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# ALIGNMENT OF A 3-D SENSOR AND A 2-D SENSOR MEASURING AZIMUTH AND ELEVATION

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## FOREWORD

The mathematical model and alignment algorithm discussed in this report were developed by the Combat System Technologies Branch (N35) of the Engineering and Technology Division (N30) and the Weapons Direction Systems Branch (G71) of the Weapons Control Division (G70). This work was funded by the Independent Research Program at the Naval Surface Warfare Center Dahlgren Division (NSWCDD). This report has been reviewed at NSWCDD in the Engineering and Technology Division and the Weapons Control Division.

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## ABSTRACT

The problem of aligning two sensors using targets of opportunity is examined in this report, where one of the sensors is a 3-D sensor that measures range, azimuth, and elevation and the other one is a 2-D sensor that measures azimuth and elevation. Both a mathematical model and an alignment algorithm are developed for this problem, where attitude and offset errors in both sensors are included in the formulation. Because of the method used to formulate the problem, it is not possible to determine the individual alignment errors at each sensor. However, it is possible to determine the differences in the respective alignment errors of the two sensors. This is sufficient to align one sensor relative to the other one. For illustrative purposes, the alignment algorithm is applied to simulated data from two sensors. The algorithm converges within 50 sec to values very close to the actual values of the alignment parameters. Using these values, it is possible to obtain a dramatic 23-fold reduction in the relative error between the tracks generated by the sensors.

## CONTENTS



## ILLUSTRATIONS

# 1.0 INTRODUCTION

In recent years, interest in integrating stand-alone sensors into multisensor systems has been increasing. The reasons for this interest include the potential for improvement of system performance and enhanced system capabilities. To effectively share the data between the sensors in the systems, all of the sensors must be accurately aligned. This is particularly important if the data from the sensors are to be fused. The presence of alignment errors will degrade the overall system performance and may lead to system performance that is worse than the performance of the individual sensors in the system. The alignment procedure must correct the effects of the alignment errors on the multisensor data.

The alignment of the sensors on a ship is usually performed when the ship is undergoing dockside maintenance. At that time, they are aligned with respect to some stationary point. Typically, this is the only time that the sensors are aligned. Once the ship returns to duty and puts out to sea, there is no way of checking the alignment of the sensors. If the errors in the alignment of each sensor fluctuate only a small amount during the ship's time at sea, there is little reason to worry about alignment errors. This is especially true when the data from the various sensors are not integrated. However, with the advent of sensors having greater accuracies and the desire to fuse the data from several sensors to obtain more accurate and reliable tracks, it is now necessary to be able to check the accuracy of the alignment of the sensors throughout the period of time the ship is at sea. For example, two sensors whose data might be fused are a 3-D radar that measures range, azimuth, and elevation and a 2-D sensor that measures azimuth and elevation (e.g., an infrared (IR) sensor). Here, the IR sensor might have angular accuracies of less than a milliradian and the radar will measure a relatively accurate range, something the IR sensor does not measure. Sensor alignment is also required when integrating data from land-based sensors. The problem of sensor misalignment in a system of land-based sensors has been documented with real data collected during the North Atlantic Treaty Organization (NATO) Antiair Warfare (AAW) experiments.[1]

One source of alignment errors is sensor calibration errors (i.e., offsets). Although the sensors are usually calibrated in an initial calibration procedure, the calibration may deteriorate over time. Another source of alignment errors is attitude errors between the sensor's reference frame and its stabilized frame. One source of these attitude errors is flexure in the platform on which the sensors are located.[2] In shipboard applications, the action of waves on a ship, the amount of loading on a ship, and the motion of a ship can all cause the ship's structure to flex. This flexing will cause the sensor's reference frame to become misaligned with respect to the ship's stabilized frame. The magnitude of this misalignment will depend on the location of the sensor on the ship; and because the various sensors are usually located at different places on the ship, the misalignment due to flexing will be different for each sensor. Platform flexure will also be a problem for sensors located on an aircraft.

Some work has been done on the removal of alignment errors in multisensor systems consisting of 3-D sensors.[3,4,5,6] One problem that has received little attention is the removal of alignment errors in dissimilar sensors (e.g., active and passive sensors, 2-D and 3-D sensors, etc.). However, the alignment of dissimilar sensors is important in applications. For example, this alignment problem occurs in the integration of a 2-D radar to a 3-D radar and in the integration of an optical sensor to a radar. The problem of aligning two sensors using targets of opportunity is examined in this report, where one of the sensors is a 3-D sensor that measures range, azimuth, and elevation and the other one is a 2-D sensor that measures azimuth and elevation. It will be assumed that the sensors are close to each other, as would occur for sensors located on the same platform, and the alignment errors do not change with time (i.e., static alignment problem).

This report is organized in the following manner: In Section 2, a mathematical model is developed for this alignment problem. Kalman filtering techniques are applied to this model in Section 3 to obtain the alignment algorithm; and this algorithm is tested with simulated data in Section 4. Finally, Section 5 summarizes the results of this study.

## 2.0 MATHEMATICAL DEVELOPMENT

The problem addressed in this report can be stated as follows: Given the measurements from two sensors (range, elevation, and azimuth from the 3-D sensor and elevation and azimuth from the 2-D sensor) over time for a specific target, estimate the parameters that will align the data from the 2-D sensor to the data from the 3-D sensor. Several assumptions will be employed in dealing with this problem. Specifically, the assumptions are as follows:

1. The distance between the sensors is small ($\leq$ 100 m).
2. Any location errors in the relative positions of the sensors are negligible.
3. The magnitudes of the alignment errors are small.
4. The sensors provide the measurement data at the same rates, and the data from the sensors are coincident in time.

The first assumption is consistent with sensors located on a single platform, and it will allow us to use the flat-Earth approximation in the transformation of data between the sensors. The second assumption is also consistent with sensors located on a single plaform. Errors in the relative locations of sensors on a single platform are negligible because the distances between the sensors can be accurately surveyed in an initial calibration procedure. Errors in the translation vector between the sensor's frames could occur because of platform flexure. This will introduce, at most, a difference in distance of several meters from the surveyed distance between the sensors. This error in distance between the sensors will produce a small relative error in the data when the data are transformed between the sensors. The small error in the distance between the sensors will be ignored because of its small effect on the data. Of course, this flexing will produce a change in the direction of the translation vector between the sensors which, in turn, will introduce errors in the relative orientation of the sensor's frames. These orientation errors will be modeled. The third assumption will allow us to employ a first-order

Taylor series approximation, and it will result in a linearized version of the alignment problem. Finally, the last assumption is made for convenience in developing the theory. This last assumption will not hold in practice. The data from the various sensors are not coincident in time, and they must be extrapolated to a common point of time. Also, the data rates from the various sensors are usually different. These two important factors will not be addressed in this report.

## 2.1 ATTITUDE ERRORS

Consider a particular sensor, say the $k^{th}$ sensor, where $k = 1, 2$. A reference frame is necessary in describing the $k^{th}$ sensor's measurements. The reference frame in which this sensor's measurements are made will be called the sensor's measurement frame. There is also a stabilized frame associated with this sensor. The stabilized frame is aligned to the true north-south horizontal line, the true east-west horizontal line, and the axis that is orthogonal to the horizontal plane formed by the north-south and east-west lines. Because of attitude (or orientation) alignment errors (e.g., from platform flexure), the measurement frame may not be aligned to the stabilized frame. Both frames have the same origin, but one frame is tilted with respect to the other one.

The stabilized frame at the $k^{th}$ sensor can be represented by the three mutually orthogonal unit vectors $\mathbf{e}_{x'k}$, $\mathbf{e}_{y'k}$, and $\mathbf{e}_{z'k}$. The subscript $k$ denotes the $k^{th}$ sensor, and the subscripts x′, y′, and z′ refer to the directions of north, east, and up, respectively. Similarly, the measurement frame at the $k^{th}$ sensor can be represented by the three mutually orthogonal unit vectors $\mathbf{e}_{xk}$, $\mathbf{e}_{yk}$, and $\mathbf{e}_{zk}$.

