Probability of Detection of Drug Users by Random Urinalysis in the U. S. Navy

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Drug use in the Navy and civilian sector, the source of new Navy personnel, continues to concern Navy managers. The Navy currently conducts a random urinalysis program to deter and detect drug abuse. Navy field commanders, who are responsible for execution of the testing program at their units, have latitude in both the frequency and rate of drug testing. The entire program is governed by strict procedural and legal safeguards. However, no systematic analysis of the sampling program has been undertaken to discover cheaper, more effective alternatives. This paper develops probabilistic models for urinalysis testing. Several observations are made concerning current practice, both with and without gaming. An alternative testing procedure is analyzed. Finally, a number of recommendations are made.

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Foreword

This report was prepared as part of the Drug Detection Screening Algorithm project (funded by program element 0909000N and work unit 91P0DD651), under the sponsorship of the Bureau of Naval Personnel (PERS-63). The objective of the project is to develop a random urinalysis methodology (algorithm) that will maximize deterrence of drug use at the lowest cost. The work described here was performed during FY92.

This work was first briefed to the sponsor on 15 January 1992. It was an invited presentation at the Tri-Service Drug Laboratory Managers' Meeting in San Antonio, TX on 15 April 1992 and it was presented at the Military Operations Research Society Symposium in Monterey, CA on 23 June 1992.

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MURRAY W. ROWE
Director, Manpower Systems Department

Summary

Problem

Drug use in the Navy and the civilian sector, the source of new Navy personnel, continues to concern Navy managers. The Navy currently conducts a random urinalysis testing program to deter and detect drug abuse. Prevalence rates reported by the Navy Drug Screening Laboratories have dropped from approximately 7 percent to 1 percent during the past 9 years. A positive rate of 1 percent could mean that the entire population (600,000 people) uses drugs 1 percent of the time, or that 10 percent of the population (60,000 people) uses drugs 10 percent of the time, or any number of other combinations.

Navy field commanders, who are responsible for execution of the testing program at their units, have latitude in both the frequency and rate of drug testing. The entire program is governed by strict procedural and legal safeguards. However, no systematic analysis of the sampling program has been undertaken to discover cheaper, more effective alternatives.

Objective

This paper describes general models for urinalysis testing. These models allow calculation of the probability of detection of drug use as well as the amount of drug use possible under simple gaming strategies. The models can be used to design drug testing policies that meet the objectives of Navy managers.

Results

A model for the probability of detection of drug users within a given time was developed. This model is based on the following assumptions: (1) a fixed testing period measured in days, (2) a simple random sample of testing days, (3) a simple random sample of people on each testing day, and (4) a member has drugs detectable in their system some fixed number of days during the testing period. For example, suppose the Navy tests 1 day per month and a member has drugs detectable in their system 6 days per month. Figure 1 shows the cumulative probability of detection versus time for 10 percent and 20 percent per month testing rates.

A simplistic drug user gaming strategy was developed. Under this gaming strategy the probability of detection is **zero** and the drug user's average usage can be calculated. This strategy is based on the following assumptions: the number of days of sample collection per time period is known to the drug user, and the drug user waits until after the last collection day to use drugs. Figure 2 shows the upper limit for the average number of days drugs could be in a member's system versus the number of sampling days. Here we assume the sampling period is monthly.

Conclusions

The current practice of random urinalysis is susceptible to gaming by drug abusers. The typical practice of testing on a fixed number of days per month (usually one) leaves a large number of days with no threat of testing. The drug abuser can simply wait until after the last sample collection day to use drugs. This allows a window of use with **zero** probability of detection.

This gaming can be eliminated by testing every day or by using an alternative urinalysis strategy. The alternative strategy is to test each day with probability strictly greater than zero and less than or equal to one. The effect of either of these solutions is to maintain a positive chance (threat) of being tested throughout the testing period.

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1.0 Introduction

In 1981 the U.S. Navy introduced a *zero tolerance* drug policy. Since then the Navy has pursued an aggressive drug abuse detection and deterrence program. The proportion of service members sampled that tested positive for drugs fell from 7 percent to 1 percent from 1983 through 1991.[1] All service members, officer and enlisted, are processed for separation for the first drug abuse incident.

The cornerstone of the deterrence program is random urinalysis testing. All officer and enlisted personnel are subject to testing on a continuing basis. Current policy (Chief of Naval Operations, 1990) directs Navy commands to test approximately 10 to 20 percent of their members each month. Commanding officers are responsible for enforcing Navy drug abuse policy. This policy allows latitude in the frequency (how often to collect samples in any given month) and rate (how many people to sample) of drug testing.

An informal survey of Navy commands was conducted to determine how the random urinalysis policy is implemented. Details of this survey are reported in Chipman, Thompson, Mosteller, Hentschel and Boyle (1992). The model of current practice developed in this paper is a result of the information learned and is consistent with all the programs surveyed.

