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Aerodynamics Technical Memorandum 390

NON-LINEAR AERODYNAMIC CHARACTERISTICS  
OBTAINED FROM THE ANALYSIS OF FLIGHT-DATA (U)

by

R.A. Felk

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**NON-LINEAR AERODYNAMIC CHARACTERISTICS  
OBTAINED FROM THE ANALYSIS OF FLIGHT DATA (U)**

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R.A. Feik

**SUMMARY**

A least squares regression analysis program has been documented and its advantages and shortcomings when used for analysing flight data have been summarised. It has been shown that the shortcomings can be largely overcome by pre-processing flight measurements via compatibility checking. A particular advantage of the least squares approach is the ability to partition data into angle of attack subsets. Application to flight data from a delta wing aircraft at  $M=0.65$  has been successful in extracting non-linear features, including a sharp drop in pitch damping at around 4 degrees angle of attack, possibly associated with the development of the leading edge vortex. Comparison with previous results, internal consistency, and small scatter all confirm the effectiveness of this approach even with moderate quality instrumentation. The methodology described has considerable potential for application to highly non-linear flight regimes. (707104), (FA)



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## 1. INTRODUCTION

The Aircraft Behaviour Studies groups at ARL have for a number of years been active in the application of System Identification methodology to the extraction of aerodynamic information from flight test data. As well as acquiring or developing a number of parameter and state estimation programs for routine dynamic flight data analysis<sup>1</sup>, work has been aimed at developing a procedure for analysing non-linear flight regimes where the aerodynamic model is uncertain. Particular examples in mind have been high angle of attack aerodynamics of aircraft such as the F-18, and spin behaviour of a basic training aircraft. Aspects of helicopter flight dynamics would also fall into this area.

Regression analysis offers a number of advantages for the analysis of non-linear data<sup>2,3</sup>. These include efficient computation, ease of varying model structure, and ability to partition data into convenient sub-sets. On the other hand serious disadvantages include the need to have measurements of all variables, and degradation in the performance of equation error methods when measurement errors are present i.e. errors in the variables. These disadvantages can be minimised by pre-processing the data to remove measurement errors, both systematic and random, and to reconstruct missing records. Flight Path Reconstruction or Compatibility Checking<sup>4,5</sup> methods were designed to achieve this through use of a non-linear state estimation technique such as an Extended Kalman Filter. Work at ARL and at the University of Newcastle has led to the development of programs for Compatibility Checking using both Maximum Likelihood and Extended Kalman Filter estimation<sup>6</sup>, and has established that successful application can be achieved even with moderate quality flight data<sup>7,8</sup>.

With the use of Compatibility Checking as a pre-processor to produce a complete set of 'error free' flight trial records, the application of regression analysis as the final stage of the procedure to obtain the desired aerodynamic characteristics, becomes an attractive option. This document reports the results achieved using a longitudinal non-steady manoeuvre of the roller coaster type and illustrates the ability of this methodology to maximise the information which can be obtained from a relatively small amount of flight data. Comparison is also made with results obtained from doublet/pulse response flight trials data. Prior to presenting the results, the regression analysis program developed for this work is documented and some remarks made as to its practical application.

## 2. METHOD

### 2.1 Basic Theory

Applied regression analysis is a well established procedure described in standard texts, <sup>9,10</sup>. The program described here draws on these and other sources to provide an interactive tool designed to allow the user considerable flexibility to specify options and test alternative models. It is assumed that the model describing the measurements is linear in the parameters and has the form

$$y(i) = \mathbf{x}^T(i)\boldsymbol{\xi} + \epsilon(i) \quad (1)$$

where

$y$	$\boldsymbol{\xi}$	$\mathbf{x}$	$\epsilon$	$i$	is the dependent variable
					is a p-dimensional vector of unknown parameters
					is a p-dimensional vector of independent variables
					is the equation error
					is the time index

For  $N$  measurements of the process given by (1) we can write

$$\mathbf{Y} = \begin{bmatrix} y(i) \\ \vdots \\ y(N) \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} \mathbf{x}^T(i) \\ \vdots \\ \mathbf{x}^T(N) \end{bmatrix} \quad (2)$$

The least squares estimate for the unknown parameters,  $\hat{\boldsymbol{\xi}}$ , is then

$$\hat{\boldsymbol{\xi}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y} \quad (3)$$

If  $\epsilon(i)$  are zero mean and independent with variance  $\sigma^2$ , then the least squares estimate is a Best Linear Unbiased Estimator with covariance of the estimates given by

$$E[(\hat{\boldsymbol{\xi}} - \boldsymbol{\xi})(\hat{\boldsymbol{\xi}} - \boldsymbol{\xi})^T] = (\mathbf{X}^T\mathbf{X})^{-1}\sigma^2 \quad (4)$$

and the value of  $\sigma^2$  can be estimated from the sum of squares of the residuals i.e.

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Regression analysis offers a number of advantages for the analysis of non-linear data<sup>2,3</sup>. These include efficient computation, ease of varying model structure, and ability to partition data into convenient sub-sets. On the other hand serious disadvantages include the need to have measurements of all variables, and degradation in the performance of equation error methods when measurement errors are present i.e. errors in the variables. These disadvantages can be minimised by pre-processing the data to remove measurement errors, both systematic and random, and to reconstruct missing records. Flight Path Reconstruction or Compatibility Checking<sup>4,5</sup> methods were designed to achieve this through use of a non-linear state estimation technique such as an Extended Kalman Filter. Work at ARL and at the University of Newcastle has led to the development of programs for Compatibility Checking using both Maximum Likelihood and Extended Kalman Filter estimation<sup>6</sup>, and has established that successful application can be achieved even with moderate quality flight data<sup>7,8</sup>.

With the use of Compatibility Checking as a pre-processor to produce a complete set of 'error free' flight trial records, the application of regression analysis as the final stage of the procedure to obtain the desired aerodynamic characteristics, becomes an attractive option. This document reports the results achieved using a longitudinal non-steady manoeuvre of the roller coaster type and illustrates the ability of this methodology to maximise the information which can be obtained from a relatively small amount of flight data. Comparison is also made with results obtained from doublet/pulse response flight trials data. Prior to presenting the results, the regression analysis program developed for this work is documented and some remarks made as to its practical application.

$$\sigma^2 = \frac{1}{N-p} (Y - X \hat{\xi})^T (Y - X \hat{\xi}) \quad (5)$$

The estimate given by (3) will be biased unless X is measured without error or, as noted above, the equation error  $\epsilon(i)$  is zero mean and white<sup>11</sup>.

