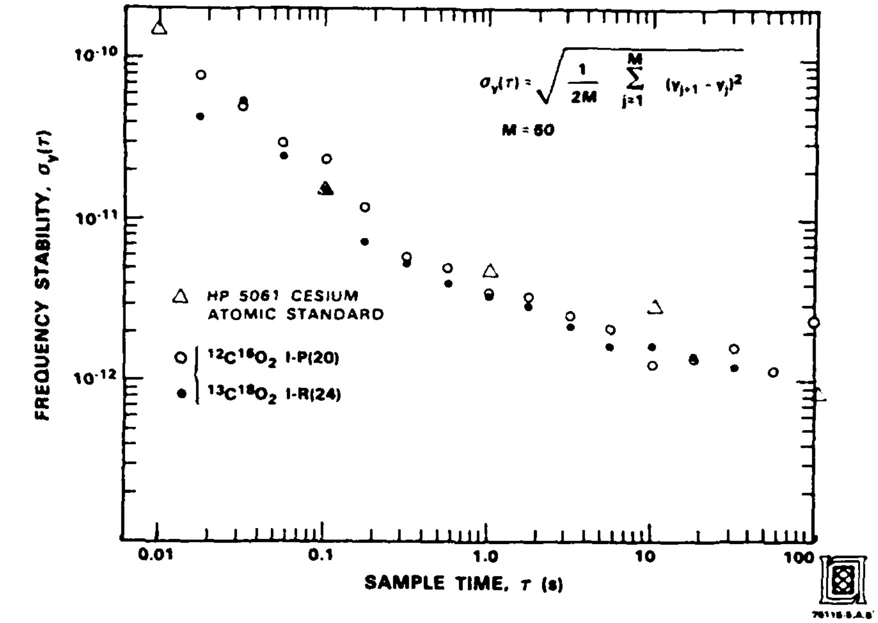


*Figure 3: Time–Domain Frequency Stability of a Fabry–Perot Cavity $CO_2$ Laser*

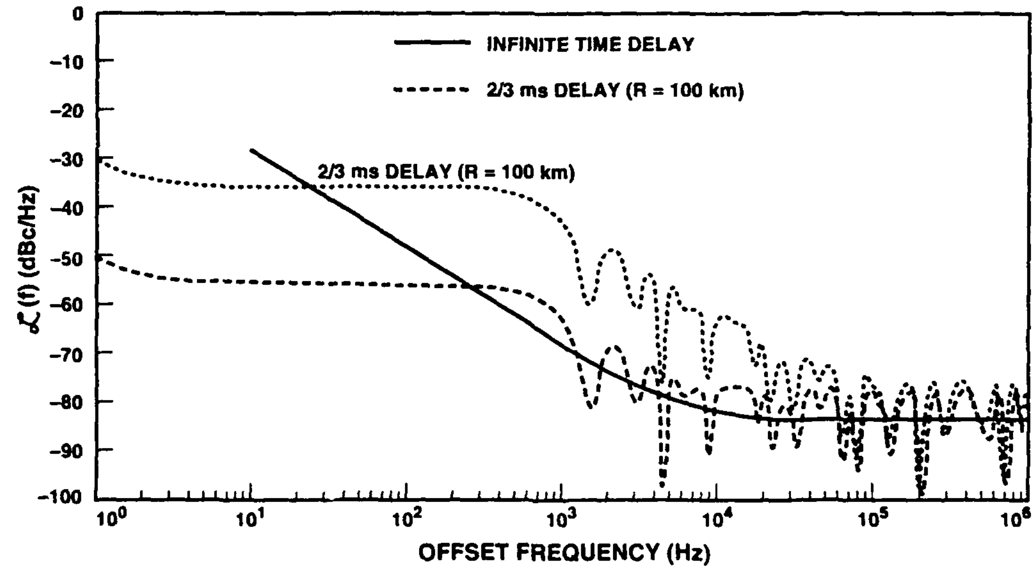


*Figure 4: SSB Phase Noise vs. Offset Frequency of Freed Lasers*

## 7. CW FM DISCRIMINATOR FOR LASER VIBRATION MEASUREMENTS

We have discussed the operation of a pulsed coherent $CO_2$ laser in which the phase noise can reduce the carrier-to-noise ratio, CNR, relative to that of a laser having a lower phase noise. We have shown that because of the reduction in CNR, the capability or dynamic range of such a laser radar is degraded. In the next few sections, we would like to change our subject of discussion and focus our attention on a CW FM laser radar that is used for remote surface vibration sensing or measurements. A typical laser radar system for vibration measurement is shown in a block diagram in Fig. 5. It is a CW frequency shifted homodyne system in which the laser is a CW $CO_2$ laser similar to one of the two $CO_2$ lasers that have just been discussed. The local oscillator power is derived from the transmitter laser beam through a beamsplitter. The local frequency is shifted from the transmitter frequency by an acousto-optic modulator frequency shifter operating at a frequency of about 40 MHz. The laser beam is sent out to a distant target in vibrational motion. The return beam contains frequency modulation produced by the target motion, which is assumed to have only the piston type of motion. It can be shown that the modulation index $\beta$ is related to the vibrational surface displacement amplitude $a$ and wavelength $\lambda$ by $\beta = \frac{4\pi a}{\lambda}$. The return signal is mixed with the local oscillator beam to produce the intermediate frequency (IF) signal. The IF signal is then considered to be the carrier signal input to an FM discriminator demodulator for demodulation.

