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Enhanced Minimum Variance Beamforming

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It is shown that both the multiple emitter signal location wavenumber spectrum estimator (MUSIC) and the minimum variance distortionless response (MVDR) beamformer are just special cases of the enhanced minimum variance (EMV) beamformer. Moreover, by separating planewave arrivals into two types, i.e., high level interferences and low (threshold) level signals, it is illustrated how the EMV beamformer simultaneously gives MUSIC-like angular resolution of an interference and MVDR-like detection of threshold signals. An additional advantage of the EMV beamformer is that the number of threshold signals need not be known a priori. As a preprocessor for the EMV beamformer, a dimensionality reducing transformation of the array data vector to a subarray beam space is proposed. This transformation is wavefront invariant and expedites the separation of source and coherent noise subspace eigenvalues.

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ENHANCED MINIMUM VARIANCE BEAMFORMING

INTRODUCTION

The minimum variance distortionless response (MVDR) criterion for achieving maximum wavefront signal-to-noise ratio (SNR) gain with a spatial sensor array beamformer is well established. The MVDR beamformer has its technical origins in the early 1960's (see [1] for bibliography). Eigenstructure based, signal subspace wavenumber spectrum analysis procedures appeared in the early 1970's [2,3] and have evolved to the present day high resolution procedures typified by the multiple emitter signal location wavenumber spectrum estimator (MUSIC) algorithm [4]. The difference between a beamformer, which can produce a directional waveform estimate, and a wavenumber analyzer, which provides a source direction estimate, is emphasized. Accordingly, the evaluation of MVDR and eigenstructure techniques has been along two parallel paths. This dual perspective has precluded the establishment of common theoretical and practical approaches that could result in the exploitation of the advantages of both techniques. The enhanced minimum variance (EMV) beamformer represents an attempt to find such a common viewpoint and to subsequently address current space-time processing technical issues from this unified perspective. Of particular concern are such issues as detection of threshold signals, determination of the number of sources present, reduction of computational burden, coherent multipath arrivals, and wideband signals.

In the next section, the commonality between the MVDR and eigenstructure methods is explored. Following that, the issue of determining the number of sources is considered within the context of two types of sources, namely, interferences and signals. Next, a wavefront invariant array data vector transformation is proposed which has the simultaneous effects of reducing signal processor computational burden and reducing the sensitivity of the eigenstructure methods to the assumption of spatially uncorrelated noise. This transformation can also be shown to include the function of presteering the spatial cross-spectral density matrix (CSDM) which then can be averaged over frequency for increased statistical stability if wideband signals are of particular concern.

THE EMV TECHNIQUE

The CSDM at frequency ω for an N-sensor array with K sources present is assumed to be

$$\underset{\sim}{R} = \sigma_s^2 \, \underset{\sim}{DED}^H + \sigma_0^2 \, \underset{\sim}{I}_N \, , \tag{1}$$

where σ_s^2 and σ_0^2 are the total autopower spectral densities for the sources and spatially uniform and uncorrelated noise, respectively, at a sensor. The superscript "H" indicates the complex conjugate transpose

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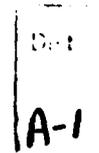
operation. The NxK matrix $\underset{\sim}{D}$ is the source direction vector matrix, $\underset{\sim}{E}$ is a KxK source complex envelope coherence matrix, and $\underset{\sim}{I}_N$ is an NxN identity matrix. The k-th column of $\underset{\sim}{D}$ is the direction vector for the k-th source. It is well known that $\underset{\sim}{R}$ can be written in factored form as

$$\underset{\sim}{R} = \sigma_s^2 \; \underset{\sim}{M} \; \underset{\sim}{\Lambda} \; \underset{\sim}{M}^H + \sigma_o^2 \; \underset{\sim}{I}_N . \tag{2}$$

where $\underset{\sim}{M}$ is an NxP ($P \leq K$) modal matrix of orthonormal eigenvectors which span the source subspace defined by the columns of $\underset{\sim}{D}$ and $\underset{\sim}{\Lambda}$ is a diagonal matrix of the rank ordered eigenvalues of $\underset{\sim}{D}\underset{\sim}{E}\underset{\sim}{D}^H$.