## 2.2 Normal Distributions

If it is assumed that  $\epsilon(i)$  are normally distributed (i.e. Y normally distributed with X noise free) then the least squares estimator (3), is a minimum variance unbiased estimator, and is normally distributed with covariance given by (4). It follows that for each element, j, of  $\xi$  the distribution of  $(\hat{\xi}_j - \xi_j) / [\text{cov}(\xi_j)]^{1/2}$  is the student t distribution on (N-p) degrees of freedom, where  $\text{cov}(\xi_j)$  is the jth diagonal element of the covariance matrix (4). Hence a  $(1-\alpha)\%$  confidence interval can be constructed for  $\xi_j$ , namely

$$\hat{\xi}_j - t_\alpha [\text{cov}(\xi_j)]^{1/2} < \xi_j < \hat{\xi}_j + t_\alpha [\text{cov}(\xi_j)]^{1/2} \quad (6)$$

where  $t_\alpha$  is the upper  $\alpha$  percentile of the t distribution for (N-p) degrees of freedom.

Once a solution,  $\hat{\xi}$ , has been obtained, the imposition of constraints can be achieved readily<sup>12</sup>. For a constraint of the general form

$$L\hat{\xi}_1 = C \quad (7)$$

the constrained solution,  $\hat{\xi}_1$ , is given by

$$\hat{\xi}_1 = \hat{\xi} - (X^T X)^{-1} L^T (L (X^T X)^{-1} L^T)^{-1} (L\hat{\xi} - C) \quad (8)$$

where L is a matrix of dimension  $s \times p$ , s being the number of constraints and p the dimension of the parameter vector, C is a vector of dimension s, and X is given by (2). For the covariance matrix of the constrained parameters, (4) is replaced by

$$E[(\hat{\xi}_1 - \xi)(\hat{\xi}_1 - \xi)^T] = (I - ML)(X^T X)^{-1} \sigma^2 \quad (9)$$

where

$$M = (X^T X)^{-1} L^T (L(X^T X)^{-1} L^T)^{-1}$$

If the sum of squares of the errors is defined as

$$S(\hat{\xi}) = (Y - X\hat{\xi})^T (Y - X\hat{\xi}) \quad (10)$$

then the quantity

$$f = \frac{[S(\hat{\xi}_1) - S(\hat{\xi})] / s}{S(\hat{\xi}) / (N-p)} \quad (11)$$

has an F distribution with  $s$  degrees of freedom in the numerator and  $(N-p)$  degrees of freedom in the denominator. Thus the F distribution can be used to test the null hypothesis given by (7). For example, if the value of  $f$  calculated from (10) is greater than the  $F_\alpha(s, N-p)$  value then the null hypothesis is rejected at the  $(1-\alpha)\%$  risk level.

The imposition of constraints as in (7) can be used to test alternative parameter models by setting selected variables to zero. An automatic procedure for doing this is that of Backward Elimination<sup>9</sup> whereby the least significant terms are systematically eliminated until further eliminations become statistically unjustifiable according to a pre-selected level of significance. The final result is a best Regression solution which retains only those terms found to be of significance at the desired level. The level of significance is a measure of the risk of error, i.e. the probability of retaining a term when it should be eliminated. An alternative procedure, Stepwise Regression, is also often used to build up a model by adding significant terms from a pre-specified set of possible items. Further details of both procedures can be found in References 9 or 10.

### 2.3 Computer Program

Equations (3) to (11) above form the basis of an interactive program, LSPROG.REG.F, written in Fortran and operational on the ARL ELXSI computer. The input to LSPROG is a file named LS.DAT, which simply consists of the problem dimensions,  $N$  and  $p$ , on the first line followed by a listing of values of the independent variables ( $p$  in number) and the dependent variable, arranged in columns. The output file is named LSOUT. The problem dimensions, which are used to set the size of various arrays, also need to be specified in a parameter statement at the beginning of the program before compilation. The program contains the following features and options:

- (a) The basic solution (3), is achieved using a robust algorithm based on the Householder transformation, which has advantages when the equations are ill-conditioned.<sup>12</sup> A direct solution using matrix inversion is also offered as an option. The square of the fit error (residuals) from (10), and the covariance of the estimates (4), are also calculated at this stage.
- (b) The Multiple Correlation Coefficient,  $RSQ$ , and the correlation matrix are also calculated and output as useful guides to the user.  $RSQ$  is defined as

$$RSQ = \frac{\sum_{i=1}^N (\hat{y}(i) - \bar{y})^2}{\sum_{i=1}^N (y(i) - \bar{y})^2} \quad (12)$$

where  $y$  and  $\hat{y}$  are the measured and calculated dependent variables respectively and  $\bar{y}$  is the mean value defined as

$$\bar{y} = \frac{1}{N} \sum_{i=1}^N y(i) \quad (13)$$

$RSQ$ , expressed as a percentage, is a criterion of the goodness of fit, a value close to 100% indicating that the variation in the data has been adequately accounted for.

Finally, the  $p \times p$  correlation matrix is obtained by non-dimensionalising the elements of the covariance matrix (4). The  $i, j$ , element is given by

$$\rho_{ij} = \frac{E[(\hat{\xi}_i - \xi_i)(\hat{\xi}_j - \xi_j)]}{[E(\hat{\xi}_i - \xi_i)^2 \cdot E(\hat{\xi}_j - \xi_j)^2]^{1/2}} \quad (14)$$

By definition, diagonal elements of the correlation matrix have the value 1 (perfect correlation) while off diagonal elements are between zero and one. Values close to one, indicating high correlation between parameters, can lead to misleading results and should be avoided.

- (c) The user is asked interactively whether the t-test confidence intervals given by (6) are required to be calculated. If so, the values of  $\alpha$  and  $t_\alpha$ , from standard statistical tables, need to be input.
- (d) If the option to constrain parameters is chosen by the user, the constraint equations (7) must be specified interactively. This can be done either by listing the coefficients of each constraint equation, or more directly for simple constraints, by listing the parameter number followed by its constrained value, one parameter per line.

The constrained solution and variances are then obtained from (8) and (9) and the fit error from (10). The multiple correlation coefficient and the correlation matrix are also calculated as in (b).

- (e) At this stage the null hypothesis  $L\xi_1 = C$  can be tested using the F-test with the f value given by (11). The program will request the user to input the F-value for  $s$ ,  $N-p$  degrees of freedom at the appropriate risk level if this option is chosen.
- (f) Finally, the Backward Elimination procedure as described in section 2.2 can be implemented with a level of significance of 5% or 1% as specified by the user. Prior to starting this, some chosen parameters can be eliminated if desired and will no longer be considered in the subsequent regression analysis. This can be useful if previous processing has shown that a particular parameter is unimportant or is highly correlated with another parameter.

It is worth noting that the *Backward Elimination* procedure does not pick out highly correlated parameters in general. Hence deceptive results can be obtained if the user relies solely on this as an automatic means of obtaining an adequate model structure. In practice it has been found that better results can be obtained by intelligent use of the interactive process in (d) and (e) and careful examination of the fit criteria provided.