Let the spherical coordinates of a point in the $k^{th}$ sensor's stabilized frame be denoted by $r'_k$, $\theta'_k$, and $\varepsilon'_k$, which represent the range, azimuth, and elevation of the point, respectively. The position vector for this point expressed in rectangular coordinates in the $k^{th}$ sensor's stabilized frame will be denoted by $\mathbf{r}'_k$, where $\mathbf{r}'_k = [\, x'_k \;\; y'_k \;\; z'_k \,]^T$ is the 3 x 1 column vector (the superscript T denotes matrix transposition) with the rectangular coordinates

$$x'_k = r'_k \cos\varepsilon'_k \sin\theta'_k, \qquad y'_k = r'_k \cos\varepsilon'_k \cos\theta'_k, \qquad z'_k = r'_k \sin\varepsilon'_k. \tag{2-1}$$

Similarly, the spherical coordinates of this point in the $k^{th}$ sensor's measurement frame are denoted by $r_k$, $\theta_k$, and $\varepsilon_k$, and $\mathbf{r}_k = [\, x_k \;\; y_k \;\; z_k \,]^T$ is the position vector in the $k^{th}$ sensor's measurement frame, where

$$x_k = r_k \cos\varepsilon_k \sin\theta_k\,, \qquad y_k = r_k \cos\varepsilon_k \cos\theta_k\,, \qquad z_k = r_k \sin\varepsilon_k\,. \tag{2-2}$$

The transformation between the $k^{th}$ sensor's stabilized frame and measurement frame can be described by a set of Eulerian angles. The xyz-convention[7] will be employed in this report. In the xyz-convention, the first rotation is the yaw angle $\phi_k$ about the stabilized frame's z-axis, the second rotation is the pitch angle $\eta_k$ about the intermediate y-axis, and the third rotation is the roll angle $\psi_k$ about the final x-axis. The transformation of the position vector from the $k^{th}$ sensor's stabilized frame to its measurement frame is given by

$$\mathbf{r}_k = \mathbf{A}_k \mathbf{r}'_k, \tag{2-3}$$

where $\mathbf{A}_k$ is the 3 x 3 orthogonal matrix given by[7]

$$\mathbf{A}_k = \begin{vmatrix} \cos\eta_k \cos\phi_k & \cos\eta_k \sin\phi_k & -\sin\eta_k \\ \sin\psi_k \sin\eta_k \cos\phi_k - \cos\psi_k \sin\phi_k & \sin\psi_k \sin\eta_k \sin\phi_k + \cos\psi_k \cos\phi_k & \cos\eta_k \sin\psi_k \\ \cos\psi_k \sin\eta_k \cos\phi_k + \sin\psi_k \sin\phi_k & \cos\psi_k \sin\eta_k \sin\phi_k - \sin\psi_k \cos\phi_k & \cos\eta_k \cos\psi_k \end{vmatrix}. \tag{2-4}$$

Since $\mathbf{A}_k$ is an orthogonal matrix, $\mathbf{A}_k^{-1} = \mathbf{A}_k^T$, and the transformation from the measurement frame to the stabilized frame is given by

$$\mathbf{r}'_k = \mathbf{R}_k \mathbf{r}_k, \tag{2-5}$$

where $\mathbf{R}_k = \mathbf{A}_k^T$.

To be specific, let the first sensor ($k$ = 1) be the 3-D sensor, and the second sensor ($k$ = 2) the 2-D sensor. Let the 2-D sensor be located at the point $\mathbf{t} = [\, t_x \;\; t_y \;\; t_z \,]^T$ in the 3-D sensor's stabilized frame. The vector $\mathbf{t}$ is assumed to be known. For example, $\mathbf{t}$ can be accurately determined in an initial calibration procedure.

The transformation from the 3-D sensor's stabilized frame to the 2-D sensor's stabilized frame is given by

$$\mathbf{r}'_{21} = \mathbf{r}'_1 - \mathbf{t}, \tag{2-6}$$

ere $\mathbf{r}'_1$ is a position vector in the 3-D sensor's stabilized frame, and $\mathbf{r}'_{21}$ is the corresponding ition vector in the 2-D sensor's stabilized frame. Using Equation 2-5 in Equation 2-6 gives

$$\mathbf{R}_2 \mathbf{r}_{21} = \mathbf{R}_1 \mathbf{r}_1 - \mathbf{t}, \tag{2-7}$$

ere $\mathbf{r}_1$ and $\mathbf{r}_{21}$ are the corresponding position vectors in the 3-D and 2-D sensor's asurement frames, respectively. Since $\mathbf{R}_2$ is an orthogonal matrix, Equation 2-7 can be ressed as

$$\mathbf{r}_{21} = \mathbf{R}\,\mathbf{r}_1 - \mathbf{R}_2^T \mathbf{t}, \tag{2-8}$$

ere

$$\mathbf{R} = \mathbf{R}_2^T \mathbf{R}_1. \tag{2-9}$$

uation 2-8 represents the transformation from the 3-D sensor's measurement frame to the 2-D sor's measurement frame, and it accounts for the attitude alignment errors in both sensors.

Assuming that the yaw, pitch, and roll angles at each sensor are small ($\leq 1°$), the small-gle approximations (i.e., first-order Taylor series expansions about zero) can be used in the onometric quantities in $\mathbf{R}_1$ and $\mathbf{R}_2$. In this case, $\mathbf{R}_k$ (where $k = 1, 2$) can be approximated by first order)

$$\mathbf{R}_k \approx \mathbf{I} + \mathbf{dR}_k, \tag{2-10}$$

ere $\mathbf{I}$ is the 3 x 3 identity matrix, and $\mathbf{dR}_k$ is the matrix differential of $\mathbf{R}_k$ given by

$$\mathbf{dR}_k = \left| \begin{array}{ccc} 0 & -\phi_k & \eta_k \\ \phi_k & 0 & -\psi_k \\ -\eta_k & \psi_k & 0 \end{array} \right|. \tag{2-11}$$

ostituting these into Equation 2-9 allows the matrix $\mathbf{R}$ to be approximated by (to first order)

$$\mathbf{R} \approx \mathbf{I} + \mathbf{dR}, \tag{2-12}$$

where

$$\mathbf{dR} = \mathbf{dR}_1 + \mathbf{dR}_2^T = \begin{vmatrix} 0 & \Delta\phi & \Delta\eta \\ -\Delta\phi & 0 & \Delta\psi \\ -\Delta\eta & -\Delta\psi & 0 \end{vmatrix}, \tag{2-13}$$

and

$$\Delta\phi = \phi_2 - \phi_1, \qquad \Delta\eta = \eta_1 - \eta_2, \qquad \Delta\psi = \psi_2 - \psi_1. \tag{2-14}$$

Using these approximations in the transformation equation appearing in Equation 2-8 gives

$$\mathbf{r}_{21} \approx \mathbf{r}_1 - \mathbf{t} + \mathbf{dR}\,\mathbf{r}_1 - \mathbf{dR}_2^T\mathbf{t}. \tag{2-15}$$

The last term appearing on the right side of Equation 2-15 is negligible under the assumptions made for this problem. This term accounts for the error in the transformation caused by the apparent rotation of the origin of the 3-D sensor's frame due to the tilting of the 2-D sensor's measurement frame, with respect to its stabilized frame. Assuming that the maximum value of the roll, pitch, and yaw at the second sensor is 1°, and the maximum separation between the sensors is 100 m, it can be shown that $\| \mathbf{dR}_2^T \mathbf{t} \| \leq 6$ m. That is, ignoring the last term on the right side of Equation 2-15 will introduce no more than 6 m of error in the transformation equation, and Equation 2-15 becomes

$$\mathbf{r}_{21} \approx \mathbf{r}_1 - \mathbf{t} + \mathbf{dR}\,\mathbf{r}_1. \tag{2-16}$$

The transformation in Equation 2-16 is accomplished by first transforming $\mathbf{r}_1$ to the 2-D sensor's frame as if there were no alignment errors (i.e., $\mathbf{r}_1 - \mathbf{t}$). Then, the effects of the attitude errors in the sensors are accounted for by adding $\mathbf{dR}\,\mathbf{r}_1$ to the previous result.

The attitude parameters in Equation 16 are the yaw parameter $\Delta\phi$, the pitch parameter $\Delta\eta$, and the roll parameter $\Delta\psi$, and they are elements of the matrix **dR**. There are only three attitude parameters because the last term on the right side of Equation 2-15 was ignored. These attitude parameters depend only on the differences in the Eulerian angles (see Equation 2-14) that describe the orientations of the measurement frames of the two sensors. In this sense, Equation 2-16 can be thought of as the transformation that *relatively aligns* the 3-D sensor's measurement frame to the 2-D sensor's measurement frame. For widely separated sensors (e.g., sensors located on separate platforms), there may be difficulties in the relative alignment of the sensor's measurement frames because it may not be valid to ignore the last term on the right side of Equation 2-15.