The enhanced CSDM,

$$\underset{\sim}{R}(e) = e\sigma_s^2 \; \underset{\sim}{M} \; \underset{\sim}{\Lambda} \; \underset{\sim}{M}^H + \sigma_o^2 \; \underset{\sim}{I}_N . \tag{3}$$

is now defined in terms of Eq. (2) and the scalar enhancement factor e. The EMV beamformer for beamsteering direction vector $\underline{d}(\Theta)$ is defined by the array filter vector

$$\underline{w}(\Theta, e) = \underset{\sim}{R}(e)^{-1} \; \underline{d}(\Theta) / (\underline{d}(\Theta)^H \; \underset{\sim}{R}(e)^{-1} \; \underline{d}(\Theta)) \tag{4}$$

and the direction-of-arrival (DOA) spectrum

$$P_{EMV}(\Theta, e) = (\underline{d}(\Theta)^H \; \underset{\sim}{R}(e)^{-1} \; \underline{d}(\Theta))^{-1} . \tag{5}$$

We have the identity

$$\underset{\sim}{R}^{-1}(e) = (\underset{\sim}{I}_N - \underset{\sim}{M}\underset{\sim}{B}(e)\underset{\sim}{M}^H)/\sigma_o^2 , \tag{6}$$

where $\underset{\sim}{B}(e)$ is a real, diagonal, and positive definite matrix defined by

$$\underset{\sim}{B}(e) = (\underset{\sim}{\Lambda} + \frac{\sigma_o^2}{e\sigma_s^2} \; \underset{\sim}{I}_p)^{-1} \underset{\sim}{\Lambda} . \tag{7}$$

For the case e = 1, the EMV beamformer is simply the MVDR beamformer. For the case e = ∞, where $\underset{\sim}{B}(\infty) = \underset{\sim}{I}_p$, the EMV beamformer is such that the DOA spectrum estimator is given by

$$P_{EMV}(\Theta, \infty) = \sigma_o^2/(N - |\underline{d}(\Theta)^H \; \underset{\sim}{M}|^2)$$

$$= P_{MUSIC}(\Theta) , \tag{8}$$

i.e., it is identical to the MUSIC DOA estimator [1]. Thus, both the MVDR and MUSIC procedures have been related to the single scalar parameter e.

SIGNALS AS THRESHOLD SOURCES

The process of estimating the CSDM matrix of Eq. (2) involves a decision as to the dimension P of the source subspace. This decision is followed by an estimation procedure for the source eigenstructure in terms of $\underset{\sim}{M}$ and $\underset{\sim}{\Lambda}$. A persistent issue with eigenstructure based source subspace methods is that of estimating the number of sources (P) that are arriving at the array [5]. This is a most difficult problem in the case of very low SNR, i.e., threshold, signals because it is tantamount to making a high probability detection on __all__ sources that are present prior to directional processing of the array. The EMV approach is to exploit the fact that the threshold signals need not be "captured" by the estimated CSDM. Alternatively, only a high level source, hereafter referred to as interference, needs to be accounted for in the estimated CSDM and that detection of the threshold signals is relegated to an EMV beamformer output process. This process shall be represented in the following as infinite time averaging of the EMV beam output power.

Let the true CSDM be

$$\underset{\sim}{R} = \sigma_s^2 [a\underline{d}\underline{d}^H + \underset{\sim}{M} \underset{\sim}{\Lambda} \underset{\sim}{M}^H] + \sigma_0^2 \underset{\sim}{I}_N, \tag{9}$$

where $a << 1$, $(N\sigma_s^2/\sigma_0^2) \geq 1$ and $\underline{d}$ is the threshold signal direction vector. The estimated CSDM is taken to be Eq. (3) because $(aN\sigma_s^2 + \sigma_0^2)$ is below a threshold level such that all eigenvalues of $\underset{\sim}{R}$ are assigned the value $\lambda = \sigma_0^2$ for $P < i \leq N$. Now we have the DOA spectrum