### 3. APPLICATION TO FLIGHT DATA

#### 3.1 Pre-processing

The flight data chosen for analysis was a roller-coaster manoeuvre of a delta wing aircraft at a nominal Mach number of 0.65 and altitude of 33,000 ft. (Actual Mach number varied over the range 0.60 to 0.70) Approximately 58 seconds of record, sampled at 60 per second, for a total of 3500 data points per variable, was used. Interpreting the measured accelerations  $a_x$ ,  $a_z$ , and pitch rate,  $q$ , as inputs, and the airspeed  $V$ , angle of attack  $\alpha$ , pitch altitude  $\theta$ , and altitude  $h$ , as outputs, a compatibility checking procedure was implemented, whereby the redundant information available in these measurements was used to identify systematic instrument bias and scale factor errors and measurement lags, and to produce smoothed estimates of the outputs  $V, \alpha$  for use in subsequent analysis. Full details are given in References 7 and 8 for the manoeuvre under consideration. The bias and scale factor errors determined in this way were then used to correct the measurements.

Some further processing was required in order to obtain records of the angle of attack derivative,  $\dot{\alpha}$ , and pitch rate derivative,  $\dot{q}$ . The latter is directly related to the pitching moment. Both were derived by numerical differentiation of the respective  $q$  and  $\alpha$  records using a recursive least squares algorithm as described in Reference 7.

A complete set of records required for the current analysis is shown in Figure 1. The Mach number change has been obtained from the airspeed record using a value of 299.3 m/s for the speed of sound. Some of the other variables have been non-dimensionalised in accordance with the equation formulations described in section 3.2.

Thus

$$\begin{aligned}
 \text{non-dimensional } q &= q\bar{c}/2V \\
 \text{non-dimensional } \dot{\alpha} &= \dot{\alpha}\bar{c}/2V \\
 \text{pitching moment coefficient } C_m &= (I_y/\frac{1}{2} \rho V^2 S \bar{c}) \dot{q} \\
 \text{Z-force coefficient } C_z &= (m/\frac{1}{2} \rho V^2 S) . a_z
 \end{aligned}$$

where  $\bar{c}$  is the mean aerodynamic chord,  $I_y$  is the moment of inertia in pitch,  $m$  is the aircraft mass,  $S$  is the reference wing area, and  $\rho$  is the air density. Apart from a small amount of random noise on the independent variables (i.e. Z-force and pitching moment) and the  $q$  record, the time histories in Figure 1 can be regarded as essentially error free. The one possible exception is the elevator control angle,  $\delta$ , which was not involved in the compatibility checking process. This emphasises the need for accurate measurements of control inputs, as errors in  $\delta$  can lead to biases in the estimated parameters.<sup>11</sup>

### 3.2 Data and Analysis Procedure

Before proceeding with the analysis it is worth examining the basic features of the data shown in Figure 1. The angle of attack range goes from approximately -4 degrees up to 12 degrees maximum, while the trim values of  $\alpha$  and  $\delta$  are about 5.2 degrees and -5.1 degrees respectively. The distribution of data over the  $\alpha$  range is indicated in Figure 2, which shows the number of samples within two degree  $\alpha$  intervals starting at -4 degrees. Apart from the expected peak around the trim value of  $\alpha$ , the data are reasonably well spread over the whole  $\alpha$  range. Figures 3 to 8 summarise the distribution of the independent and dependent variables as functions of  $\alpha$ . In general there is a reasonable spread at any particular  $\alpha$  for all the variables, although, as expected, there is a mean trend with  $\alpha$  for  $\delta$  and  $C_z$ . Also, a plot of  $q$  against  $\dot{\alpha}$  (Figure 9) indicates a correlation between these two variables. This is brought out more clearly if a limited  $\alpha$  range is examined. For example, Figure 10, which contains 600 points of data for  $\alpha$  between -4 and -1.15 degrees, clearly demonstrates this strong correlation.

An initial attempt was made to analyse the complete data set with non-linear representations for force and moment, including various powers of  $\alpha$  and  $\delta$  as well as cross terms. It soon became apparent that sensible results could not be obtained

without considerable effort being expended on developing a complex model structure, with accompanying difficulties in interpreting the results. Consequently, a simpler, and physically more meaningful, approach was adopted, involving the partitioning of data into angle of attack subsets, containing typically 300 points in each subset. For example, starting at say -4 degrees,  $\alpha$  was gradually incremented until a total of at least 300 data points was achieved. Thus between -4 and -2.2 degrees there were 307 points, between -2.2 and -1.5 there were 312 points and so on. The  $\alpha$  range in each subset was typically 1 or 2 degrees with a maximum of 3 degrees. Some subsets of 600 points, with an  $\alpha$  range of 2 or 3 degrees, were also used. Subsets were also varied by changing the starting point. For each subset, mean values of  $\alpha$ ,  $\delta$  and other variables were calculated. For example, Figures 11 and 12 indicate the variation of  $\bar{\delta}$  and  $\Delta\bar{M}$  with  $\bar{\alpha}$ , where  $\Delta\bar{M}$  is the change of Mach number from a reference value of 0.68, while the bars indicate mean values. Figure 11 reflects the mean trend of  $\delta$  with  $\alpha$  as noted in Figure 5.

Force and moment equations were formulated for each subset, with  $\alpha$  and  $\delta$  interpreted as perturbations about their respective mean values,  $\bar{\alpha}$  and  $\bar{\delta}$ , for that particular subset. The pitching moment equation becomes

$$C_m = C_{m_0} + C_{m_M} \Delta M + C_{m_\alpha} \Delta \alpha + C_{m_\alpha^2} \Delta \alpha^2 + C_{m_q} \cdot q \quad (16)$$

$$+ C_{m_\delta} \Delta \delta + C_{m_\delta^2} \Delta \delta^2 + C_{m_{\alpha\delta}} \Delta \alpha \cdot \Delta \delta$$

where  $\Delta \alpha$  is  $(\alpha - \bar{\alpha})$  and  $\Delta \delta$  is  $(\delta - \bar{\delta})$ , while the coefficients in (16) represent the non-dimensional aerodynamic derivatives, including non-linear and cross terms. In the same way the Z-force equation becomes

$$C_z = C_{z_0} + C_{z_M} \Delta M + C_{z_\alpha} \Delta \alpha + C_{z_\alpha^2} \Delta \alpha^2 + C_{z_q} \cdot q \quad (17)$$

$$+ C_{z_\delta} \Delta \delta + C_{z_\delta^2} \Delta \delta^2 + C_{z_{\alpha\delta}} \Delta \alpha \cdot \Delta \delta$$

The Least Square program described in section 2 was used to identify the coefficients in (16) and (17). The multiple correlation coefficient (12) and correlation matrix (13) provided extremely useful quantitative measures, while the ability to eliminate parameters interactively provided the flexibility to achieve a good final result. The correlation matrix indicated clearly that  $q$  and  $\dot{\alpha}$  derivatives could not be separated, and consequently only the  $q$  term is included in (16) and

(17). However,  $Cz_q$  and  $Cm_q$  should be interpreted as damping due to their combined effects. In general, because of the limited  $\alpha$  range in the partitioned data, it was found that the non-linear and cross terms in (16) and (17) could usually be eliminated without degrading the fit error or affecting the other derivative values. This makes the interpretation of the results, in the next section, particularly straightforward.