Currently Bureau of Naval Personnel (PERS-63) wants to standardize and automate the random urinalysis procedure. Their objectives include development of a methodology that provides maximum deterrence at the lowest cost. However, no systematic analysis of the sampling program has been undertaken. Some related work can be found in Evanovich (1985) and Stoloff (1985).

This report begins with a description of a model for detection based on current Navy practice. Then, implications for detection and deterrence of allowable variations in frequency and rate of testing are discussed. Analysis of gaming strategies possible under current policy are also presented. Finally, an alternative testing model, less sensitive to gaming strategies, is developed.

2.0 Modeling the Current Program

2.1 Model for Detection

In this section, a model for the probability of detection of illegal drugs is presented. The model is consistent with the current practice of random urinalysis in the Navy. The model is based on four assumptions.

1. The testing period is a fixed number of days. Since the Navy's program is conducted on a monthly basis, 30 days is used as the testing period. The model, though, considers the general case with the size of the testing period any fixed value.

2. A simple random sample of days is drawn from the set of days in the testing period. The observed size of this sample is usually one, with occasional values as high as four or five.

[1]Reflects prevalence rates reported by the Navy Drug Screening Laboratories (NDSLs). Prevalence rates are the number of positive results divided by the total samples tested for some fixed time period (e.g., week).

3. On each of the days sampled, a simple random sample of people is drawn from the total population of members at a given command. Navy policy states that 10 to 20 percent of each command shall be sampled monthly. Thus, a command desiring to test 20 percent may sample 20 percent 1 day each month, or 10 percent twice a month, and so forth.

4. A member has drugs detectable in their system for some fixed number of days during the testing period. Based on these assumptions the probability of detection of a drug user during a single testing period can be developed. A complete mathematical specification follows.

Let $Pr(\text{DET})$ be the probability of detection of illegal drugs. Let $D = \{D_1, D_2, \ldots, D_M\}$ be a set of days and let $T = \{T_1, T_2, \ldots, T_K\}$ be a simple random sample of K days from D. On each of these T_i days a simple random sample of size n_i is drawn from the population of service members. Let the member population size equal N and let $n_1 + n_2 + \ldots + n_K = n$. Note that $0 \le n_i \le N, \forall\, i$.

Suppose a given user has drugs in their system m days out of M. Let S equal the set of these m days and let Z be the number of days in the intersection of T and S. Initially we choose $n_i = n / K$; that is, all n_i are equal. Then the probability of detection conditional on $Z = k$ is:

$$Pr(\text{DET} \mid Z = k) = 1 - \left(1 - \frac{n}{KN}\right)^k . \tag{1}$$

The probability distribution of Z is hypergeometric and $Pr(Z = k)$ is:

$$Pr(Z = k) = \binom{m}{k}\binom{M-m}{K-k} \Big/ \binom{M}{K} . \tag{2}$$

Therefore:

$$Pr[\text{DET} \cap (Z = k)] = \left[1 - \left(1 - \frac{n}{KN}\right)^k\right] \binom{m}{k}\binom{M-m}{K-k} \Big/ \binom{M}{K} \tag{3}$$

and

$$Pr(\text{DET}) = \sum_{k = \max(0,\, m - M + K)}^{\min(m,\, K)} \left[1 - \left(1 - \frac{n}{KN}\right)^k\right] \binom{m}{k}\binom{M-m}{K-k} \Big/ \binom{M}{K} . \tag{4}$$

From Equation 4, Table 1 shows that as the number of days (K) when testing is conducted increases, **everything else constant**, the probability of detection decreases. As an extreme example, if a command sampled 100 percent of its personnel on 1 day, then the probability of detecting a current drug user is one. However, if another command sampled 50 percent of its personnel on each of 2 days (also 100% per month), clearly the probability of detecting a current user is less than one. As the sampling rate (n/M) increases, the probability of detection increases. However, a doubling of n/M will not double the probability of detection except when $K = 1$. These observations imply that a smaller K is better. However, as we will see in Section 2.2, there are problems with small values of K.

Table 1

Probability of Detection Within a 30-day Period for Various Sampling Strategies and Drug Usage Rates

		Probability of Detection	
Testing Days (K)	Drug Usage Days (m)	10% Sampling	20% Sampling
1	3	0.0100	0.0200
5	3	0.0100	0.0199
10	3	0.0100	0.0199
15	3	0.0100	0.0199
30	3	0.0100	0.0199
1	6	0.0200	0.0400
5	6	0.0199	0.0395
10	6	0.0198	0.0394
15	6	0.0198	0.0394
30	6	0.0198	0.0393
1	9	0.0300	0.0600
5	9	0.0297	0.0587
10	9	0.0296	0.0585
15	9	0.0296	0.0585
30	9	0.0296	0.0584
1	12	0.0400	0.0800
5	12	0.0394	0.0776
10	12	0.0393	0.0773
15	12	0.0393	0.0772
30	12	0.0393	0.0771
1	30	0.1000	0.2000
5	30	0.0961	0.1846
10	30	0.0956	0.1829
15	30	0.0955	0.1824
30	30	0.0953	0.1818

Table 1 also contains the probability of detection for values of m equal to 30. This probability is equal to the probability of being tested at least once. That is, the probability of detection when drugs are always in a user's system is just the probability of being tested.