$$P_{EMV}(\Theta,\infty) = \underline{w}(\Theta,\infty)^H \underset{\sim}{R} \underline{w}(\Theta,\infty)$$

$$= \frac{\sigma_s^2 a (N^2\ell - 2N Re\{\alpha\underline{d}^H\underset{\sim}{M}\underset{\sim}{M}^H\underline{d}(\Theta)\} + |\underline{d}(\Theta)^H\underset{\sim}{M}\underset{\sim}{M}^H\underline{d}|^2) + \sigma_0^2 (N - |\underline{d}(\Theta)\underset{\sim}{M}|^2)}{(N - |\underline{d}(\Theta)^H\underset{\sim}{M}|^2)^2}$$

$$\tag{10}$$

where $\underline{d}(\Theta)^H\underline{d} = N\alpha$ and $|\alpha|^2 = \ell$.

For a conventional beamformer (CBF) with a (time) delay-and-sum (DS) uniformly weighted aperture, the directional response is

$$P_{CBF}(\Theta) = \underline{d}(\Theta)^H\underset{\sim}{R} \underline{d}(\Theta)/N^2$$

$$= (\sigma_s^2 a N^2\ell + \sigma_s^2\underline{d}(\Theta)^H\underset{\sim}{M} \underset{\sim}{\Lambda} \underset{\sim}{M}^H\underline{d}(\Theta) + \sigma_0^2 N)/N^2. \tag{11}$$

An examination of Eqs. (10) and (11) indicates that if a source with direction vector $\underline{d}(\Theta)$ is well separated from the interference subspace, i.e., $\underline{d}(\Theta)$ is orthogonal to the columns of $\underset{\sim}{M}$, so $\underset{\sim}{M}^H\underline{d}(\Theta) = \underline{o}$, then the EMV beamformer treats the source as a CBF because Eqs. (10) and (11) are equal. This is known to be optimum. However, when the steering direction vector $\underline{d}(\Theta)$ has a significant projection onto the interference subspace mode vectors, then the denominator of Eq. (10) can become very small and

consequently $p_{EMV}(\Theta,\infty)$ can become very large. This indicates a MUSIC-like high resolution condition as in Eq. (8). In fact, for $\sigma_S^2 \ll \sigma_0^2$ in Eq. (10), $p_{EMV}(\Theta,\infty) \cong P_{MUSIC}(\Theta)$. Thus, the EMV beamformer performs nearly optimally from a resolution standpoint even though the signal was not captured by the estimated CSDM.

REDUCED DIMENSION EMV BEAMFORMING

A major assumption required by the EMV process, as indicated by the CSDM model of Eq. (1), is that the additive noise is spatially uncorrelated from sensor to sensor. In many cases, a more realistic CSDM model would be

$$\underset{\sim}{R} = \sigma_S^2 \underset{\sim}{DED}^H + \sigma_C^2 \underset{\sim}{Q} + \sigma_0^2 \underset{\sim}{I}_N, \tag{12}$$

where Q represents the CSDM for a (partially) coherent additive background noise. The variance σ_C^2 is the autopower spectral density of this coherent noise at a sensor. The difficulty with EMV beamforming for low SNR (threshold) signals in the presence of coherent noise is that strictly on the basis of observing the eigenvalues of R in Eq. (12) estimated over a finite time interval it is difficult to define separable source and noise subspaces. The noise coherence matrix Q can be of full rank $P = N$ such that for $\sigma_C^2 \gg \sigma_S^2$ the source energy may not even appear in the largest eigenvalues [6]. One solution to this problem is to whiten the noise portion $\sigma_C^2 Q + \sigma_0^2 I_N$ of the CSDM matrix [7]. This approach requires that the matrix Q be accurately parameterized and these parameters must then be estimated simultaneously with the source subspace eigenstructure.