### 3.3 Results

The pitching moment results are summarised in figures 13 to 17, while the Z-force results are given in Figures 18 to 22. In all cases results are presented as functions of  $\alpha$ . Where appropriate, comparisons are made with derivatives extracted in an earlier stability and control investigation described in Reference 13. The latter involved the analysis of perturbations about a reference trim state in response to doublet/pulse type control inputs, and provide derivative values which represent an average over the  $\alpha$  range covered during the perturbations. The particular average values presented here are for a Mach number of 0.65 with a trim  $\alpha$  value of about 5 degrees.

#### 3.3.1 Pitching Moment

The pitch stiffness derivative,  $Cm_\alpha$ , in Figure 13 has a minimum absolute value at about 2 degrees and increases slightly for lower or higher values of  $\alpha$ . It increases more rapidly at about 5 or 6 degrees reaching a maximum at around 9 degrees and then appears to drop in value. The difference from the average value from Reference 13 can be ascribed to differences in centre of gravity location. On the other hand, the elevator pitch effectiveness,  $Cm_\delta$  in Figure 14 is almost constant with  $\alpha$ , decreasing somewhat in absolute value only above 8 degrees. The agreement with the previous average value is very good.

The pitch damping,  $Cm_q$ , in Figure 15 shows a strong variation with  $\alpha$ , indicating a sharp decline in damping above about 4 degrees, associated perhaps with the leading edge vortex development typical of delta wings. This feature could not be identified in the previous small perturbation analysis, whose average value falls approximately mid way between the maximum and minimum values shown in Figure 15.

Finally, extracted values for the Mach number derivative,  $Cm_M$ , are presented in Figure 16. In general the derivative values are small and imply little compressibility effect, as may be expected at this Mach number. The apparent trend with  $\alpha$  broadly reflects the mean Mach number variation shown in Figure 12, but no conclusions regarding its significance can be drawn at this stage.

In Figure 17 the static pitching moment coefficient,  $Cm$ , is presented as a function of  $\alpha$ , for zero elevator angle,  $\delta$ . Under these conditions, with  $\alpha$  equal to  $\bar{\alpha}$ , and  $\Delta M$  and pitch rate set to zero, (16) reduces to

$$Cm = Cm_0 + Cm_{\delta} \cdot \bar{\delta} \quad (18)$$

The variation of  $Cm$  with  $\alpha$  is very well defined in Figure 17, and the slope of this curve provides an alternative estimate of  $Cm_{\alpha}$ . The comparison with  $Cm_{\alpha}$  from Figure 13 can be shown to produce excellent agreement and provides a check on the overall accuracy of the least square results.

### 3.3.2 Z-Force

The lift curve slope,  $Cz_{\alpha}$ , is shown in Figure 18 as a function of  $\alpha$ . The value of  $Cz_{\alpha}$  is seen to be roughly constant up to about 6 degrees, agreeing well with the average value from Reference 13, but appears to increase significantly above 6 degrees, due to the leading edge vortex development. The elevator derivative,  $Cz_{\delta}$ , in Figure 19, shows a slight decreasing trend in absolute value with  $\alpha$ . The relatively large amount of scatter is an indication of the difficulty in extracting this derivative. This is in agreement with the finding of Reference 13, which concluded that, for those flight measurements, insufficient information on this derivative existed to justify any substantial shift from its a priori value. This a priori value, based on wind tunnel tests, is shown on Figure 19, and indicates a significant overestimation compared with the present flight results.

The pitch rate derivative,  $Cz_q$ , in Figure 20 appears to be reasonably well defined and, while relatively constant up to about 4 degrees, decreases steadily for higher  $\alpha$ . No information on  $Cz_q$  was obtained in the earlier investigation. Finally, for completeness, results for the Mach number derivative  $Cz_M$  are given in Figure 21. The comments on  $Cm_M$ , in the previous section, apply here also.

The static Z-force coefficient is presented as a function of  $\alpha$  in Figure 22. This is obtained, in a manner analogous to the  $C_m$  vs.  $\alpha$  curve, from the equation

$$C_z = C_{z_0} + C_{z_\delta} \cdot \bar{\delta} \quad (19)$$

and shows the expected increase in slope above about 6 degrees, confirming the results in Figure 18 and providing a check on overall consistency and accuracy.

#### 4. CONCLUSIONS

This report has described the use of regression analysis procedures to obtain aerodynamic information from flight data in non-linear regimes where the aerodynamic model is uncertain. The use of moderately accurate instrumentation presents a problem with this approach due to the unfavourable effects of measurement errors on the results. In order to overcome this, flight measurements were pre-processed using compatibility checking procedures to provide a complete set of error free records.

A computer program based on the least squares regression methodology has been documented and experience in its use has been critically discussed. In particular, the ability to constrain or eliminate parameters based on quantitative measures, such as the multiple correlation coefficient and the correlation matrix, has provided a flexible means for developing an adequate model structure. A feature of the least squares approach is the ability to partition data into angle of attack subsets, thereby providing physically meaningful results which can be easily interpreted. This approach need not be limited to angle of attack partitioning, but, given sufficient data samples, alternative or additional subsets, such as control angle, pitch rate etc. could be used.

Results for force and pitching moment characteristics have been presented, based on the analysis of approximately 60 seconds of a roller-coaster type of manoeuvre of a delta wing aircraft at a nominal Mach number of 0.65. A complete set of aerodynamic derivatives has been identified, and non-linear behaviour highlighted. While many of the features can be associated with the development of the leading edge vortex typical of delta wing aircraft, the sharp decrease in pitch

damping at about 4 degrees angle-of-attack has not been noted previously. The present results are consistent with earlier flight results and this, together with their internal consistency and relatively small amount of scatter, provides a considerable degree of confidence in their accuracy.

The use of regression analysis as illustrated here, together with pre-processing via compatibility checking of flight measurements, has been shown to be a viable and effective approach to obtaining aerodynamic characteristics in non-linear flight regimes, even with instrumentation of moderate quality. It has considerable potential for application to highly non-linear behaviour, such as high angle-of-attack and spin, which have proved difficult to analyse in the past.