For sensitive measurements, the carrier-to-noise ratio (CNR) at the input to the limiter or frequency discriminator is a very important laser radar parameter. The noise here is the shot noise generated by the local oscillator at the optical detector. Thermal noise from the RF amplifier may add to the shot noise depending on the shot noise level relative to the thermal noise. When the CNR is high, a high SNR at the output of the low pass filter of the discriminator demodulator is obtained. On the other hand, when the CNR value is low, the operation of the CW FM discriminator is poor in that there are noisy impulses present at the output; these noisy impulses have been called click noise. The threshold value of CNR required for quiet FM operation is about 12 dB. In the following we will estimate the range of laser radar operating parameters or conditions within which a CW FM laser radar operates by considering some realistic situations.

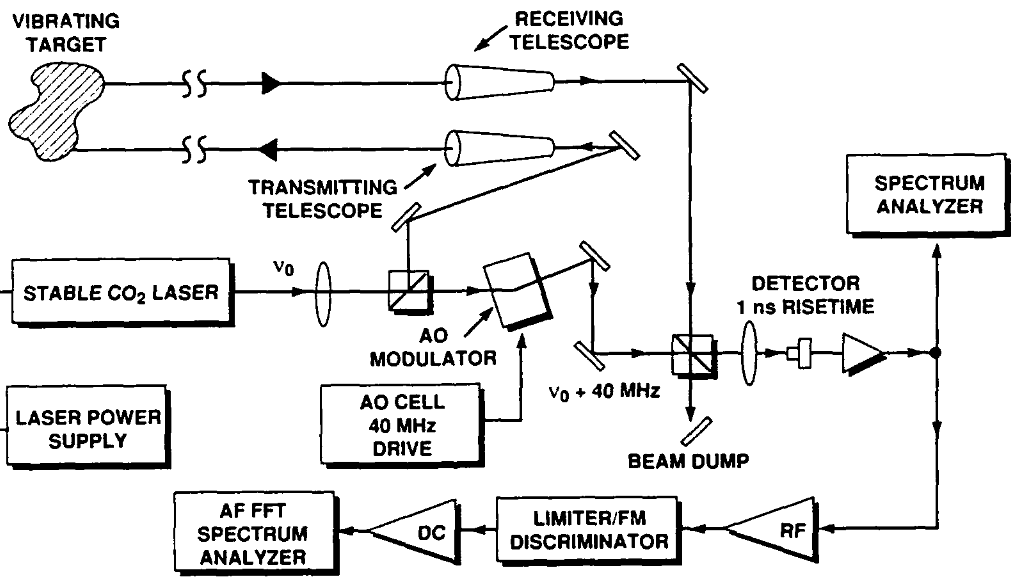


*Figure 5: $CO_2$ Laser Vibration Sensor*

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It has been mentioned in a number of places in this paper that the laser phase noise can be represented by the SSB phase noise quantity $\mathcal{L}(f)$ provided that the phase angle is small. This small modulation phase angle condition is usually satisfied easily at microwave frequency or lower. As it turns out, at optical frequency, satisfying this condition is even harder. Of the two lasers we have examined, neither one satisfies the small phase angle condition. They miss the condition by 20–40 dB. For the above reason, it is more appropriate to represent the frequency stability by the phase fluctuation function $S_\phi(f)$ rather than the SSB phase noise $\mathcal{L}(f)$. This is especially appropriate in laser radar systems using the CW FM technique. For example, in laser vibration measurements using FM demodulation technique, the laser phase fluctuation represents a noise input to the FM demodulator for signal detection consisting of a limiter, a discriminator, an envelope detector, and a low pass filter; the demodulator converts the input phase noise to amplitude noise at the output of the discriminator. Since the FM frequency discriminator demodulator is sensitive to a frequency change at the input, the phase noise or phase fluctuation must first be converted to a frequency fluctuation before it can be detected by the frequency discriminator. Accordingly, for CW FM vibration measurements, the input phase fluctuation, represented by $S_\phi(f)$, is first converted to a frequency fluctuation noise, represented by $S_\nu(f)$, before it is directed to the input of the frequency discriminator. Therefore, the relationship expressed by Eq.(8) is used

$$S_\nu(f) = f^2 S_\phi(f) \quad . \tag{8}$$

According to the operating principle of a frequency discriminator, the output noise power produced at the output of the discriminator demodulator over a filter bandwidth equal to the modulating frequency $f_m$ by a frequency fluctuation $S_\nu(f)$ at the input is

$$N_{output,\phi} = (2\pi K_{dis})^2 \int_0^{f_m} S_\nu(f)df = (2\pi K_{dis})^2 \int_0^{f_m} f^2 S_\phi(f) \times 2[1 - cos(2\pi f \tau_d)]df \quad , \tag{9}$$

where $K_{dis}$ is the discriminator response in units of volt/Hz. We have incorporated the delay function in Eq. (9) to account for the finite time delay effect on phase noise reduction.

