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The proposal dealt with several statistical inference problems in the context of univariate and multivariate linear models having effects, i.e., fixed effects as well as random effects. Most of the articles that were completed under the grant are on the following topics: (i) improved nonnegative estimation of variance components, (ii) exact and/or optimum tests for the fixed effects, and (iii) exact and/or optimum tests for variance components. The results have the potential for applications in all areas where the mixed model methodology is used and the applications are highlighted in most of the articles that were written based on the problems in the proposal.

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Final Technical Report of the research work based on the proposal AFOSR 89-0237 for the period January 1, 1989 – December 31, 1992.

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March 10, 1993

The proposal dealt with several statistical inference problems in the context of univariate and multivariate linear models having mixed effects, *i.e*, fixed effects as well as random effects. Most of the articles that were completed under the grant are on the following topics: (*i*) improved nonnegative estimation of variance components, (*ii*) exact and/or optimum tests for the fixed effects, and (*iii*) exact and/or optimum tests for variance components. The results have the potential for applications in all areas where the mixed model methodology is used and the applications are highlighted in most of the articles that were written based on the problems in the proposal. (A list of the papers that were completed under the grant is given at the end). Given below is a brief summary of the results obtained under (*i*), (*ii*) and (*iii*) above.

In our work on the estimation of variance components, we have attempted to provide a satisfactory answer to the problem of nonnegative estimation of variance components. As is well known, even though the variance components are nonnegative, some of their standard estimators (for example, the ANOVA estimator) can assume negative values, making them unacceptable for practical use. This problem has received considerable attention in the variance components literature. Even though several approaches are available to deal with this problem, the alternative estimators that are available in the literature have several drawbacks such as: very often the estimators are not quadratic forms in the data, they do not have good mean squared error (MSE) properties, and in particular, they may not provide significant reduction in MSE compared to some of the standard unbiased estimators. In a series of articles (see references # 4, 5, 6, 8, 13, 16 and 17 among the list of papers at the end), we have addressed this problem mostly for linear models involving two variance components, one variance component (say σ_1^2) corresponding to a random effect, and a second variance component (say σ^2) corresponding to the experimental errors. Generalizations to more general models have also been obtained, including some multivariate extensions. For the estimation of σ_1^2 mentioned above, we have derived nonnegative quadratic estimators (whenever they exist) uniformly better than unbiased quadratic estimators of σ_1^2. Such improved estimators do not always exist. However, quadratic estimators having uniformly smaller MSE and uniformly smaller probability of negativity, compared to quadratic unbiased estimators, always exist. We have derived such estimators and our recommendation is to truncate such improved estimators at zero, in order to arrive at nonnegative estimators. The reduction in MSE that can be achieved over the standard unbiased estimators turns out to be substantial. Our results are expected to provide a satisfactory solution to the problem of nonnegative estimation of variance components.

In applications involving the use of a mixed effects or random effects model, testing the significance of the fixed effects and variance components is an important part of data analysis. For several ANOVA models, both univariate and multivariate, we have derived both exact and optimum tests for testing such hypotheses (see references # 1, 2, 3, 9, 10, 11, 12 and 19). In most cases, optimum tests exist only under certain conditions and these conditions have been obtained for the various models considered. Our work that needs special mention in this context are the following: (*i*) derivation of exact and optimum tests in unbalanced split-plot designs under mixed and random models, (*ii*) application of the recent concept of a *generalized p-value* to obtain tests for variance components in some set ups where the standard test procedures are not applicable, and (*iii*) combining independent tests from several independent models for testing hypotheses concerning a common mean or a common variance component. Even though the split-plot design is an important and widely used experimental design in factorial experiments, its analysis has been available only in the balanced case. In our work (reference # 3), we have considered some unbalanced situations involving the split-plot design and have derived both exact and optimum tests under all possible mixed effects and random effects models.

The concept of a *generalized p-value* is due to Tsui and Weerahandi (1989, *Journal of the American Statistical Association*) and is useful for testing hypotheses in certain situations where it is difficult or impossible to identify a test of a fixed size. We have used the generalized p-value to test hypotheses in two situations (reference # 15). The first is to test the significance of a main effect variance component in a general balanced mixed model when an exact F-test does not exist. The usual procedure in this case is to use the Satterthwaite approximation. However, it is known that the Satterthwaite approximation is not always satisfactory. The approach using generalized p-value leads to a unique test in this case. Our second application deals with testing the equality of two random effects variance components in two independent mixed models. For example, the problem could be to test the equality of the treatment variance components in two independent one-way random models. An application of this type came up at the U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, where the problem was to compare two types of tubes used on military tanks for firing ammunition. It was desired to test if tube-to-tube dispersion was less in a new tube compared to a control tube, *i.e.*, to compare the two variance components representing the variability among the new tubes and the control tubes. A test having a specified size cannot be obtained for this problem except in some very special cases. The generalized p-value is quite useful in this case. In our article dealing with this topic (reference # 15), we have also analyzed some data obtained from the U. S. Army Ballistic Research Laboratory, in the context of the example mentioned above.