The probability of detection over multiple time periods is $1 - [1 - Pr(\text{DET})]^{M}$, where M is the number of time periods. Figure 1 was calculated in this manner.

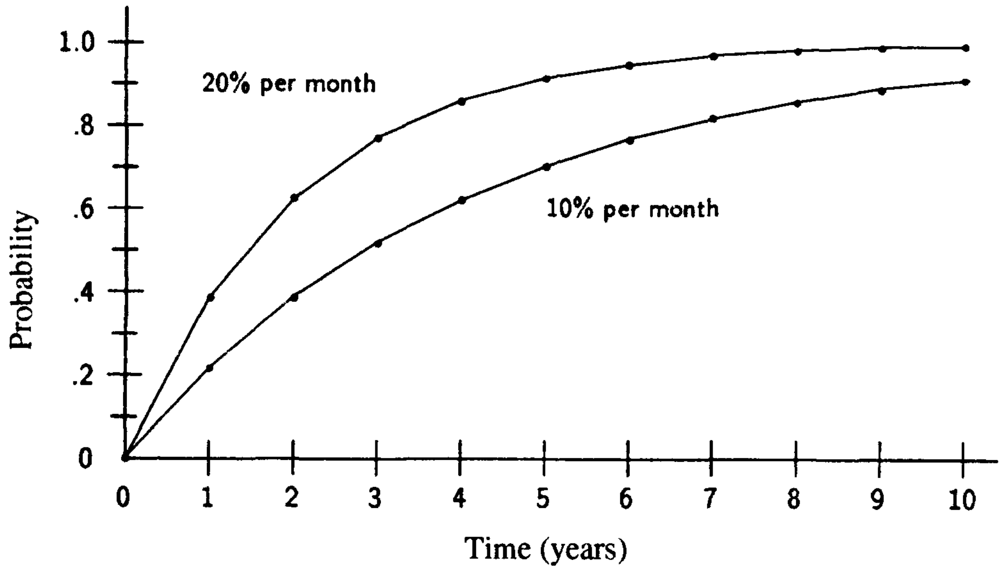

Figure 1. Cumulative probability of detection with drugs detectable in member's system 6 days per month.

2.2 Deterrence and Gaming

This section describes a simple strategy for the drug user to minimize detection. Assume the user knows the number of days testing is conducted (K) and will wait until after the last day of testing to use drugs. Also, assume drug users control their usage so that drugs are in their system for m days or for all remaining days, whichever is smaller. Then, since there is no chance of testing positive, $Pr(\text{DET}) = 0$.

We compute the expected usage, $E(y)$, as a function of K and m. Let $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(K)}$ be the order statistics from T. Then

$$E(y) = \sum_{j=0}^{M-K} Pr(T_{(K)} = M - j)\min(m, j)$$

$$= \sum_{j=0}^{M-K} \left[\binom{M-j-1}{K-1} \Big/ \binom{M}{K} \right] \min(m, j) . \tag{5}$$

From Equation 5, Table 2 clearly shows that as the number of testing days (K) increases, average usage decreases. Therefore, large values of K would be a deterrent to drug usage. The last column in Table 2 is plotted in Figure 2. The figure shows the upper limit for the average number of days drugs could be in a member's system versus the number of sampling days per month.

Table 2

The Expected Days of Drug Usage Using the Gaming Strategy for Various Values of K and m With $M = 30$

	m				
K	3	6	9	12	30
1	2.800	5.300	7.500	9.400	14.500
2	2.609	4.680	6.276	7.457	9.333
3	2.427	4.133	5.276	5.996	6.750
4	2.254	3.649	4.457	4.887	5.200
5	2.089	3.222	3.786	4.036	4.167
6	1.933	2.846	3.233	3.375	3.429
7	1.784	2.514	2.775	2.854	2.875
8	1.644	2.221	2.394	2.436	2.444
9	1.510	1.963	2.075	2.097	2.100
10	1.384	1.735	1.806	1.817	1.818
11	1.265	1.534	1.578	1.583	1.583
12	1.153	1.356	1.382	1.385	1.385
13	1.047	1.198	1.213	1.214	1.214
14	0.947	1.058	1.066	1.067	1.067
15	0.853	0.933	0.937	0.937	0.938
16	0.766	0.821	0.823	0.824	0.824
17	0.683	0.721	0.722	0.722	0.722
18	0.606	0.631	0.632	0.632	0.632
19	0.534	0.550	0.550	0.550	0.550
20	0.466	0.476	0.476	0.476	0.476
21	0.403	0.409	0.409	0.409	0.409
22	0.345	0.348	0.348	0.348	0.348
23	0.290	0.292	0.292	0.292	0.292
24	0.239	0.240	0.240	0.240	0.240
25	0.192	0.192	0.192	0.192	0.192
26	0.148	0.148	0.148	0.148	0.148
27	0.107	0.107	0.107	0.107	0.107
28	0.069	0.069	0.069	0.069	0.069
29	0.033	0.033	0.033	0.033	0.033
30	0.000	0.000	0.000	0.000	0.000