Since $S_\phi(f)$ is proportional to $\mathcal{L}(f)$ and since $\mathcal{L}(f) \times 2[1 - cos(2\pi f \tau_d)]$ is approximately flat over the frequency range of interest, $S_\phi(f) \times 2[1 - cos(2\pi f \tau_d)]$ should also be flat over the same frequency range. For the Hughes laser, it can be seen from Fig. 2 that $S_\phi(f) \times 2[1 - cos(2\pi f \tau_d)]$ should be approximately equal to $-30$ dB $Rad^2/Hz$ or $1 \times 10^{-3}$ $Rad^2/Hz$ with respect to 1 $Rad^2/Hz$ for frequency from zero to about 1 kHz, taking into account that $S_\phi(f)$ is numerically a factor of 2 larger than $\mathcal{L}(f)$.

The output signal power at the discriminator output for a frequency deviation $\Delta f$ is proportional to the frequency deviation square. Since the frequency deviation is a peak quantity, the output signal power is

$$S_{output} = \frac{(2\pi K_{dis})^2}{2} \Delta f^2 \quad . \tag{10}$$

The SNR at the output of the discriminator demodulator assuming only phase noise is present is obtained by dividing Eq.(10) by Eq. (9) as follows:

$$SNR_{output,\phi} = \frac{1}{S_\phi(f) \times 2[1 - cos(2\pi f \tau_d)]} \frac{3\Delta f^2}{2f_m^3} \approx \frac{1}{1 \times 10^{-3}} \frac{3\Delta f^2}{2f_m^3} \quad . \tag{11}$$

We have just described the noise at the discriminator demodulator output produced by the laser phase noise input. In other words, the input carrier signal has a phase fluctuation or phase noise which is first converted to a frequency fluctuation before it is sent to the frequency discriminator to obtain the amplitude noise at the discriminator demodulator output.

Now we want to examine the noise at the discriminator output due to the laser local oscillator shot noise which is also present at the input to the discriminator. Since the shot noise is Gaussian, it is flat over the frequency range of interest with a double-sideband spectral noise power density $\eta_s/2 = eI_0R/2$ [6], where $e$ is the electronic charge, $I_0$ is the laser local oscillator current, and R is the detector load resistance, usually equal to 50 $\Omega$.

The noise at the discriminator output caused by this additive noise or equivalent frequency fluctuation corrupting the input carrier signal with amplitude $A$ has been shown to be [6]

$$N_{output,shot} = \frac{2}{3} (2\pi K_{dis})^2 \frac{\eta_s f_m^3}{A^2} \quad . \tag{12}$$

The SNR at the discriminator demodulator output is obtained by taking the ratio of Eq.(10) to Eq.(12) which is seen to be very similar to Eq.(30) [6]:

$$SNR_{output,shot} = \frac{3}{2}\frac{\Delta f^2}{f_m^2}\frac{S_i}{\eta_s f_m} \ , \tag{13}$$

where $S_i = A^2/2$ is the carrier signal power at the input to the discriminator. Equation(13) can be expressed in terms of the modulation index $\beta$ and the CNR input to the discriminator as follows:

$$SNR_{output,shot} = \frac{3}{2}\frac{\Delta f^2}{f_m^2}\frac{S_i}{\eta_s f_m} = \frac{3}{2}\frac{\Delta f^2}{f_m^2}\frac{S_i}{\eta_s \Delta f}\frac{\Delta f}{f_m} = \frac{3}{2}\beta^3 CNR \ , \tag{14}$$

where $\beta = \Delta f / f_m$ and the definition of CNR is

$$CNR = \frac{S_i}{\eta_s \Delta f} \ , \tag{15}$$

Let the ratio of $SNR_{output,shot}$ to $SNR_{output,\phi}$ be defined as $R_{s/\phi}$, we then have

$$R_{s/\phi} = CNR \times \Delta f \times S_\phi(f) \times 2[1 - cos(2\pi f \tau_d)] = CNR\frac{\Delta f}{10^3} \ . \tag{16}$$

For CNR+15.85, $\Delta f$=10 kHz, and $f_m$=300 Hz, the phase noise causes the output SNR to decrease by about 22 dB in comparison with that for the shot noise case. For higher values of CNR and frequency deviation, the output SNR deterioration becomes greater.

For lasers with lower phase noise or for smaller time delays, the deterioration is not as severe. As a matter of fact, if the Fabry–Perot cavity ultrastable laser is used, $R_{s/\phi}$ becomes approximately unity. This means that the shot-noise-limited condition is close to being attainable.

Note even though there is a large decrease in the output SNR due to the presence of the laser phase noise, the output SNR nevertheless is still very large. This large output SNR can only be attributed to the superior quiet detection scheme inherent in an FM system with a large frequency deviation and an above–threshold operation.

The case in which there are two signals close to each other in frequency but having different values of $\beta$ is examined next. Assuming that the first signal is denoted by $S_1$ and the second by $S_2$; the corresponding $\beta's$ are $\beta_1$ and $\beta_2$, respectively.