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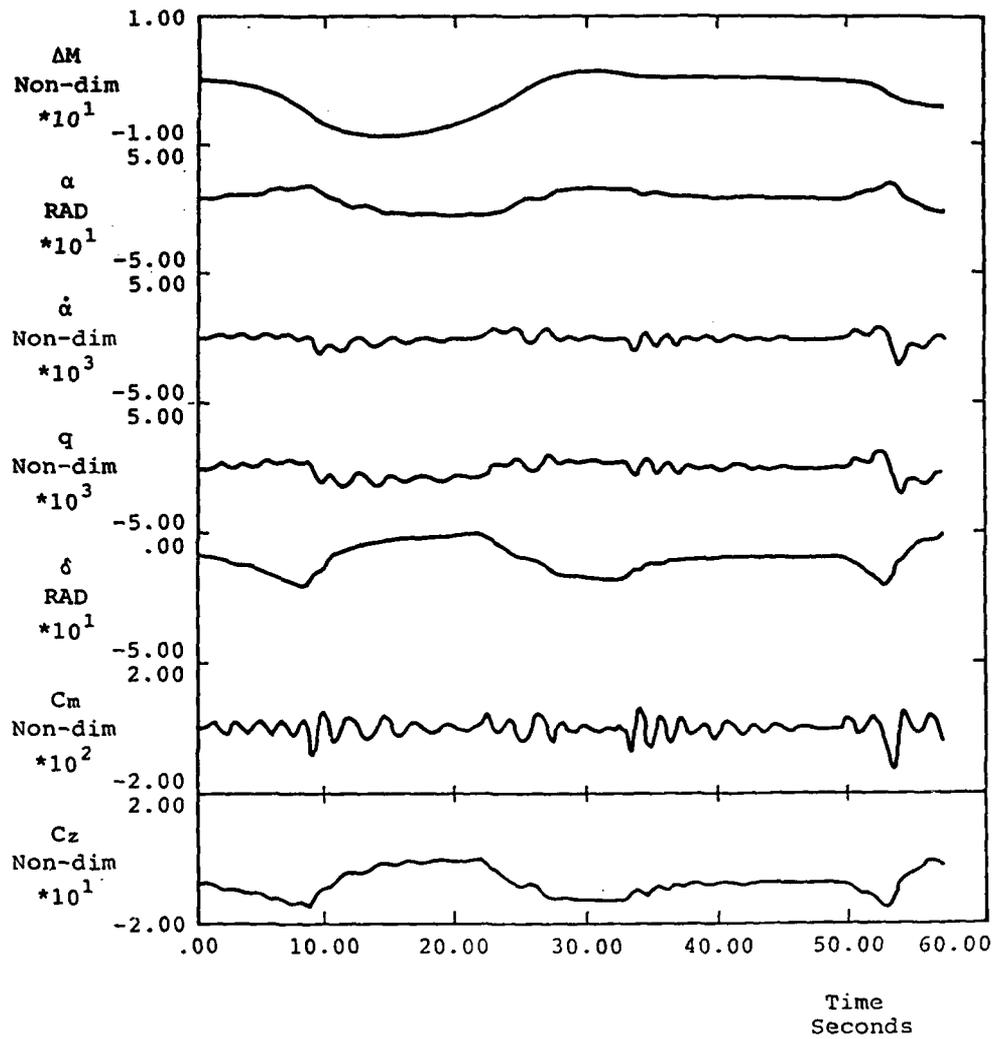