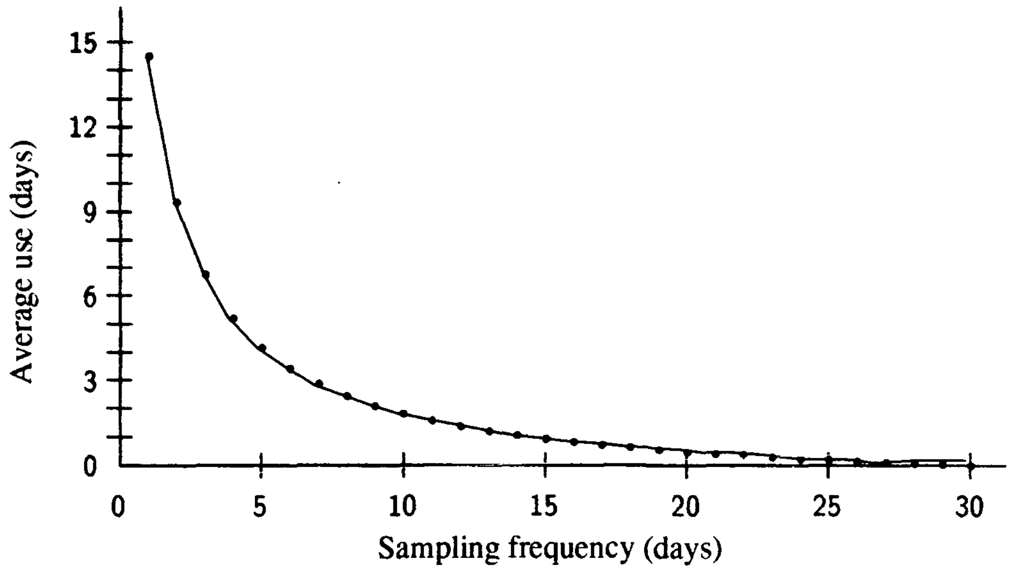

Figure 2. Upper limit on average number of possible drug use days given the number of sampling days per month. Probability of detection is zero under the gaming strategy.

While this strategy does give zero probability of detection, a user may be willing to trade off probability of detection for a higher average use. Regardless of the gaming strategy, increasing the number of testing days will either deter use or increase the probability of detection.

2.3 Interpretation of Prevalence Rates

Prevalence rates reported by the NDSLs are cited as evidence of reduced drug abuse in the Navy. These rates are simply the number of positive results divided by the total samples tested for some fixed time period (e.g., week). From 1983 through 1991 these rates have dropped from approximately 7 percent to 1 percent. In this section a statistical model developed to help interpret these prevalence rates is presented.

Given a population of N people, let fN of them have drugs detectable in their system m days out of M. One day is chosen randomly and a sample of size n is chosen from the N people. Define the sampling fraction $r = n/N$. Let W equal the number of people with drugs in their system on the day of the sample. W is distributed binomially and

$$Pr(W = w) = \binom{fN}{w} \left(\frac{m}{M}\right)^{w} \left(1 - \frac{m}{M}\right)^{fN-w} \tag{6}$$

for

$$w = 0, 1, \ldots, fN.$$

Now let X equal the number of drug users detected, that is, the number of drug users included in the sample. The distribution of X given $W = w$ is hypergeometric and

$$Pr(X = x|W = w) = \binom{w}{x}\binom{N-w}{rN-x}\bigg/\binom{N}{rN} \tag{7}$$

for

$$x = \max(0, w - N + rN), \ldots, \min(w, rN).$$

The distribution of X follows from Equations 6 and 7. See Appendix B.

The expected value of X is given by:

$$E(X) = E[E(X|W)] = E\left[\frac{rNW}{N}\right]$$

$$= r\,E[W] = \frac{rmfN}{M}. \tag{8}$$

Therefore, the expected value of the sample proportion is:

$$E\left(\frac{X}{rN}\right) = \frac{mf}{M}. \tag{9}$$

The prevalence rates can be interpreted as a product of f, the proportion of the population that are drug users, and m / M, the proportion of time drugs are detectable in their systems. Figure 3 plots m / M versus f with $mf / M = .01$. This figure shows that a prevalence rate of 1 percent could mean that the entire population uses drugs 1 percent of the time, or that 10 percent of the population uses drugs 10 percent of the time, or any number of other combinations. Thus, the decline in the prevalence rate from 7 percent to 1 percent could represent a decline in drug users or drug usage by the same number of users.