It is also possible to relatively align the 2-D sensor's measurement frame to the 3-D sensor's measurement frame. Proceeding in a manner similar to the one above, it can be shown that the transformation for this alignment is given by

$$\mathbf{r}_{12} = \mathbf{R}^T \mathbf{r}_2 + \mathbf{R}_1^T \mathbf{t} \approx \mathbf{r}_2 + \mathbf{t} + \mathbf{dR}^T \mathbf{r}_2, \tag{2-17}$$

where $\mathbf{r}_2$ is a position vector in the 2-D sensor's measurement frame, and $\mathbf{r}_{12}$ is the corresponding position vector in the 3-D sensor's measurement frame.

## 2.2 OFFSET ERRORS

In addition to the attitude errors in the sensor's reference frames, the sensor's measurements may also contain offset errors (or biases). The 3-D sensor provides measurements of the range $r_1$, azimuth $\theta_1$, and elevation $\varepsilon_1$ (all expressed in the 3-D sensor's measurement frame), and the 2-D sensor provides measurements of the azimuth $\theta_2$ and elevation $\varepsilon_2$ (expressed in the 2-D sensor's measurement frame). The errors in the measurements will be modeled as follows: the measured value is equal to the sum of the true value, the offset error, and the random measurement error.

For the 3-D sensor, the measured values of the range $r_1$, azimuth $\theta_1$, and elevation $\varepsilon_1$ are given by

$$r_1 = r_{1T} + dr_1 + e_{r1}, \quad \theta_1 = \theta_{1T} + d\theta_1 + e_{\theta 1}, \quad \varepsilon_1 = \varepsilon_{1T} + d\varepsilon_1 + e_{\varepsilon 1}, \tag{2-18}$$

where $r_{1T}$, $\theta_{1T}$, and $\varepsilon_{1T}$ are the true values of the range, azimuth, and elevation; $dr_1$, $d\theta_1$, and $d\varepsilon_1$ are the 3-D sensor's offset errors in the range, azimuth, and elevation; and $e_{r1}$, $e_{\theta 1}$, and $e_{\varepsilon 1}$ are the random errors in the 3-D sensor's measurements. The first-order Taylor series expansion of the true position vector $\mathbf{r}_{1T}$ in the 3-D sensor's measurement frame about the measured position vector $\mathbf{r}_1$ in the 3-D sensor's measurement frame is given by

$$\mathbf{r}_{1T} \approx \mathbf{r}_1 - \mathbf{dr}_1 - \mathbf{e}_1, \tag{2-19}$$

where the differential $\mathbf{dr}_1$ and the random error vector $\mathbf{e}_1$ are given by

$$\mathbf{dr}_1 = \mathbf{f}(\theta_1,\varepsilon_1)\ dr_1 + \mathbf{g}(r_1,\theta_1,\varepsilon_1)\ d\theta_1 + \mathbf{h}(r_1,\theta_1,\varepsilon_1)\ d\varepsilon_1, \tag{2-20}$$

$$\mathbf{e}_1 = \mathbf{f}(\theta_1,\varepsilon_1)\ e_{r1} + \mathbf{g}(r_1,\theta_1,\varepsilon_1)\ e_{\theta 1} + \mathbf{h}(r_1,\theta_1,\varepsilon_1)\ e_{\varepsilon 1}. \tag{2-21}$$

The vector-valued functions **f**, **g**, and **h** are defined by

$$\mathbf{f}(\theta, \varepsilon) = \frac{\partial \mathbf{r}}{\partial r} = [\ cos\ \varepsilon\ sin\ \theta \quad cos\ \varepsilon\ cos\ \theta \quad sin\ \varepsilon\ ]^T = \frac{\mathbf{r}}{r}, \tag{2-22}$$

$$\mathbf{g}(r, \theta, \varepsilon) = \frac{\partial \mathbf{r}}{\partial \theta} = [\ r\ cos\ \varepsilon\ cos\ \theta \quad -r\ cos\ \varepsilon\ sin\ \theta \quad 0\ ]^T = [\ y \quad -x \quad 0\ ]^T, \tag{2-23}$$

$$\mathbf{h}(r, \theta, \varepsilon) = \frac{\partial \mathbf{r}}{\partial \varepsilon} = [\ -r\ sin\ \varepsilon\ sin\ \theta \quad -r\ sin\ \varepsilon\ cos\ \theta \quad r\ cos\ \varepsilon\ ]^T, \tag{2-24}$$

where $\mathbf{r} = [\ x \quad y \quad z\ ]^T$ is the position vector containing the rectangular coordinates of a point, and $r$, $\theta$, and $\varepsilon$ are the corresponding spherical coordinates. In Equations 2-20 and 2-21, the functions **f**, **g**, and **h** are evaluated at the 3-D sensor's measurements $r_1$, $\theta_1$, and $\varepsilon_1$.

For the 2-D sensor, the measured values of the azimuth $\theta_2$ and elevation $\varepsilon_2$ are given by

$$\theta_2 = \theta_{2T} + d\theta_2 + e_{\theta 2}, \qquad \varepsilon_2 = \varepsilon_{2T} + d\varepsilon_2 + e_{\varepsilon 2}, \tag{2-25}$$

where $\theta_{2T}$ and $\varepsilon_{2T}$ are the true values of the azimuth and elevation; $d\theta_2$ and $d\varepsilon_2$ are the 2-D sensor's offset errors in azimuth and elevation; and $e_{\theta 2}$ and $e_{\varepsilon 2}$ are the random errors in the 2-D sensor's measurements. Note that there is no equation relating $r_2$ to $r_{2T}$ for the 2-D sensor because the 2-D sensor does not measure the range $r_2$. Below, it will be shown how the 3-D sensor's range can be assigned to the 2-D sensor's measurement frame.

## 2.3 POSITION VECTOR IN THE 2-D SENSOR'S MEASUREMENT FRAME

Equation 2-16 can be used to obtain the range $r_2$ in the 2-D sensor's measurement frame. Using Equation 2-12 in Equation 2-16 allows Equation 2-16 to be expressed as

$$\mathbf{r}_{21} \approx \mathbf{R}\,\mathbf{r}_1 - \mathbf{t}, \tag{2-26}$$

where $\mathbf{r}_1$ is a position vector in the 3-D sensor's measurement frame and $\mathbf{r}_{21}$ is the corresponding position vector in the 2-D sensor's measurement frame. Then

$$r_{21}^2 = \mathbf{r}_{21}^T\mathbf{r}_{21} = (\mathbf{R}\,\mathbf{r}_1 - \mathbf{t})^T(\mathbf{R}\,\mathbf{r}_1 - \mathbf{t}) = r_1^2 + t^2 - 2\,\mathbf{r}_1^T\mathbf{R}^T\mathbf{t}, \tag{2-27}$$

where

$$r_1^2 = \mathbf{r}_1^T\mathbf{r}_1, \qquad t^2 = \mathbf{t}^T\mathbf{t} = t_x^2 + t_y^2 + t_z^2. \tag{2-28}$$

The quantity $r_{21}$ is the 3-D sensor's range measurement expressed in the 2-D sensor's measurement frame. Assign $r_{21}$ to the 2-D sensor's measurements (i.e., let $r_2 = r_{21}$). Using Equation 2-27, $r_2$ can be expressed as

$$r_2 = (r_1^2 + t^2 - 2\,\mathbf{r}_1^T\mathbf{t} - 2\,\mathbf{r}_1^T\mathbf{dR}^T\mathbf{t})^{1/2}, \tag{2-29}$$

where Equation 2-12 has been used to eliminate $\mathbf{R}$. In Equation 2-29, all of the terms on the right side are known except for the last one. Assuming that $r_1$ is the dominant term on the right side of Equation 2-29, the binomial expansion can be used to approximate $r_2$ by

$$r_2 \approx r_1 + \frac{t^2}{2\,r_1} - \frac{\mathbf{r}_1{}^T\mathbf{t}}{r_1} - \frac{\mathbf{r}_1{}^T\mathbf{dR}^T\mathbf{t}}{r_1}. \tag{2-30}$$

Assuming that the maximum values of the roll, pitch, and yaw for each sensor is 1° and the maximum separation between the sensors is 100 m, it can be shown that $\| \mathbf{r}_1^T\,\mathbf{dR}^T\,\mathbf{t}/r_1 \| \le 12$ m, and this is independent of the value of the range. That is, neglecting the last term on the right side of Equation 2-30 will introduce less than 12 m of error in the range $r_2$. Similarly, if the range $r_1 = 1000$ m, then $| t^2/2r_1 | \le 5$ m (and decreases as $r_1$ increases). Targets of interest will have $r_1 \ge 1000$ m. These two terms will be neglected in Equation 2-30, so that $r_2$ is approximated by

$$r_2 \approx r_1 - \frac{\mathbf{r}_1^T \mathbf{t}}{r_1}, \tag{2-31}$$

where all of the terms on the right side are known quantities. Now, the position vector $\mathbf{r}_2$ in the 2-D sensor's measurement frame can be calculated using the azimuth $\theta_2$ and elevation $\varepsilon_2$ measured by the 2-D sensor, and the range $r_2$ from Equation 2-31, which is obtained from the 3-D sensor's measurements.