FIG. 1. FLIGHT DATA FOR ROLLER-COASTER MANOEUVRE,  $M=0.65$ , 33000 ft

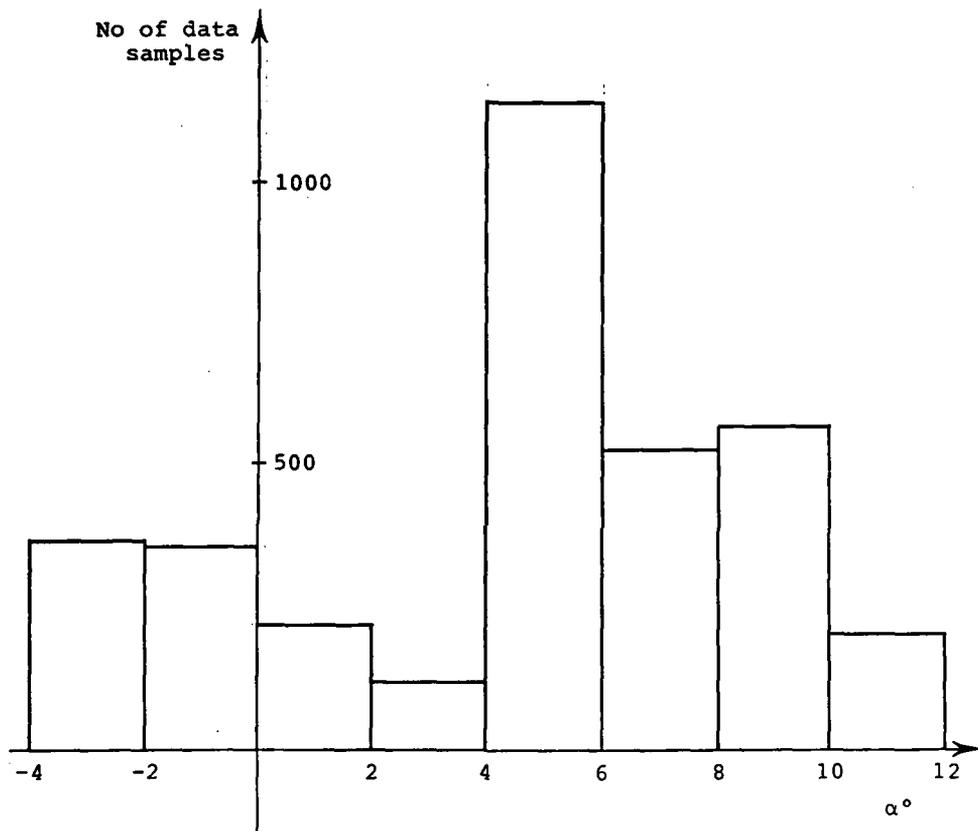


FIG. 2. DISTRIBUTION OF DATA OVER ANGLE OF ATTACK RANGE

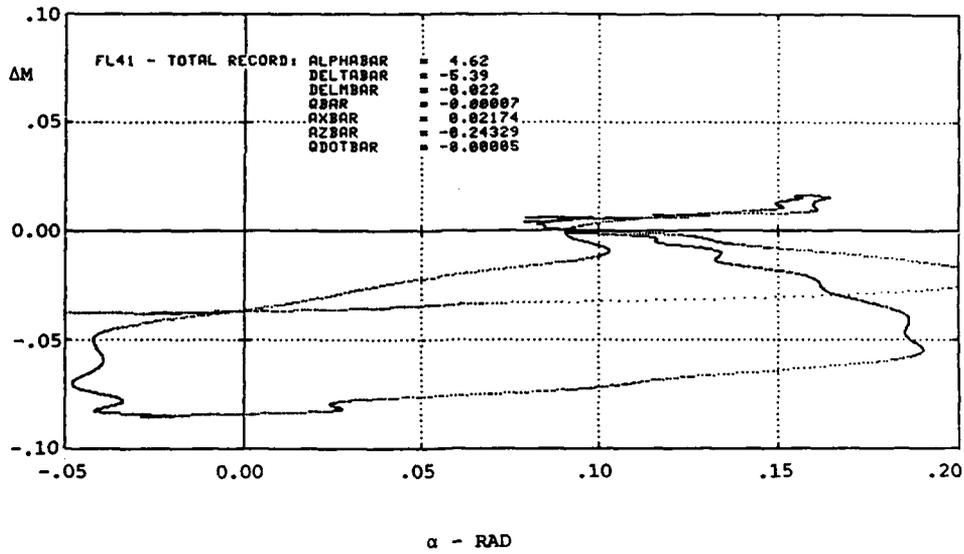


FIG. 3. DISTRIBUTION OF MACH NUMBER AS A FUNCTION OF ANGLE OF ATTACK

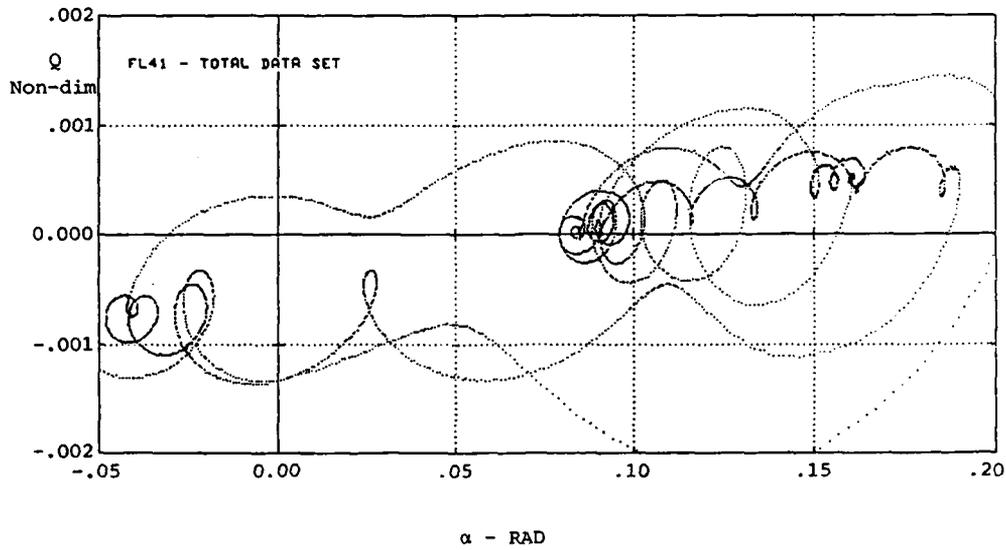


FIG. 4. DISTRIBUTION OF PITCH RATE AS A FUNCTION OF ANGLE OF ATTACK

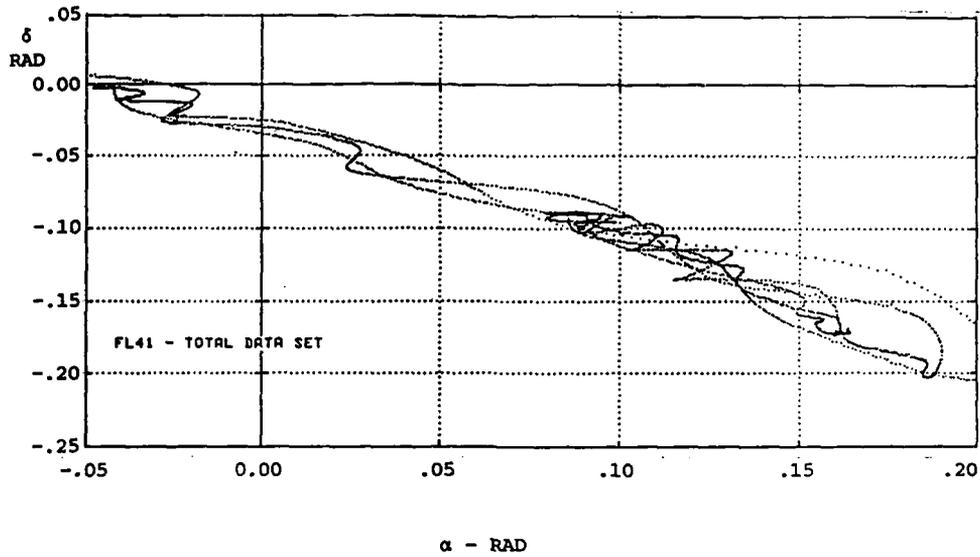


FIG. 5. DISTRIBUTION OF ELEVATOR ANGLE AS A FUNCTION OF ANGLE OF ATTACK