For the shot noise limited case, we let $\beta_1 = 33.3$ and CNR=15.85, the SNR at the calculated output is $8.3 \times 10^5$. If there is a weaker signal with a modulating frequency slightly different from $f_m$=300 Hz but with the same CNR (the input CNR does not change since it depends on the transmitter power and target reflectance) and a much smaller frequency deviation(the frequency deviation is target dependent) denoted by $\beta_2 = 0.444$. The output SNR is calculated to be SNR=$0.83 \times 2^3$=6.6, which means that the second signal can still be detected.

For the phase noise limited case, we let $\Delta f_1$=10 kHz and $\Delta f_2$=200 Hz. The calculated SNR's are $SNR_1$=5,556 and $SNR_2$=0.556×4=2.2, respectively. It means that the second signal with a smaller frequency deviation is barely detectable. Should $S_\phi(f)$ becomes larger, say $1\times10^{-2}$ instead of $1\times10^{-3}$, the effect of phase noise would become a lot more evident.

We see from the above illustrations that as the frequency deviation $\Delta f$ becomes smaller, the signal becomes smaller and eventually reaches an undetectable level. This is especially true when the input noise is phase noise.

## 7.1. The Addition of Thermal Noise to Shot Noise

In the preceding discussion on CW FM discriminator for laser vibration measurements, we had assumed that the shot noise is much higher than the thermal noise. If this is not the case, the thermal noise can be taken in consideration by modifying Eq.(12) to read

$$N_{output,thermal} = \frac{2}{3}\,(2\pi K_{dis})^2\,\frac{\eta_t f_m^3}{A^2}\ , \tag{17}$$

where $\eta_t = \frac{kT}{2}$ denotes the two-sided thermal noise spectral power density. This is to be compared with the two sided shot noise spectral power density at low frequencies given by $\eta_{shot}/2 = eI_0R/4$.

Furthermore, by dividing Eq.(10) by the sum of Eqs.(12) and (17), and defining

$$CNR' = \frac{S_i}{(\eta_s + \eta_t)\Delta f}\ , \tag{18}$$

the output SNR changes from that shown in Eq.(14) to

$$SNR_{output,shot+thermal} = \frac{3}{2}\frac{\Delta f^2}{f_m^2}\frac{S_i}{(\eta_s + \eta_t)f_m} = \frac{3}{2}\frac{\Delta f^2}{f_m^2}\frac{S_i}{(\eta_s + \eta_t)\Delta f}\frac{\Delta f}{f_m} = \frac{3}{2}\beta^3 CNR'\ . \tag{19}$$

The reason that the thermal and shot noises can be summed is that they are both power spectral density quantities and they are independent. Hence both noises can be considered to be additive Gaussian white noises.

## 8. LOW CNR′ AND CLICK NOISE

In the preceding sections, we have assumed that the input CNR is high. When this is not the case, the output SNR is different because the FM noise for low CNR′ is higher due to click noise. Neglecting the effect of the modulation on the carrier, the amount of click noise power at the discriminator output has been determined to be [6]

$$N_{output,click} = \frac{(2\pi K_{dis})^2 \Delta f \cdot f_m \cdot erfc(\sqrt{CNR'})}{\sqrt{3}}\ , \tag{20}$$

where $\Delta f$ is considered to be the $IF$ bandwidth and $erfc$ is the complementary error function. Combining Eq.(20), Eq.(12), and Eq.(17) leads to the expression for the total noise due to thermal noise, shot noise, and click noise

$$N_{output,thermal+shot+click} = (2\pi K_{dis})^2 \times \left[\ \frac{2(\eta_s + \eta_t)f_m^3}{3A^2} + \frac{\Delta f \cdot f_m \cdot erfc(\sqrt{CNR'})}{\sqrt{3}}\right]\ . \tag{21}$$

The complementary error function $erfc(x)$ has the properties that it is unity at $x$=0; it becomes 0.157 at $x = 1$ and $7.21 \times 10^{-3}$ at $x = 2$ or CNR=4. At larger values of $x$, it can be approximated by $e^{x^2}/(x\sqrt{\pi})$. Therefore, at a CNR value of 12 dB or CNR=15.85, we have $x = \sqrt{15.85} \approx 4$ and $erfc(x) = 1.5 \times 10^{-8}$, which says that the click noise as defined in Eq.(20) automatically becomes very small.