The first-order Taylor series expansion of the true position vector $\mathbf{r}_{2T}$ in the 2-D sensor's measurement frame about the measured position vector $\mathbf{r}_2$ in the 2-D sensor's measurement frame is given by

$$\mathbf{r}_{2T} \approx \mathbf{r}_2 - \mathbf{dr}_2 - \mathbf{e}_2, \tag{2-32}$$

where the differential $\mathbf{dr}_2$ and the random error vector $\mathbf{e}_2$ are given by

$$\mathbf{dr}_2 = \mathbf{f}(\theta_2,\varepsilon_2)\ dr_2 + \mathbf{g}(r_2,\theta_2,\varepsilon_2)\ d\theta_2 + \mathbf{h}(r_2,\theta_2,\varepsilon_2)\ d\varepsilon_2, \tag{2-33}$$

$$\mathbf{e}_2 = \mathbf{f}(\theta_2,\varepsilon_2)\ e_{r2} + \mathbf{g}(r_2,\theta_2,\varepsilon_2)\ e_{\theta 2} + \mathbf{h}(r_2,\theta_2,\varepsilon_2)\ e_{\varepsilon 2}. \tag{2-34}$$

In Equations 2-33 and 2-34, the offset error $dr_2$ and random error $e_{r2}$ in the range $r_2$ depend on the offset and random errors in the 3-D sensor. Using Equation 2-31, the first-order Taylor series expansion of the true range $r_{2T}$ about the measured range $r_2$ is given by

$$r_{2T} \approx r_2 - dr_2 - e_{r2}, \tag{2-35}$$

where $r_2$ is given by Equation 2-31, and the offset error $dr_2$ and random error $e_{r2}$ are given by

$$dr_2 = dr_1 - \frac{\mathbf{g}^T(r_1,\theta_1,\varepsilon_1)\,\mathbf{t}}{r_1}\ d\theta_1 - \frac{\mathbf{h}^T(r_1,\theta_1,\varepsilon_1)\,\mathbf{t}}{r_1}\ d\varepsilon_1, \tag{2-36}$$

$$e_{r2} = e_{r1} - \frac{\mathbf{g}^T(r_1,\theta_1,\varepsilon_1)\,\mathbf{t}}{r_1}\ e_{\theta 1} - \frac{\mathbf{h}^T(r_1,\theta_1,\varepsilon_1)\,\mathbf{t}}{r_1}\ e_{\varepsilon 1}. \tag{2-37}$$

## 2.4 ALIGNMENT EQUATION

As. ne that both sensors are tracking a common target. The 3-D sensor provides measurements of the range $r_1$, azimuth $\theta_1$, and elevation $\varepsilon_1$ for the target (expressed in the 3-D sensor's measurement frame). The target's position vector $\mathbf{r}_1$ in the 3-D sensor's measurement frame can be calculated using these measurements. Similarly, the 2-D sensor provides measurements of the azimuth $\theta_2$ and elevation $\varepsilon_2$ for the target (expressed in the 2-D sensor's measurement frame). Equation 2-31 is used to calculate the target's range $r_2$ in the 2-D sensor's measurement frame. Thus, the target's position vector $\mathbf{r}_2$ in the 2-D sensor's measurement frame can also be calculated.

Let $\mathbf{r}_{1T}$ denote the target's true position vector in the 3-D sensor's measurement frame and $\mathbf{r}_{2T}$ the target's true position vector in the 2-D sensor's measurement frame. Since $\mathbf{r}_{2T}$ is a position vector in the 2-D sensor's measurement frame, Equation 2-17 can be used to transform $\mathbf{r}_{2T}$ to the 3-D sensor's measurement frame. That is,

$$\mathbf{r}_{12T} = \mathbf{r}_{2T} + \mathbf{t} + \mathbf{dR}^{\mathrm{T}}\mathbf{r}_{2T}, \tag{2-38}$$

where $\mathbf{r}_{12T}$ is the target's true position vector $\mathbf{r}_{2T}$ from the 2-D sensor's measurement frame, but expressed in the 3-D sensor's measurement frame. By definition,

$$\mathbf{r}_{1T} = \mathbf{r}_{12T}. \tag{2-39}$$

This gives

$$\mathbf{r}_{1T} = \mathbf{r}_{2T} + \mathbf{t} + \mathbf{dR}^{\mathrm{T}}\mathbf{r}_{2T}. \tag{2-40}$$

Using Equations 2-19 and 2-32 to eliminate $\mathbf{r}_{1T}$ and $\mathbf{r}_{2T}$, and only keeping first-order terms, gives

$$\mathbf{r}_1 \approx \mathbf{r}_2 + \mathbf{t} + \mathbf{a} + \mathbf{e}, \tag{2-41}$$

where the alignment vector $\mathbf{a}$ and the random error vector $\mathbf{e}$ are given by

$$\mathbf{a} = \mathbf{dr}_1 - \mathbf{dr}_2 + \mathbf{dR}^{\mathrm{T}}\mathbf{r}_2, \qquad \mathbf{e} = \mathbf{e}_1 - \mathbf{e}_2. \tag{2-42}$$

Equation 2-41 is the transformation that aligns the position vector $\mathbf{r}_2$ from the 2-D sensor's measurement frame to the position vector $\mathbf{r}_1$ in the 3-D sensor's measurement frame. Thus, Equation 2-41 represents the alignment relative to the 3-D sensor's measurement frame. The alignment is accomplished by first transforming $\mathbf{r}_2$ to the 3-D sensor's frame as if there were no alignment errors (i.e., $\mathbf{r}_2 + \mathbf{t}$). This result is then aligned to $\mathbf{r}_1$ by applying $\mathbf{a}$. The term e represents the random errors in the alignment process.

Equation 2-41 can also be manipulated to obtain the alignment relative to the 2-D sensor's measurement frame. This is accomplished by solving Equation 2-42 for $\mathbf{r}_2$, which gives

$$\mathbf{r}_2 = \mathbf{r}_1 - \mathbf{t} - \mathbf{a} - \mathbf{e}. \tag{2-43}$$

In Equation 2-43, $\mathbf{r}_1$ from the 3-D sensor's measurement frame is aligned to $\mathbf{r}_2$ in the 2-D sensor's measurement frame. Note that this alignment requires the same information as the alignment relative to the 3-D sensor's measurement frame. In both cases, it is necessary to determine the alignment vector $\mathbf{a}$. Thus, it is arbitrary as to which sensor is chosen as the master sensor.

The problem is to find the alignment vector $\mathbf{a}$. Using Equations 2-13, 2-20, and 2-33 in Equation 2-42, and Equation 2-36 to eliminate $dr_2$, the alignment vector $\mathbf{a}$ can be expressed as

$$\begin{aligned}\mathbf{a} = {} & [\,\mathbf{f}(\theta_1,\varepsilon_1) - \mathbf{f}(\theta_2,\varepsilon_2)\,]\, dr_1 + \left[\,\mathbf{g}(r_1,\theta_1,\varepsilon_1) - \frac{\mathbf{g}^T(r_1,\theta_1,\varepsilon_1)\,\mathbf{t}}{r_1}\,\mathbf{f}(\theta_2,\varepsilon_2)\,\right] d\theta_1 \\ & + \left[\,\mathbf{h}(r_1,\theta_1,\varepsilon_1) - \frac{\mathbf{h}^T(r_1,\theta_1,\varepsilon_1)\,\mathbf{t}}{r_1}\,\mathbf{f}(\theta_2,\varepsilon_2)\,\right] d\varepsilon_1 - \mathbf{g}(r_2,\theta_2,\varepsilon_2)\,(d\theta_2 + \Delta\phi) \\ & - \mathbf{h}(r_2,\theta_2,\varepsilon_2)\, d\varepsilon_2 + \mathbf{c}_1\,\Delta\eta + \mathbf{c}_2\,\Delta\psi,\end{aligned} \tag{2-44}$$

where

$$\mathbf{c}_1 = [\,-z_2 \quad 0 \quad x_2\,]^T, \qquad \mathbf{c}_2 = [\,0 \quad -z_2 \quad y_2\,]^T. \tag{2-45}$$