Comprehensive policy and procedures for the Navy's random urinalysis program is contained in Opnav Instruction 5350.4B (Chief of Naval Operations, 1990). This instruction states:

> "... testing programs should be designed so that a service member's chance of selection remains constant throughout the testing period."

However, none of the commands visited (Chipman, et al., 1992) have programs that strictly follow this guidance. A urinalysis strategy that does keep a service member's chance of selection constant throughout the testing period is described in the next section. This strategy is resistant to the gaming presented here.

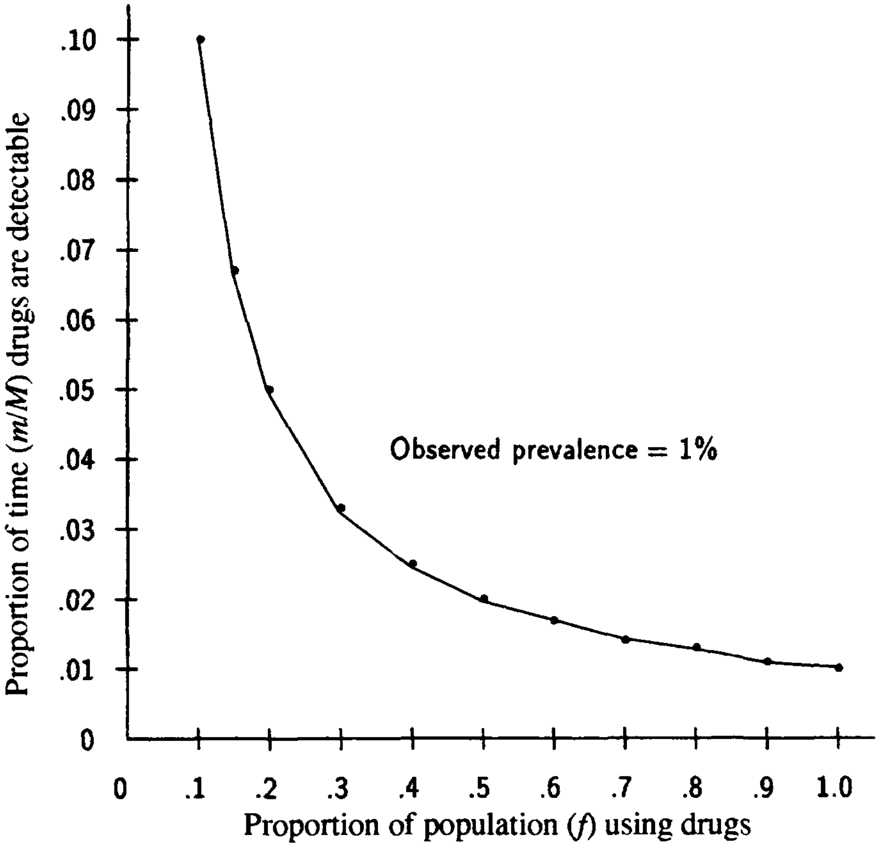

Figure 3. Proportion of time drugs are detectable versus proportion of population that are drug users.

3.0 Alternative Urinalysis Strategy

This section describes another model for the probability of detection of illegal drugs. The model is based on an alternative urinalysis strategy. Like the previous one, it assumes four conditions, but the second assumption differs. The assumptions are: (1) the testing period is a fixed number of days, (2) testing is conducted on each day in the testing period with some fixed probability, (3) on each of the testing days, a si- -le random sample of people is drawn from the total population of members at a given command, (4) a member has drugs detectable in their system some fixed number of days during the testing period. Based on these assumptions, the probability of detection of a drug user during a single testing period can be developed. The mathematical derivation follows.

Instead of choosing a fixed K days randomly from M days, we treat each day independently and sample on a given day with probability p_i, $i = 1, 2, \ldots, M$. Let $X_i = 1$ if we sample on the ith day, and $X_i = 0$ otherwise. Also let n_i, the sample size on the ith day, equal $X_i r_i N$, with $0 \le r_i \le 1$. Then r_i is the sampling fraction on the ith day.