In addition, at low values of CNR$'$, the output signal has to be modified to read

$$S_0 = \frac{(2\pi K_{dis}\Delta f)^2}{2}(1 - e^{-CNR'}) \ . \tag{22}$$

The output SNR is obtained by taking the ratio of Eq.(22) to Eq.(21) as follows:

$$SNR_{output,thermal+shot+click} = \frac{\frac{(\Delta f)^2}{2}(1 - e^{-CNR'})}{\frac{2(\eta_s+\eta_t)f_m^3}{3A^2} + \frac{\Delta f \cdot f_m \cdot erfc(\sqrt{CNR'})}{\sqrt{3}}} \ . \tag{23}$$

The output SNR including the laser phase noise can also be obtained by including it in Eq.(23) to read:

$$SNR_{out,thermal+shot+click+\phi} = \frac{\frac{(\Delta f)^2}{2}(1 - e^{-CNR'})}{\frac{2(\eta_s+\eta_t)f_m^3}{3A^2} + 4\int_0^{f_m} f^2 S_\phi(f) sin^2(\pi f \tau_d) df + \frac{\Delta f \cdot f_m \cdot erfc(\sqrt{CNR'})}{\sqrt{3}}} \ . \tag{24}$$

Substituting $S_i$ for $A^2/2$ into Eq.(24), using Eq.(18), and the definition for the modulation index $\beta = \Delta f / f_m$, Eq.(24) becomes

$$SNR_{out,thermal+shot+click+\phi} = \frac{\frac{3}{2}\beta^3 CNR'(1 - e^{-CNR'})}{1 + 12\frac{\beta CNR'}{f_m^2}\int_0^{f_m} f^2 S_\phi(f) sin^2(\pi f \tau_d) df + \sqrt{3}CNR'\beta^2 \cdot erfc(\sqrt{CNR'})} \ . \tag{25}$$

Equation(25) says that at the shot–noise–limited operation, the SNR is given by the familiar $\frac{3}{2}\beta^3 CNR'$ when operating above threshold.

Figure 6 shows that plots of the calculated SNR using Eq.(25) assuming that the range of CNR$'$ value is achieved for different vibrometer operating conditions at ranges of 20 km and 200 km; at 20 km, the phase noise of the waveguide laser is sufficiently low so that the SNR(dashed curve) is essentially close to that for the shot–noise–limited case(solid curve) for a modulation frequency of 500 Hz and a modulation index of 5.

However, when the range is increased to 100 km, the output SNR(dotted curve) is deteriorated by a factor as high as 20 for the same modulation index of 5 due to the laser oscillator phase noise. Here we again assume that the the range of CNR$'$ value is also achieved for this longer range of vibrometer operating conditions. Maintaining the range of CNR$'$ values can be done by either increasing the target reflectance or increasing the transmitted power.

Since it has been shown that the Fabry–Perot cavity ultrastable $CO_2$ laser has a lower phase noise, we would like to compare its SNR with that for the waveguide laser. First, at a range of 10 km, there is only a small deterioration in the output SNR for the waveguide laser(dotted curve) and there is no measurable deterioration of the SNR for the Fabry–Perot cavity laser. On the other hand, at a range of 200 km, a modulation frequency of 500 Hz, and a modulation index of 5, Fig. 7 shows that for the ultrastable laser, the deterioration in SNR(dashed curve) is still quite small, less than about 30 % or 2 dB, which is very much smaller than the deterioration factor of about 38 or 16 dB for the waveguide laser(see the dotted–dashed curve)!

Thus, the above figures clearly illustrate the effect of the operating parameters on the output SNR of a CW laser vibrometer using an FM discriminator for signal demodulation. The range, in particular, is a very critical parameter once it exceeds some nominal value which is dependent on the laser phase noise or frequency instability.

## 9. SPECKLE NOISE

When the target surface is not smooth, the laser return signal contains speckle noise so that at the output of the discriminator we must add the speckle noise in addition to the click noise. The speckle noise has a bandwidth which depends on the type of target motion. It can be as wide as tens of kHz for a distant rotating object or a nearby object going through a very rapid tilting motion.

The combined thermal shot, click and speckle noise is [7]

$$N_{output,c} = N_{output,thermal,shot} + N_{output,click} + N_{output,speckle}$$

$$= (2\pi K_{dis})^2 \times \{\frac{2(\eta_s + \eta_t)f_m^3}{3A^2} + \frac{f_m}{\sqrt{3}}[\Delta f \cdot erfc(\sqrt{CNR'}) + B_s \cdot erf(\sqrt{CNR'})]\} \quad , \tag{26}$$

where $N_{output,click}$ and $N_{output,speckle}$ are the click noise and speckle noise, respectively, and $erf$ and $erfc$ denote the error function and complementary error function, respectively. The terms containing the expression CNR$'$ explicitly have been taken from [7].

In order to determine the speckle noise, the speckle bandwidth $B_s$ of the target has to be known. For a target at range R that is tilting at an angular rate of $\dot{\theta}$, the speckle bandwidth for a collimated Gaussian beam propagating from the transmitter where the beam waist radius is w can be calculated using the formula

$$B_s = \frac{\dot{\theta}R}{w} \quad . \tag{27}$$

## 10. OVERALL SNR AT FM DISCRIMINATOR DEMODULATOR OUTPUT

The overall SNR at the output of the FM discriminator demodulator can be determined by taking the ratio of the signal to the sum of all noises. The sum of all noises include the thermal noise, the local oscillator shot noise, the laser phase noise, the FM click noise at low CNR$'$, and the dynamic speckle noise due to target motion. At the discriminator output, the noises can be determined using Eq.(9) for the phase noise and Eq.(26) for the thermal, shot, click, and speckle noises. Therefore, the total noise (normalized to $(2\pi K_{dis})^2$ can be written as

$$\frac{N_{output,total}}{(2\pi K_{dis})^2} = \frac{2(\eta_s + \eta_t)f_m^3}{3A^2} + \int_0^{f_m} f^2 S_\phi(f) \times 2[1 - cos(2\pi f \tau_d)]df$$
$$+ \frac{f_m}{\sqrt{3}}[\Delta f \cdot erfc(\sqrt{CNR'}) + B_s \cdot erf(\sqrt{CNR'})] \quad . \tag{28}$$