In a similar manner, the random error vector $\mathbf{e}$ can be expressed as

$$\mathbf{e} = [\, \mathbf{f}(\theta_1,\varepsilon_1) - \mathbf{f}(\theta_2,\varepsilon_2)\,]\, e_{r1} + [\, \mathbf{g}(r_1,\theta_1,\varepsilon_1) - \frac{\mathbf{g}^T(r_1,\theta_1,\varepsilon_1)\,\mathbf{t}}{r_1}\, \mathbf{f}(\theta_2,\varepsilon_2)\,]\, e_{\theta 1}$$

$$+ [\, \mathbf{h}(r_1,\theta_1,\varepsilon_1) - \frac{\mathbf{h}^T(r_1,\theta_1,\varepsilon_1)\,\mathbf{t}}{r_1}\, \mathbf{f}(\theta_2,\varepsilon_2)\,]\, e_{\varepsilon 1} - \mathbf{g}(r_2,\theta_2,\varepsilon_2)\, e_{\theta 2}$$

$$- \mathbf{h}(r_2,\theta_2,\varepsilon_2)\, e_{\varepsilon 2}. \tag{2-46}$$

Equations 2-44 and 2-46 can be simplified by using the assumptions made for this problem. It can be shown that

$$\| (\, \mathbf{g}^T(r_1,\theta_1,\varepsilon_1)\,\mathbf{t} / r_1 \,)\, \mathbf{f}(\theta_2,\varepsilon_2)\; d\theta_1 \| \le \| \mathbf{t} \|\, | d\theta_1 | , \tag{2-47}$$

$$\| (\, \mathbf{h}^T(r_1,\theta_1,\varepsilon_1)\,\mathbf{t} / r_1 \,)\, \mathbf{f}(\theta_2,\varepsilon_2)\; d\varepsilon_1 \| \le \| \mathbf{t} \|\, | d\varepsilon_1 | . \tag{2-48}$$

Since $\| \mathbf{t} \| \le 100$ m, $| d\theta_1 | \le 1°$, and $| d\varepsilon_1 | \le 1°$, the expression in Equation 2-47 will contribute less than 1.75 m to the alignment vector **a**. Similarly, the expression in Equation 2-48 will also contribute less than 1.75 m to **a**. That is, ignoring both of these terms will introduce an error in **a** of less than 3.5 m. In this case, Equation 2-44 becomes

$$\mathbf{a} = [\, \mathbf{f}(\theta_1,\varepsilon_1) - \mathbf{f}(\theta_2,\varepsilon_2)\,]\, dr_1 + \mathbf{g}(r_1,\theta_1,\varepsilon_1)\, d\theta_1 + \mathbf{h}(r_1,\theta_1,\varepsilon_1)\, d\varepsilon_1$$

$$- \mathbf{g}(r_2,\theta_2,\varepsilon_2)\,(d\theta_2 + \Delta\phi) - \mathbf{h}(r_2,\theta_2,\varepsilon_2)\, d\varepsilon_2 + \mathbf{c}_1\, \Delta\eta + \mathbf{c}_2\, \Delta\psi. \tag{2-49}$$

In a similar manner, the random error vector **e** can be expressed as

$$\mathbf{e} = [\, \mathbf{f}(\theta_1,\varepsilon_1) - \mathbf{f}(\theta_2,\varepsilon_2)\,]\, e_{r1} + \mathbf{g}(r_1,\theta_1,\varepsilon_1)\, e_{\theta 1} + \mathbf{h}(r_1,\theta_1,\varepsilon_1)\, e_{\varepsilon 1}$$

$$- \mathbf{g}(r_2,\theta_2,\varepsilon_2)\; e_{\theta 2} - \mathbf{h}(r_2,\theta_2,\varepsilon_2)\; e_{\varepsilon 2}. \tag{2-50}$$

The alignment vector **a** in Equation 2-49 depends on seven parameters: $dr_1$, $d\theta_1$, $d\varepsilon_1$, $d\theta_2 + \Delta\phi$, $d\varepsilon_2$, $\Delta\eta$, and $\Delta\psi$. Note that the effect of the yaw alignment parameter $\Delta\phi$ cannot be separated from the effect of the azimuth offset $d\theta_2$ in the 2-D sensor. Since the sensors are close to each other (much closer than the distances to targets of interest) and the alignment errors are assumed to be small, it may be difficult to separate the effects of the remaining seven parameters in the data. That is, observability problems may be encountered if an attempt is made to estimate all seven parameters. This occurs because some of the vectors multiplying the parameters in Equation 2-49 are nearly parallel when the sensors are close to each other. In particular, if the sensors are close to each other and the alignment errors are small, $\mathbf{f}(\theta_1,\varepsilon_1)$ will be nearly parallel to $\mathbf{f}(\theta_2,\varepsilon_2)$, $\mathbf{g}(r_1,\theta_1,\varepsilon_1)$ will be nearly parallel to $\mathbf{g}(r_2,\theta_2,\varepsilon_2)$, and $\mathbf{h}(r_1,\theta_1,\varepsilon_1)$ will be nearly parallel to $\mathbf{h}(r_2,\theta_2,\varepsilon_2)$. In addition, $\mathbf{f}(\theta_1,\varepsilon_1)$ and $\mathbf{f}(\theta_2,\varepsilon_2)$ have the same length (because both are unit vectors), $\mathbf{g}(r_1,\theta_1,\varepsilon_1)$ and $\mathbf{g}(r_2,\theta_2,\varepsilon_2)$ have nearly the same length, and $\mathbf{h}(r_1,\theta_1,\varepsilon_1)$ and $\mathbf{h}(r_2,\theta_2,\varepsilon_2)$ have nearly the same length. Thus, the alignment vector $\mathbf{a}$ in Equation 2-49 and the random error vector $\mathbf{e}$ in Equation 2-50 may be simplified by replacing $\mathbf{f}(\theta_1,\varepsilon_1)$ with $\mathbf{f}(\theta_2,\varepsilon_2)$, $\mathbf{g}(r_1,\theta_1,\varepsilon_1)$ with $\mathbf{g}(r_2,\theta_2,\varepsilon_2)$, and $\mathbf{h}(r_1,\theta_1,\varepsilon_1)$ with $\mathbf{h}(r_2,\theta_2,\varepsilon_2)$. This gives

$$\mathbf{a} = \mathbf{c}_1\,\Delta\eta + \mathbf{c}_2\,\Delta\psi + \mathbf{c}_3\,\Delta\theta + \mathbf{c}_4\,\Delta\varepsilon, \tag{2-51}$$

and

$$\mathbf{e} = \mathbf{c}_3\,e_\theta + \mathbf{c}_4\,e_\varepsilon, \tag{2-52}$$

where

$$\Delta\theta = d\theta_1 - d\theta_2 - \Delta\phi, \quad \Delta\varepsilon = d\varepsilon_1 - d\varepsilon_2, \quad e_\theta = e_{\theta 1} - e_{\theta 2}, \quad e_\varepsilon = e_{\varepsilon 1} - e_{\varepsilon 2}, \tag{2-53}$$

and

$$\mathbf{c}_3 \equiv \mathbf{g}(r_2,\theta_2,\varepsilon_2) = [\,y_2 \quad -x_2 \quad 0\,]^T, \tag{2-54}$$

$$\mathbf{c}_4 \equiv \mathbf{h}(r_2,\theta_2,\varepsilon_2) = [\,-z_2 \sin\theta_2 \quad -z_2 \cos\theta_2 \quad r_2 \cos\varepsilon_2\,]^T. \tag{2-55}$$

The alignment vector $\mathbf{a}$ in Equation 2-51 is expressed in terms of $\mathbf{g}(r_2,\theta_2,\varepsilon_2)$ and $\mathbf{h}(r_2,\theta_2,\varepsilon_2)$ from the 2-D sensor, rather than $\mathbf{g}(r_1,\theta_1,\varepsilon_1)$ and $\mathbf{h}(r_1,\theta_1,\varepsilon_1)$ from the 3-D sensor, because the 2-D sensor's measurements are usually more accurate than the 3-D sensor's measurements (e.g., the 2-D sensor may be an optical sensor that provides very accurate angular information). Of course, the range information assigned to the 2-D sensor depends on the 3-D sensor's measurements.