Again we assume drugs are in a user's system m days out of M. Let $\mathcal{K}$ be the number of sampling days. Note $\mathcal{K}$ is a random variable and:

$$\mathcal{K} = \sum_{i=1}^{M} X_i , \tag{10}$$

$$\mathrm{E}\,(\mathcal{K}) = \sum_{i=1}^{M} p_i = K, \text{ and} \tag{11}$$

$$\mathrm{VAR}\,(\mathcal{K}) = \sum_{i=1}^{M} p_i\,(1 - p_i)\,. \tag{12}$$

Assume all the p_i are equal (i.e., $p_i = K / M$). Then $\mathcal{K}$ has a *binomial* distribution with parameters K / M and M. Let $Z_i = 1$ if sampling is occurring on the ith day ($X_i = 1$) **and** a given user has drugs in his system on the ith day; $Z_i = 0$, otherwise. Let the total number of sampling days when a user has drugs in their system, Z, equal $\sum_{i=1}^{M} Z_i$. Then Z has a binomial distribution with parameters K / M and m:

$$Pr(Z = z) = \binom{m}{z} \left(\frac{K}{M}\right)^{z} \left(1 - \frac{K}{M}\right)^{m-z} \tag{13}$$

for

$$z = 0, 1, \ldots m.$$

Assume all the r_i are equal (i.e., $r_i = r = n / K N$). Then

$$Pr(\mathrm{DET} \mid Z = z) = 1 - (1 - r)^{z}. \tag{14}$$

Therefore

$$Pr(\mathrm{DET}) = \sum_{z=o}^{m} [1 - (1\text{-}r)^{z}] \binom{m}{z} \left(\frac{K}{M}\right)^{z} \left(1 - \frac{K}{M}\right)^{m-z} . \tag{15}$$

With an application of the binomial theorem, Equation 15 can be shown to equal:

$$\begin{aligned} Pr(\mathrm{DET}) &= 1 - \left(1 - \frac{rK}{M}\right)^{m} \\ &= 1 - \left(1 - \frac{n}{NM}\right)^{m} . \end{aligned} \tag{16}$$

Note that the Pr(DET) is unaffected by changes in the value of K and that Equation 16 is equivalent to Equation 4 when $K = M$. The application of Equation 16 yields probabilities of detection presented in Tables 3 and 4. For comparison purposes we also include the probability of detection calculated from Equation 4.

Table 3

Comparison of Probabilities of Detection for the Current and Alternative Urinalysis Strategies. (Testing Period is 30 Days and the Overall Sampling Rate is 10%)

Testing Days (K)	Sampling Fraction (r)	Drug Usage (m)	Probability of Detection: Current Strategy	Probability of Detection: Alternative Strategy
1	0.100	3	0.0100	0.0100
5	0.020	3	0.0100	0.0100
10	0.010	3	0.0100	0.0100
15	0.007	3	0.0100	0.0100
30	0.003	3	0.0100	0.0100
1	0.100	6	0.0200	0.0198
5	0.020	6	0.0199	0.0198
10	0.010	6	0.0198	0.0198
15	0.007	6	0.0198	0.0198
30	0.003	6	0.0198	0.0198
1	0.100	9	0.0300	0.0296
5	0.020	9	0.0297	0.0296
10	0.010	9	0.0296	0.0296
15	0.007	9	0.0296	0.0296
30	0.003	9	0.0296	0.0296
1	0.100	12	0.0400	0.0393
5	0.020	12	0.0394	0.0393
10	0.010	12	0.0393	0.0393
15	0.007	12	0.0393	0.0393
30	0.003	12	0.0393	0.0393
1	0.100	30	0.1000	0.0953
5	0.020	30	0.0961	0.0953
10	0.010	30	0.0956	0.0953
15	0.007	30	0.0955	0.0953
30	0.003	30	0.0953	0.0953

Table 4

Comparison of Probabilities of Detection for the Current and Alternative Urinalysis Strategies (Testing Period is 30 Days and the Overall Sampling Rate is 20%)

Testing Days (K)	Sampling Fraction (r)	Drug Usage (m)	Probability of Detection: Current Strategy	Probability of Detection: Alternative Strategy
1	0.200	3	0.0200	0.0199
5	0.040	3	0.0199	0.0199
10	0.020	3	0.0199	0.0199
15	0.013	3	0.0199	0.0199
30	0.007	3	0.0199	0.0199
1	0.200	6	0.0400	0.0393
5	0.040	6	0.0395	0.0393
10	0.020	6	0.0394	0.0393
15	0.013	6	0.0394	0.0393
30	0.007	6	0.0393	0.0393
1	0.200	9	0.0600	0.0584
5	0.040	9	0.0587	0.0584
10	0.020	9	0.0585	0.0584
15	0.013	9	0.0585	0.0584
30	0.007	9	0.0584	0.0584
1	0.200	12	0.0800	0.0771
5	0.040	12	0.0776	0.0771
10	0.020	12	0.0773	0.0771
15	0.013	12	0.0772	0.0771
30	0.007	12	0.0771	0.0771
1	0.200	30	0.2000	0.1818
5	0.040	30	0.1846	0.1818
10	0.020	30	0.1829	0.1818
15	0.013	30	0.1824	0.1818
30	0.007	30	0.1818	0.1818

The major advantage of this alternative model is that gaming as described in the previous section, with zero chance of detection, is impossible. See Appendix A for a proof. This urinalysis strategy also conforms to the Navy recommended policy of keeping a members chance of selection constant throughout the testing period. As is shown in Tables 3 and 4 the probability of detection is slightly less for the alternative strategy except when testing every day under the current strategy. It is interesting to note that gaming can be eliminated by either testing every day under the current strategy or by using the alternative strategy. These two choices yield the same probability of detection. Therefore, the alternative strategy eliminates gaming without requiring daily testing.