An alternative approach to treating coherent noise is to transform the N-dimension array data vector $\underline{x}$ at frequency ω to an L-dimensional (L<<N) vector $\underline{y}$ according to [8]

$$\underline{y} = S^H \underline{x} . \tag{13}$$

The CSDM for this reduced dimension data vector is

$$R = E[\underline{y}\underline{y}^H]$$
$$= \underset{\sim}{S}^H \underset{\sim}{R} \underset{\sim}{S} . \tag{14}$$

In this new transformation space, it is desired that the eigenvalues corresponding to source and noise be more readily separable than before the transformation. This will occur if the transformation of an N-dimension source direction vector $\underline{d}_N(\Theta)$ in the space of $\underline{x}$ is invariant with respect to a constant modulus of the elements of a generalized direction vector. This invariance is expressed as

$$c\underline{d}_L(\Theta) = \underset{\sim}{S}^H \underline{d}_N(\Theta), \qquad |\Theta - \phi| \leq \frac{B}{2} , \tag{15}$$

where the n-th element of a direction vector $\underline{d}_L(\Theta)$ of dimension L is of

the known form $\exp(-j\omega\tau_{kL}(\Theta))$. This simply says that constant amplitude wavefronts are preserved under the transformation $\underset{\sim}{S}$ over an angular sector of width B centered at $\Theta = \emptyset$. The parameter c must be real if the transformation is to be distortionless and $c > 1$ if source and noise eigenvalue separation is to occur.

A transformation which meets the above requirements is the NxL matrix

$$\underset{\sim}{S}(\Theta) = \begin{bmatrix} \underset{\sim}{W}_1\underset{\sim}{d}_N(\Theta) & \underline{o} & \cdots & \underline{o} \\ \underline{o} & \underset{\sim}{W}_2\underset{\sim}{d}_N(\Theta) & \cdots & \underline{o} \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \underline{o} & \underline{o} & \cdots & \underset{\sim}{W}_L\underset{\sim}{d}_N(\Theta) \end{bmatrix} , \tag{16}$$

where $\underset{\sim}{W}_i$ is an (N/L) x N sensor amplitude weighting and subarray selection matrix and $\underline{o}$ is a (N/L)-vector of all zeroes. The vector $\underset{\sim}{W}_i\underset{\sim}{d}_N(\Theta)$ forms a beam from the ith subarray consisting of sensors $((i-1)N/L) + 1$ to iN/L steered in direction Θ. The width of the subarray beam β would be such that $\beta \ll B$, which would ensure the wavefront invariance property. The subarray provides a coherent addition for sources from the region $|\Theta - \emptyset| \le B/2$ with a gain proportional to N/L. Simultaneously, discrimination against the coherent noise occurs if the subarray phase centers are widely spaced relative to the coherence distance of the noise. Because the dimension of the subarray space is such that L<N, there is a reduction of the dimension of the noise subspace and a preservation of the dimension of the signal subspace. This makes the issue of determining the number of interference sources relatively less difficult.

In effect, subarray preprocessing reduces high resolution beamforming from N problems of complexity proportional to N^2 to N problems proportional to L^2. Accordingly, processor complexity is reduced from order N^3 to order NL^2.

EMV BEAMFORMER PERFORMANCE

In this section, the performance of the EMV beamformer for a threshold signal in the presence of a single point interfering noise source is examined. The metric of performance is the improvement in array SNR at the beamformer output relative to a single sensor as compared to a conventional time DS beamformer. This metric is called the array gain improvement, AGI.

For the conditions described, the true CSDM matrix is

$$\underset{\sim}{R} = \sigma_s^2\underline{d}\underline{d}^H + \sigma_I^2\underline{d}_I\underline{d}_I^H + \sigma_0^2\underset{\sim}{I}_N \tag{17}$$

and the enhanced estimated CSDM is

$$\hat{\underset{\sim}{R}}(e) = e\sigma_I^2\underline{d}_I\underline{d}_I^H + \sigma_0^2\underset{\sim}{I}_N . \tag{18}$$