In Equation 2-51, there are four alignment parameters that have to be determined: the pitch parameter $\Delta\eta$, the roll parameter $\Delta\psi$, the azimuth parameter $\Delta\theta$, and the elevation parameter $\Delta\varepsilon$. Note that it is not necessary to know the individual alignment parameters in each sensor to align the target's position vector from each sensor. Rather, it is only necessary to determine the differences in the respective alignment parameters. The alignment vector **a** in Equation 2-51 can be expressed in matrix notation as

$$\mathbf{a} = \mathbf{C}\,\mathbf{b}, \tag{2-56}$$

where the 4 x 1 alignment parameter vec⁺:.r, **b**, and the 3 x 4 matrix, **C**, are given by

$$\mathbf{b} = [\,\Delta\eta \quad \Delta\psi \quad \Delta\theta \quad \Delta\varepsilon\,]^T, \qquad \mathbf{C} = [\,\mathbf{c}_1 \quad \mathbf{c}_2 \quad \mathbf{c}_3 \quad \mathbf{c}_4\,]. \tag{2-57}$$

The elements of the matrix **C** are known and are related to the sensor's measurements. However, the alignment parameter vector **b** is not known and it must be determined. If the 3-D sensor is chosen as the master sensor, Equation 2-41 is the appropriate alignment equation and it can be expressed as

$$\mathbf{r}_1 = \mathbf{r}_2 + \mathbf{t} + \mathbf{C}\,\mathbf{b} + \mathbf{e}. \tag{2-58}$$

If the 2-D sensor is chosen as the master sensor, then Equation 2-43 is the appropriate alignment equation and it can be expressed as

$$\mathbf{r}_2 = \mathbf{r}_1 - \mathbf{t} - \mathbf{C}\,\mathbf{b} - \mathbf{e}. \tag{2-59}$$

Equations 2-58 and 2-59 account for the effects of the attitude and offset errors in both sensors. Equation 2-58 is the transformation that aligns the 2-D sensor relative to the 3-D sensor, and Equation 2-59 aligns the 3-D sensor relative to the 2-D sensor.

# 3.0 ALGORITHM DEVELOPMENT

To align the position vectors using Equation 2-58 or Equation 2-59, the alignment parameter vector **b** must be known. Assuming that both sensors are tracking a common target (so that $\mathbf{r}_1$ and $\mathbf{r}_2$ can be calculated), it would seem that Equation 2-58 or Equation 2-59 could be used to directly estimate **b**. That is, the equation

$$\mathbf{r}_1 - \mathbf{r}_2 - \mathbf{t} = \mathbf{C}\,\mathbf{b} + \mathbf{e}, \tag{3-1}$$

(where $\mathbf{r}_1 - \mathbf{r}_2 - \mathbf{t}$ is known) would serve as the measurement equation, and it would be used to obtain the estimate of **b**. However, there is a problem with this approach, which can be seen as follows. The random error vector **e** in Equation 2-52 can be expressed as

$$\mathbf{e} = \mathbf{A}\,\mathbf{e}', \tag{3-2}$$

where $\mathbf{A} = [\,\mathbf{c}_3 \quad \mathbf{c}_4\,]$ is a 3 x 2 matrix, and $\mathbf{e}' = [\,e_\theta \quad e_\varepsilon\,]^T$ is a 2 x 1 random measurement error vector. The covariance of $\mathbf{e}'$ is given by the 2 x 2 matrix **W**, where

$$\mathbf{W} = \mathrm{diag}\,(\sigma_\theta^2\ ,\ \sigma_\varepsilon^2), \tag{3-3}$$

and

$$\sigma_\theta^2 = \sigma_{\theta 1}^2 + \sigma_{\theta 2}^2, \qquad \sigma_\varepsilon^2 = \sigma_{\varepsilon 1}^2 + \sigma_{\varepsilon 2}^2. \tag{3-4}$$

Here, $\sigma_{\theta 1}$ and $\sigma_{\varepsilon 1}$ are the standard deviations in the 3-D sensor's azimuth and elevation measurements, and $\sigma_{\theta 2}$ and $\sigma_{\varepsilon 2}$ are the standard deviations in the 2-D sensor's azimuth and elevation measurements. The covariance of **e** is the 3 x 3 matrix given by

$$\mathrm{Cov}\,(\mathbf{e}) = \mathbf{A}\,\mathbf{W}\,\mathbf{A}^T. \tag{3-5}$$

Equation 3-5 is the source of the difficulty because the 3 x 3 matrix Cov (e) cannot have a rank that is larger than the rank of **W** (which has rank two). Therefore, Cov (e) is not an invertible matrix. This problem occurs because the 3 x 2 matrix **A** cannot have full row rank. It can be shown that

$$(\sin\theta_2) \times (\text{Row 1 of A}) + (\cos\theta_2) \times (\text{Row 2 of A}) + (\tan\varepsilon_2) \times (\text{Row 3 of A}) = \mathbf{0}, \qquad (3\text{-}6)$$

and the same relationship holds for the rows of C. That is, the rows of **A** are linearly dependent, and the rows of **C** have the same linear dependence. This means that Equation 3-1 contains a redundant relationship. Thus, one of the relationships in Equation 3-1 will have to be eliminated. The vector equation in Equation 3-1 can be expressed as the following three scalar equations

$$x_1 = x_2 + t_x - z_2\,\Delta\eta + y_2\,\Delta\theta - (z_2 \sin\theta_2)\,\Delta\varepsilon + y_2\,e_\theta - (z_2 \sin\theta_2)\,e_\varepsilon, \qquad (3\text{-}7)$$

$$y_1 = y_2 + t_y - z_2\,\Delta\psi - x_2\,\Delta\theta - (z_2 \cos\theta_2)\,\Delta\varepsilon - x_2\,e_\theta - (z_2 \cos\theta_2)\,e_\varepsilon, \qquad (3\text{-}8)$$

$$z_1 = z_2 + t_z + x_2\,\Delta\eta + y_2\,\Delta\psi + (r_2 \cos\varepsilon_2)\,\Delta\varepsilon + (r_2 \cos\varepsilon_2)\,e_\varepsilon. \qquad (3\text{-}9)$$

Equations 3-7 and 3-8 can be combined into a single relationship using $\tan\theta_1 = x_1/y_1$, or $x_1 \cos\theta_1 = y_1 \sin\theta_1$ (which holds even when $\theta_1 = 90°, 270°$). Using this gives

$$\begin{aligned} m_1 = {} & (\sin\varepsilon_2 \cos\theta_1)\,\Delta\eta - (\sin\varepsilon_2 \sin\theta_1)\,\Delta\psi - (\cos\varepsilon_2 \cos(\theta_2 - \theta_1))\,\Delta\theta \\ & + (\sin\varepsilon_2 \sin(\theta_2 - \theta_1))\,\Delta\varepsilon - (\cos\varepsilon_2 \cos(\theta_2 - \theta_1))\,e_\theta \\ & + (\sin\varepsilon_2 \sin(\theta_2 - \theta_1))\,e_\varepsilon, \end{aligned} \qquad (3\text{-}10)$$

where

$$m_1 = \cos\varepsilon_2 \sin(\theta_2 - \theta_1) + \frac{t_x \cos\theta_1 - t_y \sin\theta_1}{r_2}. \qquad (3\text{-}11)$$

Equation 3-9 can be expressed as

$$m_2 = (\cos\varepsilon_2 \sin\theta_2)\,\Delta\eta + (\cos\varepsilon_2 \cos\theta_2)\,\Delta\psi + (\cos\varepsilon_2)\,\Delta\varepsilon + (\cos\varepsilon_2)\,e_\varepsilon, \qquad (3\text{-}12)$$

where

$$m_2 = \frac{r_1 \sin \varepsilon_1}{r_2} - \sin \varepsilon_2 - \frac{t_z}{r_2}. \tag{3-13}$$

Equations 3-10 and 3-12 can be expressed in matrix notation as

$$\mathbf{m} = \mathbf{H}\,\mathbf{b} + \mathbf{e}_m, \tag{3-14}$$

where $\mathbf{m} = [\, m_1 \quad m_2 \,]^T$, $\mathbf{H} = [\, \mathbf{h}_1 \quad \mathbf{h}_2 \quad \mathbf{h}_3 \quad \mathbf{h}_4 \,]$ is a 2 x 4 matrix, $\mathbf{e}_m = \mathbf{D}\,\mathbf{e}'$ is a random error vector, $\mathbf{D} = [\, \mathbf{h}_3 \quad \mathbf{h}_4 \,]$ is a 2 x 2 matrix, and

$$\mathbf{h}_1 = [\, \sin \varepsilon_2 \cos \theta_1 \quad \cos \varepsilon_2 \sin \theta_2 \,]^T, \tag{3-15}$$

$$\mathbf{h}_2 = [\, -\sin \varepsilon_2 \sin \theta_1 \quad \cos \varepsilon_2 \cos \theta_2 \,]^T, \tag{3-16}$$

$$\mathbf{h}_3 = [\, -\cos \varepsilon_2 \cos (\theta_2 - \theta_1) \quad 0 \,]^T, \tag{3-17}$$

$$\mathbf{h}_4 = [\, \sin \varepsilon_2 \sin (\theta_2 - \theta_1) \quad \cos \varepsilon_2 \,]^T. \tag{3-18}$$