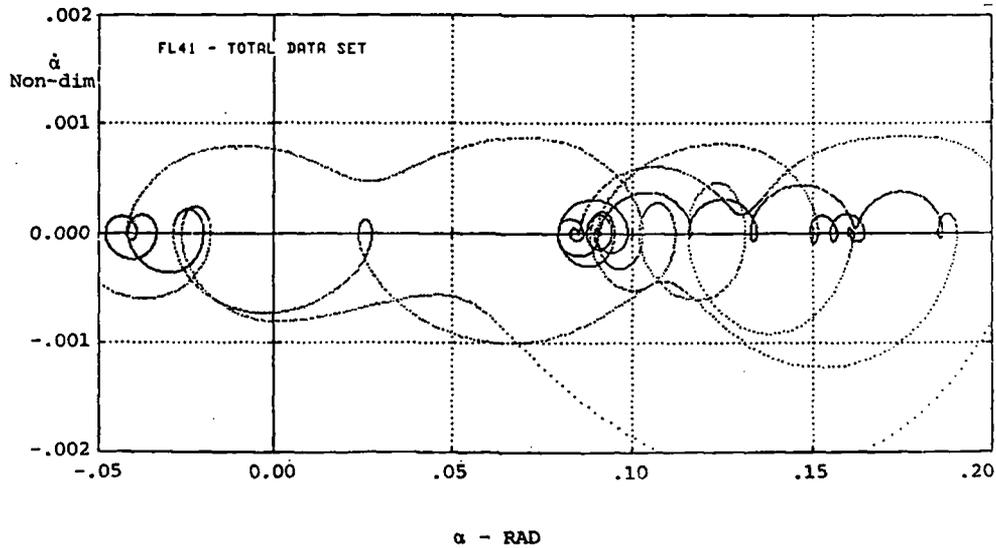


FIG. 6. DISTRIBUTION OF ANGULAR RATE AS A FUNCTION OF ANGLE OF ATTACK

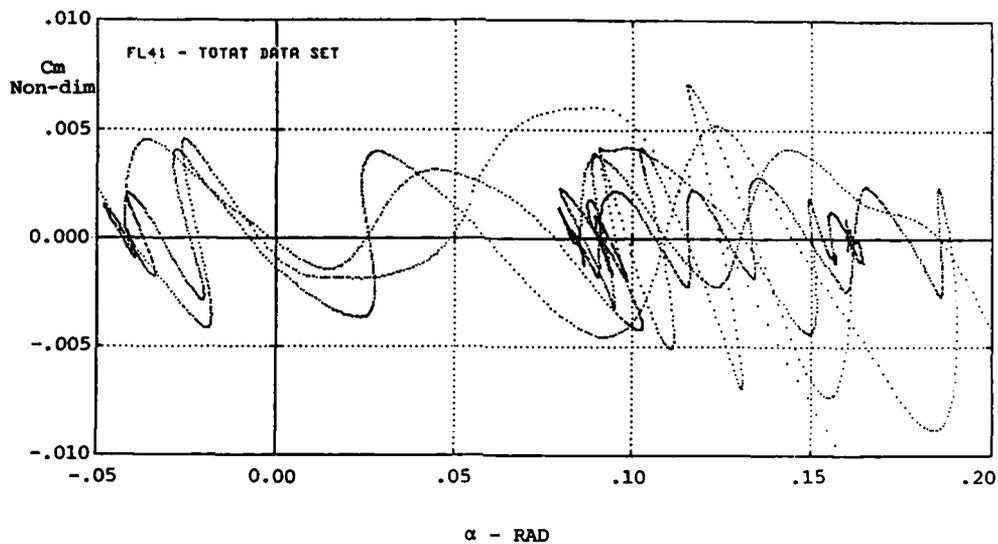


FIG. 7. DISTRIBUTION OF PITCHING MOMENT AS A FUNCTION OF ANGLE OF ATTACK

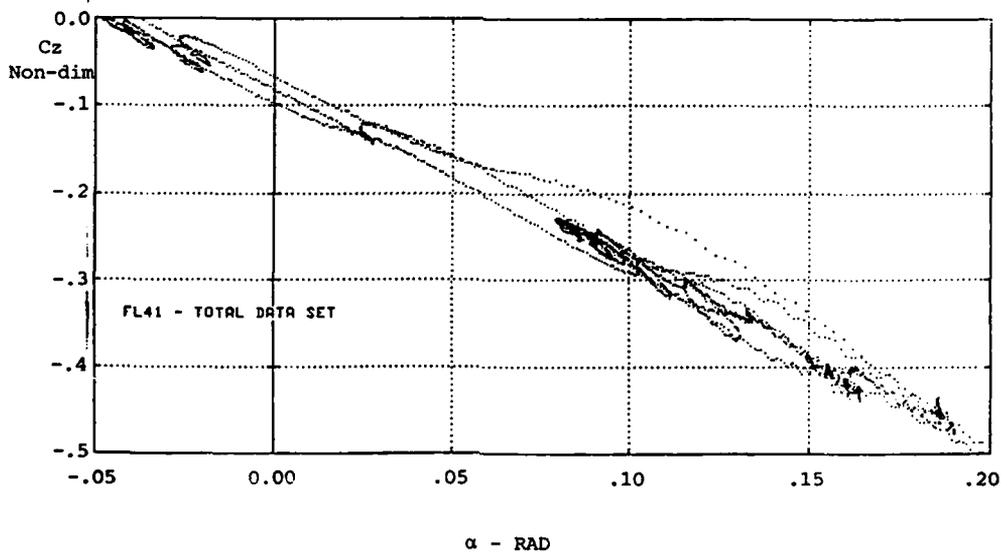


FIG. 8. DISTRIBUTION OF Z-FORCE AS A FUNCTION OF ANGLE OF ATTACK

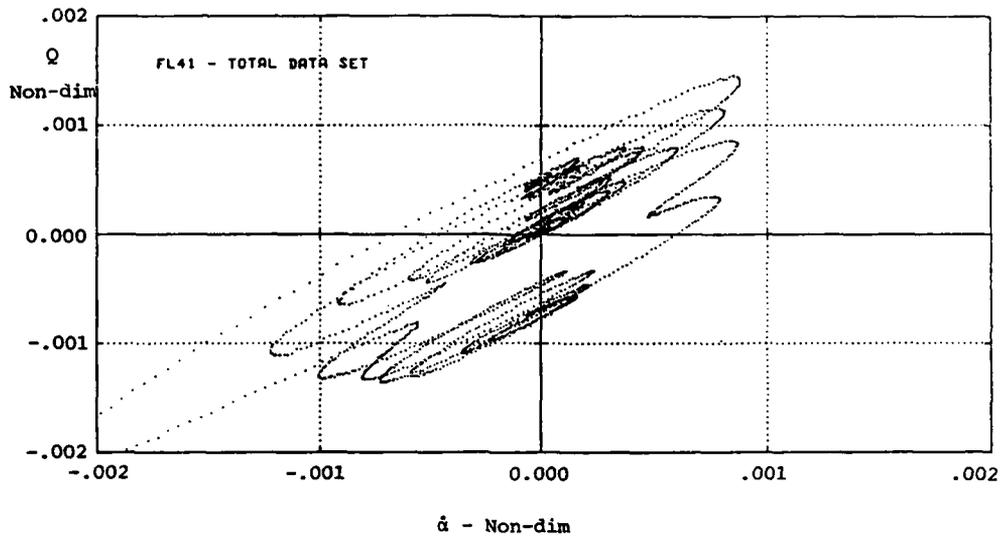


FIG. 9. DISTRIBUTION OF PITCH RATE AS A FUNCTION OF ANGULAR RATE

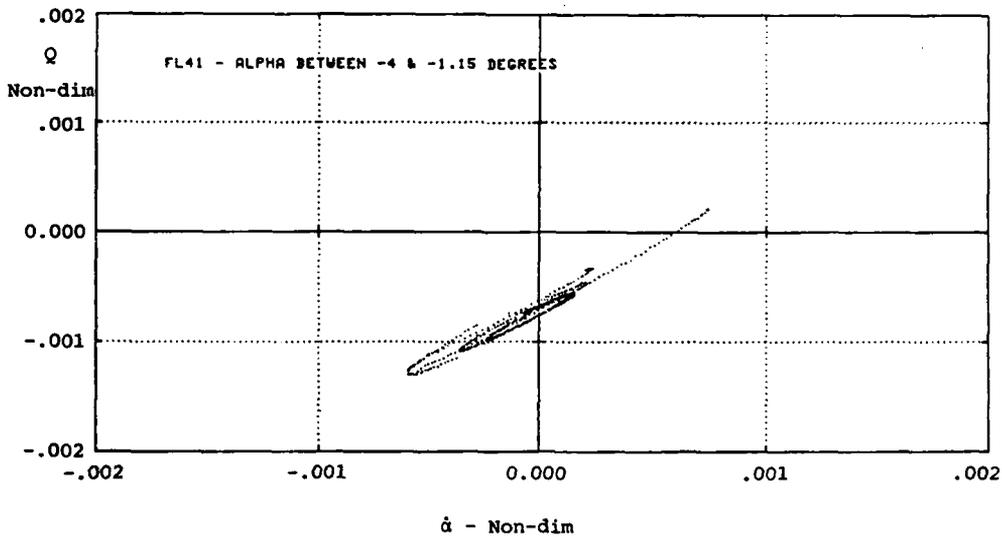