4.0 Conclusions and Recommendation

We have developed the methodology to calculate the probability of detection of drug users by random urinalysis. This information can be used to help set testing levels. For example, if the Navy wants to detect 95 percent of the casual[2] drug users within a 4-year enlistment, the Navy would have to test roughly 60.5 percent of its personnel each month.

The current practice of random urinalysis is susceptible to gaming by drug abusers. The typical practice of testing on a fixed number of days per month (usually one) leaves a large number of days with no threat of testing. The drug user can simply wait until after the last sample collection day to use drugs. This allows a window of use with **zero** probability of detection. We have developed the methodology to calculate average usage possible, with zero probability of detection, under various urinalysis testing strategies.

This gaming can be eliminated by testing every day or by using an alternative urinalysis strategy. The alternative strategy is to test each day with positive probability. The effect of either of these solutions is to maintain a positive chance (threat) of being tested throughout the testing period. To eliminate the gaming described here without requiring testing every day, we recommend the alternative strategy. A disadvantage of this new strategy is that the amount of testing conducted monthly is not fixed but is a random variable. Therefore, a fixed testing rate in a given month cannot be guaranteed. The average rate over a number of months will be close to whatever value is chosen. Given the large number of activities in the Navy, the Navy-wide monthly testing rate will not vary significantly.

[2]Drugs detectable in a user's system 3 days a month.

5.0 References

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Appendix A

Gaming Under the Alternative Model

Gaming Under the Alternative Model

Here we present a strategy, similar to the strategy in Section 2.2, for gaming under the alternative model. Suppose a drug user will wait until after the κth day of testing to use drugs. Here, we also assume drug users control their usage so that drugs are in their system for m days or for all remaining days, whichever is smaller. Let the event that the κth day of testing equals day $D_{\kappa+j}$ be denoted A_j. Then for $j = 0, \ldots, M - \kappa$:

$$A_j = \{ X_i : X_1 + X_2 + \ldots + X_{\kappa+j-1} = \kappa - 1 \} \cap \{ X_i : X_{\kappa+j} = 1 \}. \tag{A1}$$

Recall $X_i = 1$ if we sample on the ith day, and $X_i = 0$ otherwise. The A_j's are mutually exclusive and the gaming strategy implies that detection will only occur when $\mathcal{K} \geq \kappa$. Therefore

$$Pr(\mathrm{DET}) = \sum_{j=0}^{M-\kappa} Pr(\mathrm{DET} \mid A_j)\, Pr(A_j). \tag{A2}$$

Using Equation 16,

$$Pr(\mathrm{DET} \mid A_j) = 1 - \left(1 - \frac{n}{NM}\right)^{\min(m,\, M-\kappa-j)} \tag{A3}$$

and

$$Pr(A_j) = \binom{\kappa+j-1}{\kappa-1} \left(\frac{K}{M}\right)^{\kappa} \left(1 - \frac{K}{M}\right)^{j}. \tag{A4}$$

Therefore, the probability of detection, given the above gaming strategy, is

$$Pr(\mathrm{DET}) = \sum_{j=0}^{M-\kappa} \left[1 - \left(1 - \frac{n}{MN}\right)^{\min(m,\, M-\kappa-j)} \right] \binom{\kappa+j-1}{\kappa-1} \left(\frac{K}{M}\right)^{\kappa} \left(1 - \frac{K}{M}\right)^{j}. \tag{A5}$$

Note the probability of detection is greater than zero except when usage is zero (i.e., $\kappa = M$). This alternative testing scheme cannot be gamed with zero probability of detection. Table A-1 lists these probabilities when $\kappa = K$, M = 30, and the sampling rate is 20 percent.