Since Equation 3-14 represents two equations with four unknowns, it will be necessary to observe at least two targets at the same time, or a single target (which is not stationary) that is being tracked over time, in order to obtain a solution for these four unknowns. If two or more targets are being observed, there will be an **H** matrix, a **D** matrix, and an $\mathbf{e}_m$ vector for each target. In this case, all of these matrices can be augmented into a single large system of the form presented in Equation 3-14. That is,

$$\mathbf{z} = \mathbf{F}\,\mathbf{b} + \mathbf{v}, \tag{3-19}$$

where

$$\mathbf{z} = [\, \mathbf{m}_1^T \ \cdots \ \mathbf{m}_N^T \,]^T, \tag{3-20}$$

$$\mathbf{F} = [\, \mathbf{H}_1^T \ \cdots \ \mathbf{H}_N^T \,]^T, \tag{3-21}$$

$$\mathbf{v} = [\, \mathbf{e}_{m1}^T \ \cdots \ \mathbf{e}_{mN}^T \,]^T, \tag{3-22}$$

and N is the total number of targets. Equation 3-19 represents 2N equations with four unknowns and it can be used to estimate **b**. Once **b** is determined, then Equation 2-58 or Equation 2-59 can be used to align the position vectors.

Kalman filtering techniques will be applied to Equation 3-19 to obtain the estimate of **b**. The vector **b** will be assumed to be a constant that is driven by zero-mean white noise; that is, the dynamics for **b** are given by

$$\mathbf{b}_j = \mathbf{b}_{j-1} + \mathbf{w}_{j-1}, \tag{3-23}$$

where $\mathbf{w}_{j-1}$ is the zero-mean white noise term with covariance matrix $\mathbf{Q}_j$, and the subscript $j$ refers to the $j^{th}$ update on the targets. The measurement of **b** at the $j^{th}$ update is given by Equation 3-19,

$$\mathbf{z}_j = \mathbf{F}_j \mathbf{b}_j + \mathbf{v}_j. \tag{3-24}$$

The estimate of **b** for the $j^{th}$ update, denoted by $\hat{\mathbf{b}}_j$, is generated by the difference equation

$$\hat{\mathbf{b}}_j = \hat{\mathbf{b}}_{j-1} + \mathbf{K}_j ( \mathbf{z}_j - \mathbf{F}_j \hat{\mathbf{b}}_{j-1} ), \tag{3-25}$$

where the Kalman gain $\mathbf{K}_j$ is given by

$$\mathbf{K}_j = \mathbf{P}_{j|j-1} \mathbf{F}_j^{\mathrm{T}} [ \mathbf{F}_j \mathbf{P}_{j|j-1} \mathbf{F}_j^{\mathrm{T}} + \mathbf{S}_j ]^{-1}, \tag{3-26}$$

and $\mathbf{S}_j$ is the covariance matrix for $\mathbf{v}_j$. The one-step predicted covariance $\mathbf{P}_{j|j-1}$ is given by

$$\mathbf{P}_{j|j-1} = \mathbf{P}_{j-1|j-1} + \mathbf{Q}_{j-1}, \tag{3-27}$$

and the covariance matrix $\mathbf{P}_{j|j}$ is generated by

$$\mathbf{P}_{j|j} = [ \mathbf{I} - \mathbf{K}_j \mathbf{F}_j ] \mathbf{P}_{j|j-1}, \tag{3-28}$$

where **I** is the identity matrix. The Kalman filter algorithm requires an initial estimate of **b** and an initial covariance matrix $\mathbf{P}_{0|0}$. Since there may not be an initial estimate of **b**, the simplest procedure is to let $\hat{\mathbf{b}}_0 = \mathbf{0}$. Theoretically, the initial covariance matrix $\mathbf{P}_{0|0}$ is infinite because there is no *a priori* information about **b**. However, in practice, this problem can be dealt with by choosing a sufficiently large $\mathbf{P}_{0|0}$.

# 4.0 SIMULATION RESULTS

This algorithm was tested by generating a single track and including alignment and random errors in the track data. The 3-D sensor is located at the origin, and the 2-D sensor is located at $t_x = t_y = 25$ m and $t_z = 10$ m. The standard deviations in the sensor's measurements are given by

$$\sigma_{r1} = 10\,\mathrm{m}, \qquad \sigma_{\theta 1} = \sigma_{\varepsilon 1} = 0.1°, \qquad \sigma_{\theta 2} = \sigma_{\varepsilon 2} = 0.02°. \tag{4-1}$$

The following alignment errors were included in the 3-D sensor's data

$$dr_1 = 50\,\mathrm{m}, \quad d\theta_1 = 1°, \quad d\varepsilon_1 = 0.5°, \quad \phi_1 = \eta_1 = \psi_1 = 1°, \tag{4-2}$$

and the alignment errors included in the 2-D sensor's data are given by

$$d\theta_2 = -1°, \quad d\varepsilon_2 = -0.5°, \quad \phi_2 = \eta_2 = \psi_2 = -1°. \tag{4-3}$$

Using these values in Equations 2-14 and 2-53 gives the following values for the four alignment parameters,

$$\Delta\eta = 2°, \quad \Delta\psi = -2°, \quad \Delta\theta = 4°, \quad \Delta\varepsilon = 1°. \tag{4-4}$$

The 3-D sensor's measurements are used to obtain the target's position vector in the 3-D sensor's frame. The range measured by the 3-D sensor is used in Equation 2-31 to calculate the target's range in the 2-D sensor's measurement frame, and the target's position vector in the 2-D sensor's frame can be obtained. The data are assumed to be time coincident, and both sensors are reporting data at regular intervals of 0.5 sec. The Kalman filter described in the previous section was implemented to obtain the estimates of **b**. The initial estimate of **b** was taken to be zero, and the initial covariance matrix used was given by $\mathbf{P}_{0|0} = \sigma_0^2\,\mathbf{I}$, where $\sigma_0 = 0.17$ and **I** is the identity matrix. Also, the covariance matrix $\mathbf{Q}_j$ for the input noise $\mathbf{w}_j$ was assumed to be constant with $Q_j = \sigma^2 I$, where $\sigma = 0.001 \times T$ and T is the update interval in seconds.

The track coordinates of the target (which is a closing target) before and after alignment are presented in Figure 4-1. The solid line is the target's track as measured by the 3-D sensor, and the dotted line is the target's track as measured by the 2-D sensor, but expressed in the 3-D sensor's frame. The separation between the tracks before alignment is due to the alignment errors in the sensors. The estimates of the alignment parameters from the Kalman filter were used in Equation 2-58 to align the 2-D sensor's track to the 3-D sensor's track. To the scale of the graphs in Figure 4-1, the tracks after alignment are indistinguishable.

The measured azimuth and elevation angles before and after alignment are presented in Figure 4-2, where all of the data are expressed in the 3-D sensor's measurement frame. The solid lines represent the 3-D sensor's measurements, and the dotted line the 2-D sensor's measurements. The measurements are indistinguishable after alignment.

The estimates of the four alignment parameters from the Kalman filter are presented in Figure 4-3. The dotted lines in Figure 4-3 represent the true values of the alignment parameters, which are also given in Equation 4-4. It takes approximately 100 updates (50 sec) before the estimates converge to values near their true values. However, the estimate of the azimuth parameter converges much more quickly than the others. The distance between the target's track as reported by the 3-D sensor and the 2-D sensor is presented in Figure 4-4. The distance before alignment is quite large (an average of 969 m for the 200 updates). After alignment, the average distance over the 200 updates is 42 m, or a factor of 23 times smaller than the before alignment distance. Thus, the alignment algorithm performs well in estimating the alignment errors and aligning the tracks.