Substituting $S_i$ for $A^2/2$ into Eq.(28) and using the relation

$$\frac{(\eta_s + \eta_t)f_m^3}{3S_i} = \frac{f_m^2}{3\beta CNR'} \quad , \tag{29}$$

Eq.(28) becomes

$$\frac{N_{output,total}}{(2\pi K_{dis})^2} = \frac{f_m^2}{3\beta CNR'} + \int_0^{f_m} f^2 S_\phi(f) \times 2[1 - cos(2\pi f \tau_d)]df$$
$$+ \frac{f_m}{\sqrt{3}}[\Delta f \cdot erfc(\sqrt{CNR'}) + B_s \cdot erf(\sqrt{CNR'})] \quad . \tag{30}$$

The final SNR at the output of the discriminator, $SNR_0$, is obtained by taken the ratio of Eq.(22) to Eq.(30) which is

$$SNR_0 = \frac{\frac{\Delta f^2}{2}(1 - e^{-CNR'})}{\frac{f_m^2}{3\beta \cdot CNR'} + 4\int_0^{f_m} f^2 S_\phi(f) sin^2(\pi f \tau_d) df + \frac{f_m}{\sqrt{3}}[\Delta f \cdot erfc(\sqrt{CNR'}) + B_s \cdot erf(\sqrt{CNR'})]} \quad . \tag{31}$$

Because the amplitude of the return signal is Rayleigh distributed, unless the signal is hard-limited, the amplitude fluctuation is very large. Even though a limiter is used, it is still more convenient to deal with the average CNR. To do this average, the amplitude is still considered to be Rayleigh distributed after limiting. Using the relationship $erfc(x) = 1 - erf(x)$, the (normalized) expected total noise was determined to be given approximately by [7]

$$\frac{E\{N_T\}}{(2\pi K_{dis})^2} \approx \frac{f_m^2}{3\beta \overline{CNR'}} + 4\int_0^{f_m} f^2 S_\phi(f) sin^2(\pi f \tau_d) df$$
$$+ \frac{f_m}{\sqrt{3}}[\Delta f \cdot erfc(\sqrt{\overline{CNR'}}) + B_s \cdot erf(\sqrt{\overline{CNR'}})] \quad . \tag{32}$$

The output SNR when the noise is treated to be an expected noise is [7]

$$\overline{SNR_0} \approx \frac{\frac{\Delta f}{2}(1 - e^{-\overline{CNR'}})}{\frac{f_m^2}{3\beta \overline{CNR'}} + 4\int_0^{f_m} f^2 S_\phi(f) sin^2(\pi f \tau_d) df + \frac{f_m}{\sqrt{3}}[B_s + \frac{\Delta f - B_s}{2\overline{CNR'}\sqrt{1+1/\overline{CNR'}}}]} \quad . \tag{33}$$

It can be put in a slightly different form as follows:

$$\overline{SNR_0} \approx \frac{\frac{3}{2}\beta^3 \overline{CNR'}(1 - e^{-\overline{CNR'}})}{1 + \frac{12\beta \overline{CNR'}}{f_m^2}\int_0^{f_m} f^2 S_\phi(f) sin^2(\pi f \tau_d) df + \frac{\sqrt{3}\beta \overline{CNR'}}{f_m}[B_s + \frac{\Delta f - B_s}{2\overline{CNR'}\sqrt{1+1/\overline{CNR'}}}]} \quad . \tag{34}$$

The present expression for the expected SNR as represented by Eq.(34) differs from the results reported by Barr[7] in that both the shot+thermal term and the phase noise term were absent in the above report[7].

Figure 8 illustrates the output SNR of the laser vibrometer using the waveguide laser as a function of the input CNR for various changes in the operating parameters such as modulation frequency, modulation index, and speckle frequency bandwidth for a fixed range of 100 km. For $f_m = 500$ Hz, $\beta = 5$, and $B_s = 1000$ Hz, the deterioration of the SNR by the speckle noise is very large. The deterioration factor is about 20 from the shot-noise-limited value to the value in the presence of phase noise and about 15 from the value in the phase noise alone to the value including both the phase noise and the speckle noise .

Judging from the preceding discussions, In any particular application, it is perhaps advisable to examine the parameters more closely for a more definitive evaluation. Nevertheless, the figures serve to illustrate many important points in laser sensing of target vibrations.