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If the following definitions are made:

$$r = N\sigma_I^2/\sigma_0^2, \tag{19.a}$$

$$\bar{r} = er, \tag{19.b}$$

$$|\underline{d}_I^H \underline{d}|^2 = N^2\ell, \qquad 0 \le \ell \le 1, \tag{19.c}$$

then the array gain for the EMV beamformer as a function of e can be shown to be

$$G_{EMV}(e) = N(1 + \frac{r\ell - \bar{r}^2\ell(1 - \ell) + (\sigma_I^2/\sigma_0^2)\,[1 + \bar{r}\,(1 - \ell)^2]}{1 + r\ell + \bar{r}\,[2 + \bar{r}]\,(1 - \ell)}). \tag{20}$$

Similarly, for a conventional beamformer the array gain is

$$G_{CBF} = N(1 + \frac{(\sigma_I^2/\sigma_0^2) - r\ell}{1 + r\ell}) \tag{21}$$

such that the AGI is

$$AGI_{EMV}(e) = G_{EMV}(e)/G_{CBF}$$

$$= \frac{[1 + \bar{r}\,(1 - \ell)]^2\,(1 + r\ell)}{r\ell + (1 + \bar{r})^2 - \bar{r}(2 + \bar{r})\ell} \tag{22}$$

and

$$AGI_{EMV}(\infty) \cong 1 + \ell\,(r[1 - \ell] - 1)\ . \tag{23}$$

The AGI for the optimum MVDR beamformer is known to be [9]

$$AGI_{MV} = 1 + \frac{r^2\ell\,(1 - \ell)}{1 + r} \tag{24}$$

where the parameter ℓ is the normalized response level to the interference for a DS beam steered directly at the signal.

A comparison of Eq. (23) and (24) shows that for large r, i.e., high level interference, $AGI_{EMV}(\infty) \cong AGI_{MV}$. Moreover, it can be shown that

$$AGI_{EMV}(\infty) = AGI_{MV} - d, \tag{25}$$

where

$$0 \le d = \frac{\ell(1 + r\ell)}{1 + r} < 1\ . \tag{26}$$

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Thus, from the viewpoint of beamformer output signal to total background interference plus noise ratio the MVDR provides an upper bound on the EMV beamformer. However, the potentially higher resolution capability of the EMV beamformer in conjunction with the relaxation of the requirement to know the exact number of sources makes the EMV beamformer attractive.

COHERENT THRESHOLD SIGNALS

The situation of two coherent sources resulting in suppression of both sources when either MVDR or an eigenstructure technique is used is well established [1]. For example, such a condition would occur in a coherent multipath environment. While this is a valid concern for high level interfering sources it is not an issue for the so-called threshold signals introduced above. It can easily be shown that the relative amount of source power suppression in an MVDR beam output due to two perfectly coherent equal strength sources is given by the suppression factor

$$s.f. = 1 - 1/(1 + (1/r)), \tag{27}$$

where r is the beam output source power to background noise ratio. Thus, for $r < 1$ (0 dB) there will be less than 3 dB of source suppression. In Eq. (9), a threshold signal has been defined such that the corresponding beam output SNR will be less than 0 dB. Accordingly, only interfering sources with $r > 1$ will experience suppression. This is completely acceptable.

SUMMARY

The EMV beamformer is based on the estimated eigenstructure rather than the inverse of the cross-spectral density matrix. The EMV beamformer requires only the particular eigenstructure for the high level interfering sources and yet provides nearly optimum detection performance for threshold signals. Estimation of the eigenstructure associated only with the large eigenvalues can be accomplished very rapidly with simplified hardware structures using a systolic array multiplier to implement a power method algorithm [10] in a subarray beam space [11]. Power method based algorithms for large eigenvalue eigenstructure estimation can be accomplished in order N machine cycles for N^3 operations required. Finally, the presteering operation associated with a transformation of the array data vector to subarray beam space is consistent with the steered covariance matrix approach [12, 13]. The estimated steered covariance matrices can be averaged in the frequency domain to obtain increased stability of the estimated eigenstructure when wideband signals are of particular interest.

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