Now we let y equal the number of days of drug usage when applying the above gaming strategy. The expected value of y is calculated as

$$E(y) = \sum_{j=0}^{M-\kappa} \min(m, M - \kappa - j) \binom{\kappa+j-1}{\kappa-1} \left(\frac{K}{M}\right)^{\kappa} \left(1 - \frac{K}{M}\right)^{j}. \tag{A6}$$

Suppose the drug user knows the value of K and picks $\kappa = K$. The expected drug usage under this strategy is presented in Table A-2. Average drug usage again decreases as K increases. A comparison with Table 2 shows average usage is now less for small values of κ and K and average usage is higher for larger values of K. The major advantage of this alternative model is that gaming, with zero chance of detection, is impossible

Table A-1

The Probability of Detection Using the Gaming Strategy for $\kappa = K$ and $M = 30$ (Overall Sampling Rate is 20%)

	m				
$k = K$	3	6	9	12	30
1	0.012	0.023	0.033	0.042	0.068
2	0.011	0.021	0.029	0.036	0.050
3	0.011	0.020	0.027	0.033	0.041
4	0.010	0.019	0.025	0.030	0.035
5	0.010	0.018	0.024	0.028	0.031
6	0.010	0.017	0.023	0.026	0.028
7	0.010	0.017	0.021	0.024	0.025
8	0.010	0.016	0.020	0.022	0.023
9	0.009	0.016	0.019	0.021	0.021
10	0.009	0.015	0.018	0.020	0.020
11	0.009	0.015	0.017	0.018	0.019
12	0.009	0.014	0.017	0.017	0.017
13	0.009	0.014	0.016	0.016	0.016
14	0.009	0.013	0.015	0.015	0.015
15	0.008	0.013	0.014	0.014	0.014
16	0.008	0.012	0.013	0.013	0.013
17	0.008	0.012	0.012	0.012	0.012
18	0.008	0.011	0.012	0.012	0.012
19	0.008	0.010	0.011	0.011	0.011
20	0.008	0.010	0.010	0.010	0.010
21	0.007	0.009	0.009	0.009	0.009
22	0.007	0.009	0.009	0.009	0.009
23	0.007	0.008	0.008	0.008	0.008
24	0.006	0.007	0.007	0.007	0.007
25	0.006	0.006	0.006	0.006	0.006
26	0.005	0.006	0.006	0.006	0.006
27	0.005	0.005	0.005	0.005	0.005
28	0.004	0.004	0.004	0.004	0.004
29	0.002	0.002	0.002	0.002	0.002
30	0.000	0.000	0.000	0.000	0.000

Table A-2

The Expected Days of Drug Usage Using the Gaming Strategy for Various Values of κ and *m* With *M* = 30

	m				
k = K	3	6	9	12	30
1	1.838	3.553	5.129	6.553	10.850
2	1.695	3.205	4.514	5.607	7.843
3	1.621	3.009	4.149	5.033	6.374
4	1.570	2.866	3.875	4.601	5.454
5	1.531	2.749	3.649	4.247	4.803
6	1.497	2.647	3.451	3.942	4.307
7	1.468	2.555	3.272	3.673	3.911
8	1.441	2.469	3.106	3.429	3.582
9	1.416	2.387	2.950	3.207	3.303
10	1.393	2.308	2.802	3.002	3.060
11	1.369	2.230	2.660	2.812	2.846
12	1.346	2.153	2.523	2.635	2.653
13	1.323	2.076	2.389	2.468	2.478
14	1.300	1.999	2.259	2.312	2.316
15	1.276	1.920	2.131	2.166	2.167
16	1.251	1.840	2.007	2.027	2.027
17	1.225	1.758	1.884	1.895	1.895
18	1.197	1.672	1.764	1.769	1.769
19	1.168	1.584	1.646	1.647	1.647
20	1.135	1.492	1.530	1.530	1.530
21	1.100	1.395	1.416	1.416	1.416
22	1.060	1.294	1.303	1.303	1.303
23	1.014	1.188	1.190	1.190	1.190
24	0.961	1.077	1.077	1.077	1.077
25	0.896	0.961	0.961	0.961	0.961
26	0.815	0.839	0.839	0.839	0.839
27	0.708	0.708	0.708	0.708	0.708
28	0.560	0.560	0.560	0.560	0.560
29	0.374	0.374	0.374	0.374	0.374
30	0.000	0.000	0.000	0.000	0.000

Appendix B

Distribution of *X*

Distribution of X

Recall X equals the number of drug users detected and W equals the number of people with drugs in their system on the day of the sample. Then

$$P(X = x\,,\, W = w) = P(X = x \mid W = w)\, P(W = w). \tag{B1}$$

From Equations 6 and 7

$$P(X = x\,,\, W = w) = \left[\binom{w}{x}\binom{N-w}{rN-x}\Big/\binom{N}{rN}\right]\binom{fN}{w}\left(\frac{m}{M}\right)^{w}\left(1-\frac{m}{M}\right)^{fN-w}. \tag{B2}$$

Summing over all values for W we obtain

$$P(\mathrm{X} = \mathrm{x}) = \sum_{w=x}^{\min(fN,\, x+N-rN)} \left[\binom{w}{x}\binom{N-w}{rN-x}\Big/\binom{N}{rN}\right]\binom{fN}{w}\left(\frac{m}{M}\right)^{w}\left(1-\frac{m}{M}\right)^{fN-w} \tag{B3}$$

$x = 0, 1, \ldots, \min(rN, fN).$

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