## 11. Laser Oscillator Phase Noise Requirement for a CW Linear FM Laser Radar

For a symmetrical linear frequency modulation (LFM) laser radar, the laser oscillator stability requirement has been examined. For illustration purposes, the results of our study on a short range imaging

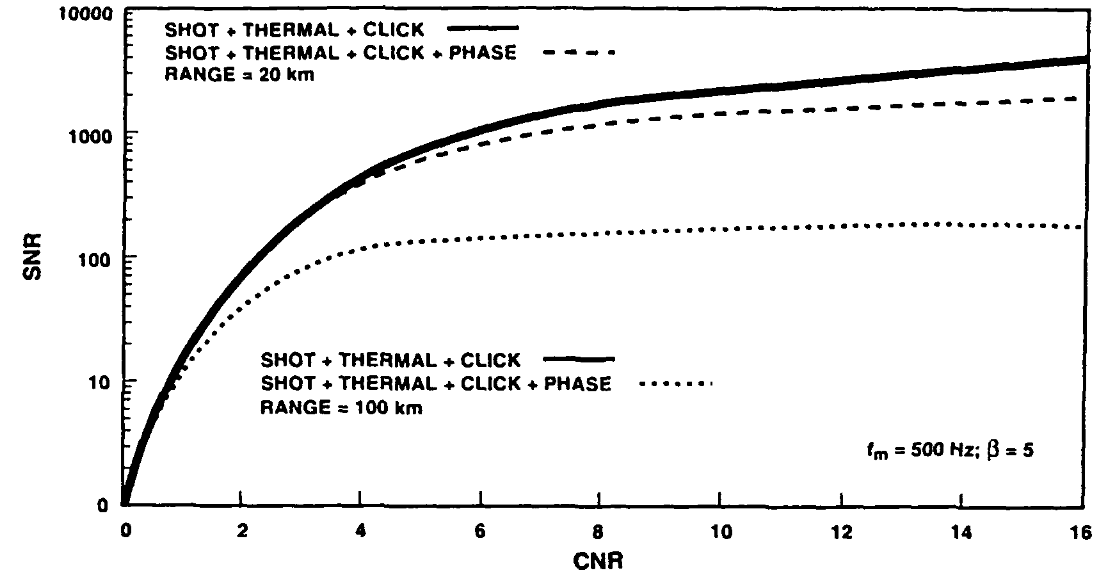


*Figure 6: SNR of Laser Vibrometer using Waveguide Laser (Glint Target): $f_m$=500 Hz; $\beta$=5; range=20 km and 100 km*

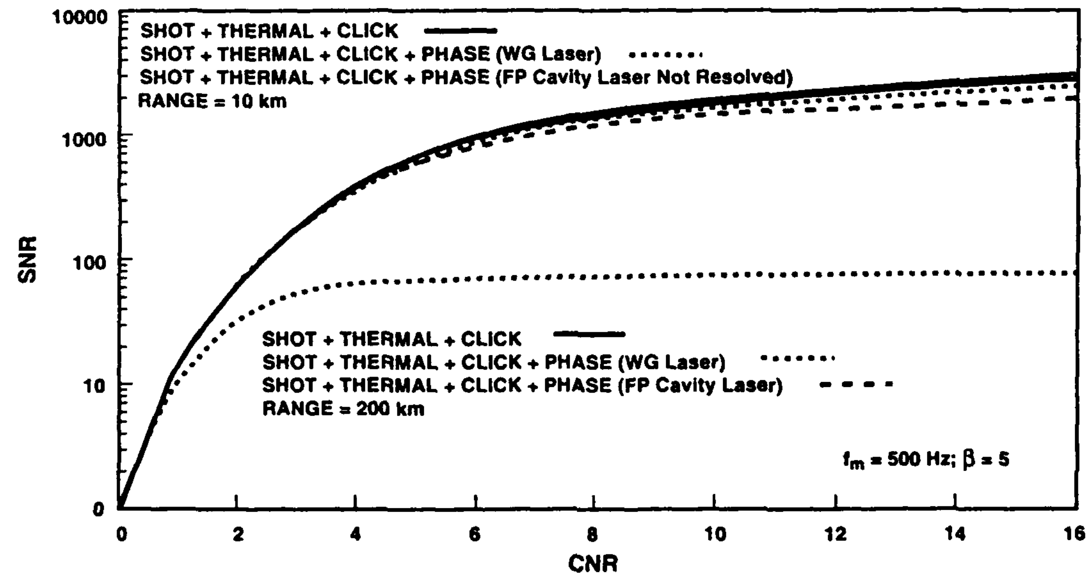


*Figure 7: SNR of Laser Vibrometer using Different Lasers (Glint Target): $f_m$=500 Hz; $\beta$=5; range=10 km and 200 km*

laser radar using a CW diode laser is presented here. The operating range is 1 km; the laser power is 10 mW. The symmetrical LFM waveform is to have a frequency deviation of 10 GHz and a period of 40 $\mu$s. Assuming that the laser radar is in a scanning mode of operation, the pixel dwell time is 20 $\mu$s and there are 50 pixels per frame. The post detection bandwidth is assumed to be 10 MHz. The delay is 6.67 $\mu$s and the frequency deviation or offset corresponding to this delay is 3.33 GHz. For this combination of delay and frequency offset, it can be shown that there is no reduction in the laser phase noise and $K^2$ is set equal to 4. Using a procedure similar to that used in the microwave region [8], the SSB phase noise at the offset