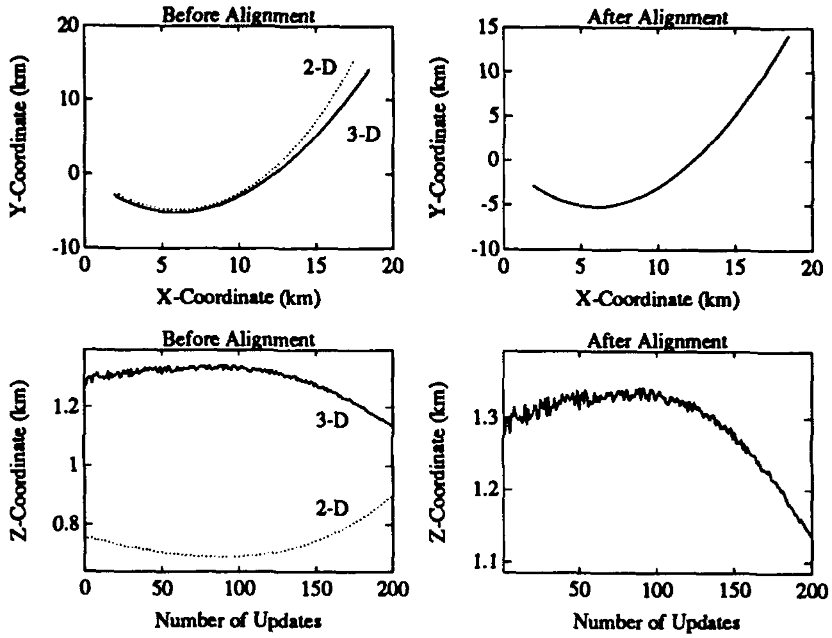


Figure 4-1. Track Coordinates Before and After Applying the Alignment Algorithm to the Track Data

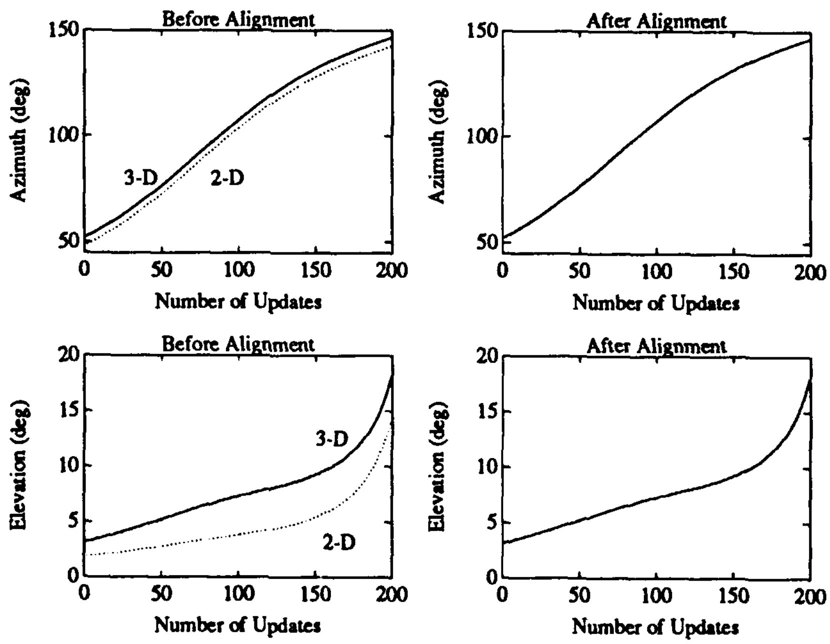


Figure 4-2. Azimuths and Elevations of the Track Before and After Applying the Alignment Algorithm to the Track Data

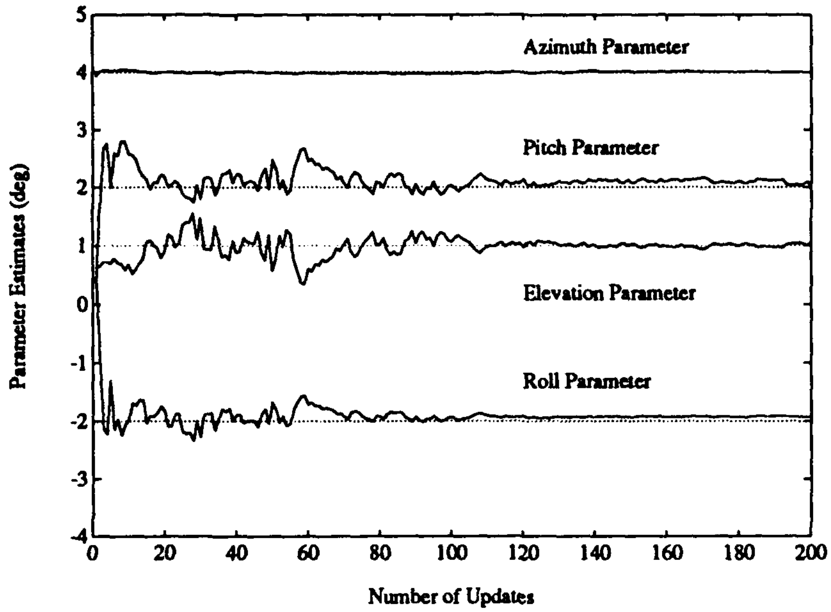


Figure 4-3. Estimates of the Alignment Parameters

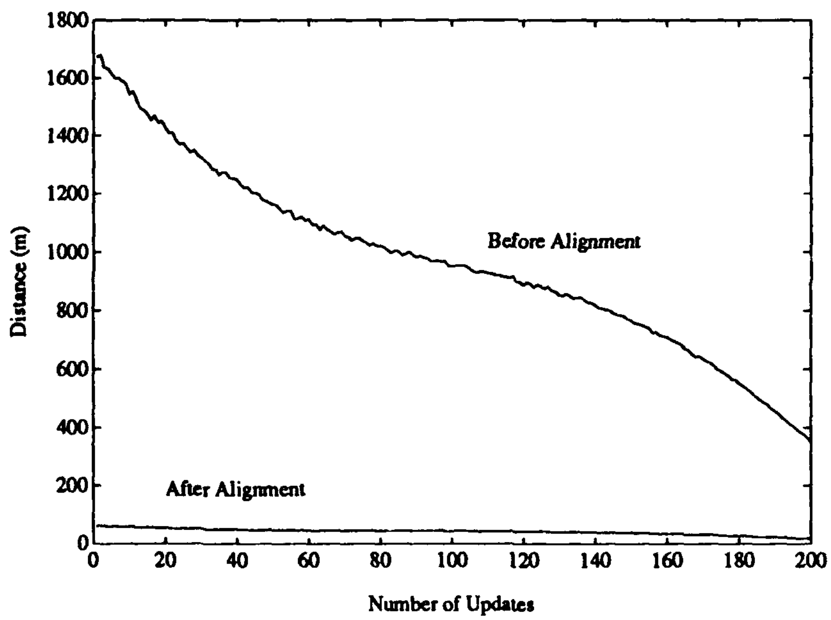


Figure 4-4. Distance Between the Tracks Before and After Applying the Alignment Algorithm to the Track Data

## 5.0 SUMMARY

The problem of aligning two sensors using targets of opportunity was examined in this report, where one of the sensors is a 3-D sensor that measures range, azimuth, and elevation and the other one is a 2-D sensor that measures azimuth and elevation. Both a mathematical model and an alignment algorithm were developed for this problem, where attitude and offset errors in both sensors are included in the formulation. Because of the method used to formulate the problem, it is not possible to determine the individual alignment errors at each sensor. However, it is possible to determine the differences in the respective alignment errors of the two sensors; that is, one sensor appears to be perfectly aligned and it is called the master sensor. The algorithm developed in this report aligns the other sensor relative to the master sensor. The problem is formulated to allow either sensor to serve as the master sensor. Of course, both sensors cannot be the master sensor at the same time. The alignment algorithm is applicable to those situations where the distance between the sensors is small, the magnitude of the alignment errors is small, and the alignment errors do not change with time (stationary).

For illustrative purposes, the alignment algorithm was applied to simulated data from two sensors that were tracking a target undergoing a simple maneuver. Each of the sensors had realistic values for their measurement errors. The filter converged within 50 sec to values of the azimuth, elevation, pitch, and roll alignment errors that are very close to their actual values. Utilizing these values, it was possible to obtain a dramatic 23-fold reduction in the relative error between the tracks generated by the sensors. For further verification of this algorithm, it should first be applied to a large number of simulated tracks following different trajectories and then to simultaneous data from real target tracks.

## 6.0 REFERENCES

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The problem of aligning two sensors using targets of opportunity is examined in this report, where one of the sensors is a 3-D sensor that measures range, azimuth, and elevation and the other one is a 2-D sensor that measures azimuth and elevation. Both a mathematical model and an alignment algorithm are developed for this problem, where attitude and offset errors in both sensors are included in the formulation. Because of the method used to formulate the problem, it is not possible to determine the individual alignment errors at each sensor. However, it is possible to determine the differences in the respective alignment errors of the two sensors. This is sufficient to align one sensor relative to the other one. For illustrative purposes, the alignment algorithm is applied to simulated data from two sensors. The algorithm converges within 50 sec to values very close to the actual values of the alignment parameters. Using these values, it is possible to obtain a dramatic 23-fold reduction in the relative error between the tracks generated by the sensors.

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