Statistical inference problems dealing with combining information from several independent sources has been an active area of research. The area of *Meta-Analysis* deals with this issue and has important applications in the social and behavioral sciences. A rather comprehensive account of this is available in the book by Hedges and Olkin (1985, *Statistical Methods for Meta-Analysis*). The issue of efficiently combining information also arises in the interblock analysis of block designs. The purpose of combining the results from several studies could be to obtain a point estimate, or to test a hypothesis, for a parameter that is common across the studies. In a series of papers (see reference ≠ 9, 11, 12, 19), we have considered the hypothesis testing issue when there are several independent linear models having a common vector parameter in their means and the problem is to test hypothesis concerning this common parameter. Cohen and Sackrowitz (1989, *Journal of the American Statistical Association*) have considered this problem in the context of the inter-block analysis of a balanced incomplete block design. We have extended the Cohen-Sackrowitz test and have suggested several other tests for this problem. Our procedures are also applicable for testing hypotheses concerning a common variance component. A numerical comparison of the powers of the various tests have been carried out and it has been possible to make specific recommendations regarding the choice of the test to be used in practical applications. The procedure has been extended to the multivariate set up as well.

In addition to the research work mentioned above, I have started working on a book, jointly with Bimal Sinha and Andre' Khuri. The book is tentatively titled *Statistical Tests in Mixed Linear Models*, and we are currently negotiating with the various publishers. We already have a contract from Springer-Verlag.

The Graduate Student, Mr. Leping Zhou, was partially supported under the grant and he graduated with a Ph. D. in Statistics in the summer of 1992.

Research Papers Prepared Under Grant AFOSR 89-0237

Published papers

1. Mathew, T. (1989). Optimum tests in mixed linear models with two variance components. In *Statistical Data Analysis and Inference*, Proceedings of a Conference in honor of C. R. Rao held in Neuchatel, Switzerland (ed. Y. Dodge), pp. 381-388, North-Holland.

2. Mathew, T. and Sinha, B. K. (1991). Towards an optimum test for non-additivity in Tukey's model. *Journal of Multivariate Analysis* **36**, 68-94.

3. Mathew, T. and Sinha, B. K. (1992). Exact and optimum tests in unbalanced split-plot designs under mixed and random models. *Journal of the American Statistical Association* **86**, 192-200.

4. Mathew, T., Sinha, B. K. and Sutradhar, B. (1992). Nonnegative estimation of random effects variance components in general balanced mixed models. In *Nonparametric Statistics and Related Topics* (ed. A. K. Md. E. Saleh), North-Holland, pp. 281-295.

5. Mathew, T., Sinha, B. K. and Sutradhar, B. (1992). Nonnegative estimation of variance components in unbalanced mixed linear models with two variance components. *Journal of Multivariate Analysis* **42**, 77-101.

6. Mathew, T., Sinha, B. K. and Sutradhar, B. (1992). Improved estimation of error variance in general balanced mixed models. *Statistics and Decisions* **10**, 227-238.

7. Mathew, T. and Nordstrom, K. (1993). Least squares and least absolute deviation procedures in approximately linear models. *Statistics and Probability Letters* **16**, 153-158.

Papers accepted for publication

8. Kelly, R. J. and Mathew, T. Improved estimators of variance components with smaller probability of negativity. To appear in *Journal of the Royal Statistical Society, Series B* (1993).

9. Zhou, L. and Mathew, T. Combining independent tests in linear models. To appear in *Journal of the American Statistical Association* (1993).

10. Zhou, L. and Mathew, T. Hypotheses tests for variance components in some multivariate mixed models. To appear in *Journal of Statistical Planning and Inference* (1993).

11. Mathew, T., Sinha, B. K. and Zhou, L. Some statistical procedures for combining independent tests. To appear in *Journal of the American Statistical Association* (1993).

12. Zhou, L. and Mathew, T. A note on combining independent tests for a common mean. To appear in *Journal of Combinatorics, Information and System Sciences* (1993).

13. Mathew, T. and Sinha, B. K. Nonnegative estimation of variance components in general mixed linear models: a survey. To appear in *Festschrift for R. R. Bahadur*.

Papers submitted for publication

14. Mathew, T. and Bhaumik, D. Minimaxity of randomized optimal designs with respect to a general optimality criterion.

15. Zhou, L. and Mathew, T. Some tests for variance components using generalized p-values.

16. Kelly, R. J. and Mathew, T. Improved nonnegative estimation of variance components in some mixed models with unbalanced data.

17. Mathew, T., Niyogi, A. and Sinha, B. K. Improved nonnegative estimation of variance components in balanced multivariate mixed models.

18. Mathew, T. and Kasala, S. An exact confidence region in multivariate calibration.

19. Zhou, L. and Mathew, T. Combining independent tests in multivariate linear models.

20. Khuri, A. I. and Mathew, T. A test to detect inadequacy of multivariate Satterthwaite approximation.