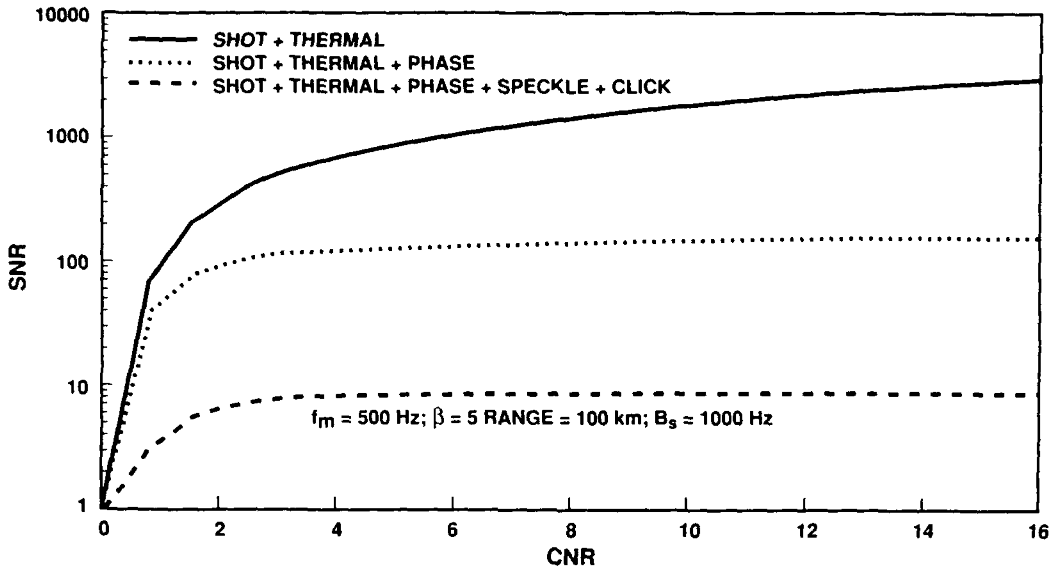


*Figure 8: SNR of Laser Vibrometer using Waveguide Laser (Diffuse Target): $f_m$=500 Hz; $\beta$=5; range=100 km; $B_s$=1000 Hz*

frequency of 3.33 GHz is

$$\mathcal{L}(f) = 10[log(\frac{r_{target}}{r_{clutter}}) - log(SNR) - log(K^2) - log(B) = -10 - 17 - 6 - 70 = -103 dBc/Hz \quad (35)$$

Here a background clutter return $r_{clutter}$ or interference level is assumed to be 10 dB higher than the laser radar return from the target, $r_{target}$ and the required SNR is assumed to be 17 dB. the required laser linewidth (FWHM) for this CW coherent laser radar is calculated to be approximately 24 kHz if the phase noise of the laser diode oscillator can be represented adequately and simply by a Lorentzian lineshape; that is, the phase noise falls off quadratically with the offset frequency measured from the carrier position.

## 12. Summary and conclusions

- Laser phase noise has been examined for two stable $CO_2$ lasers for different time delays;
- Methodology has been developed for examining laser frequency stability requirement for laser radars;
- Phase noise has been shown to limit Doppler resolution in a Doppler laser radar;
- Laser phase noise degrades measurement sensitivity, making quantum-noise–limited detection of glint target surface vibration unattainable;
- For diffuse targets in dynamic motions, further degradation is likely so that laser phase noise requirement is less stringent;
- The laser phase noise requirement for a laser radar using a symmetrical LFM waveform has also been investigated.

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## 14. REFERENCES


1. J. H. Shoaf, "Specification and Measurement of Frequency Stability, "National Bureau of Standards Report No. NBC Report NBSIR 74-396, Nov., 1974.

2. R. J. Nordstrom, L. Berg, H. Grieneisen, and A. Javan, "Remote Oscillator/Clock (ROCK) Review and Update," Paper presented at the 1990 Active IRIS Conference, Silver Spring, MD. October 16–18, 1990.

3. F. L. Walls, J. Gary, A. O'Gallagher, R. Sweet, and L. Sweet, "Time Domain Frequency Stability Calculated from the Frequency Domain Description," Nat. Inst. of Standards and Technol., NISTIR-89-3916Revised, Sept. 1991.

4. S. J. Goldman, Phase Noise Analysis in Radar Systems Using Personal Computers, J. Wiley and Sons, New York, 1989, Chapter 2.

5. C. Freed, "Ultrastable $CO_2$ Lasers," The Lincoln Laboratory Journal 3, 479(1990).

6. H. Taub and R. L. Schilling, Principles of Communications and Systems, McGraw Hill, New York, 1971, Chapter 4.

7. D. N. Barr, "Speckle Effects on Laser Vibration Sensor Noise," Army Night Vision Laboratory Report, 1985.

8. W. P. Robins, "Phase Noise in Signal Sources," Peter Peregrinus Ltd., London, UK, 1982. Pages 224–232.