FIG. 10. PITCH RATE AS ANGULAR RATE FOR LIMITED ANGLE OF ATTACK RANGE

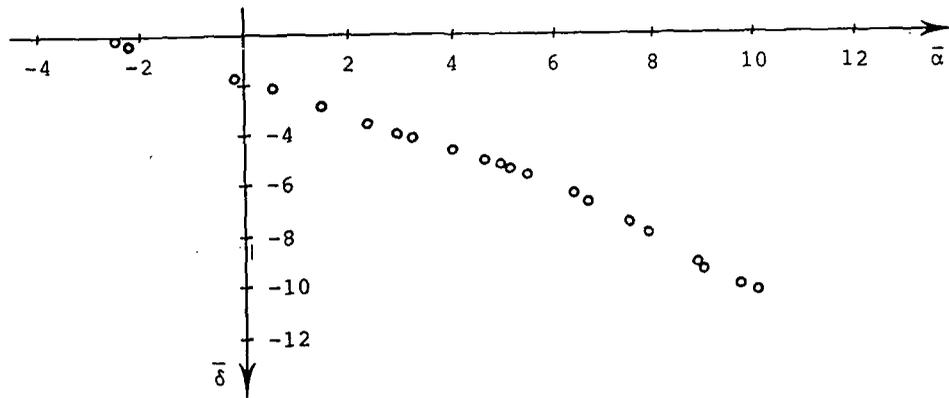


FIG. 11. VARIATION OF ELEVATOR ANGLE WITH ANGLE OF ATTACK (MEAN VALUES)

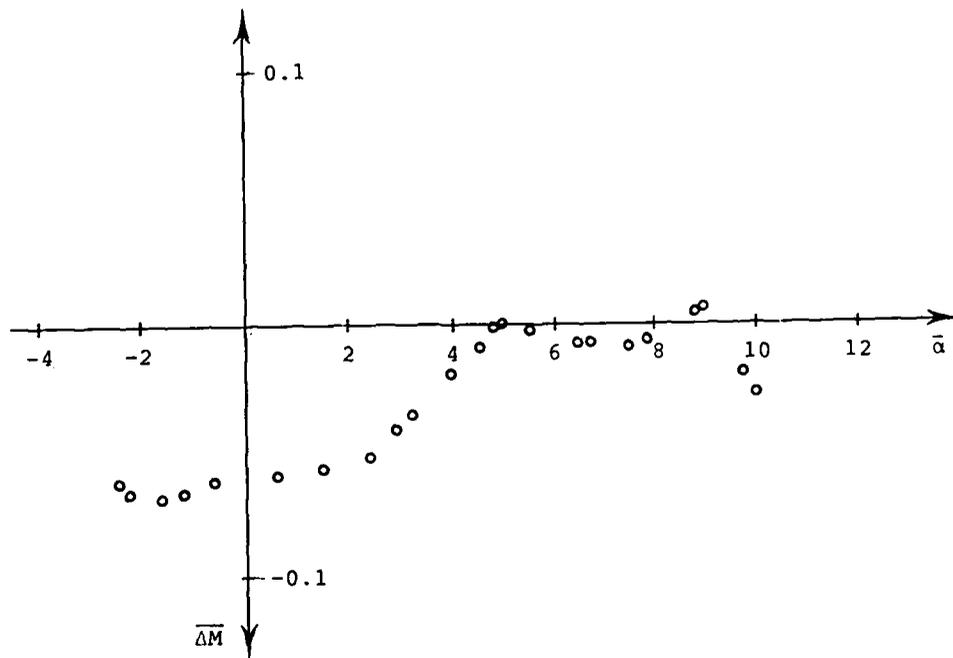


FIG. 12. VARIATION OF MACH NUMBER WITH ANGLE OF ATTACK (MEAN VALUES)

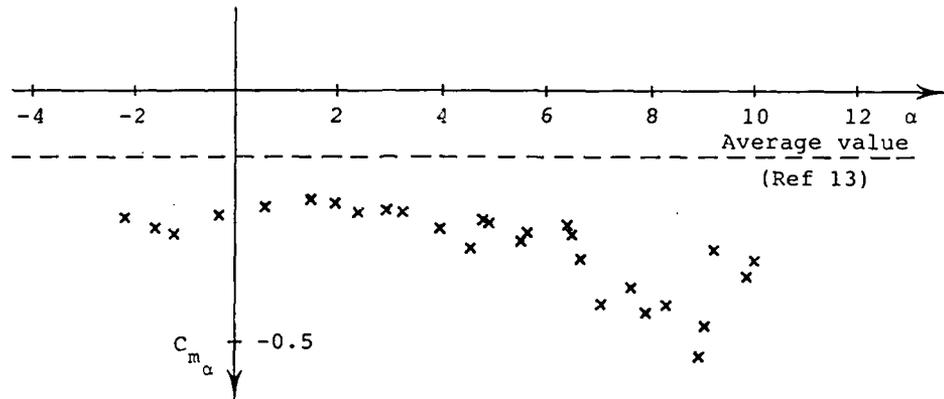


FIG. 13. PITCH STIFFNESS DERIVATIVE VARIATION WITH ANGLE OF ATTACK

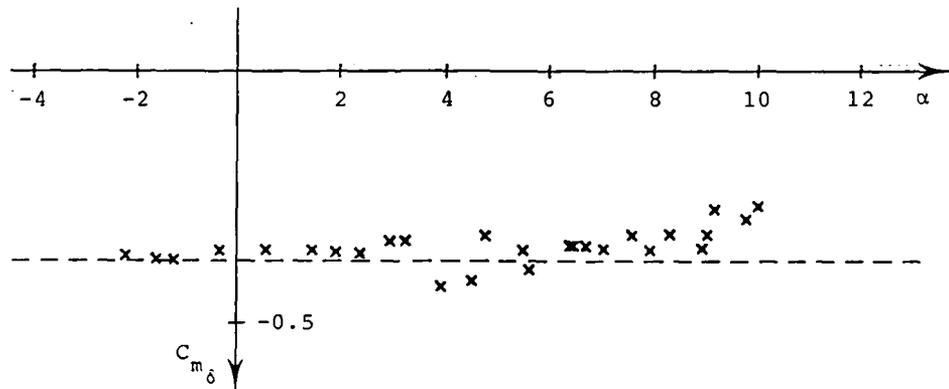


FIG. 14. ELEVATOR PITCH DERIVATIVE VARIATION WITH ANGLE OF ATTACK

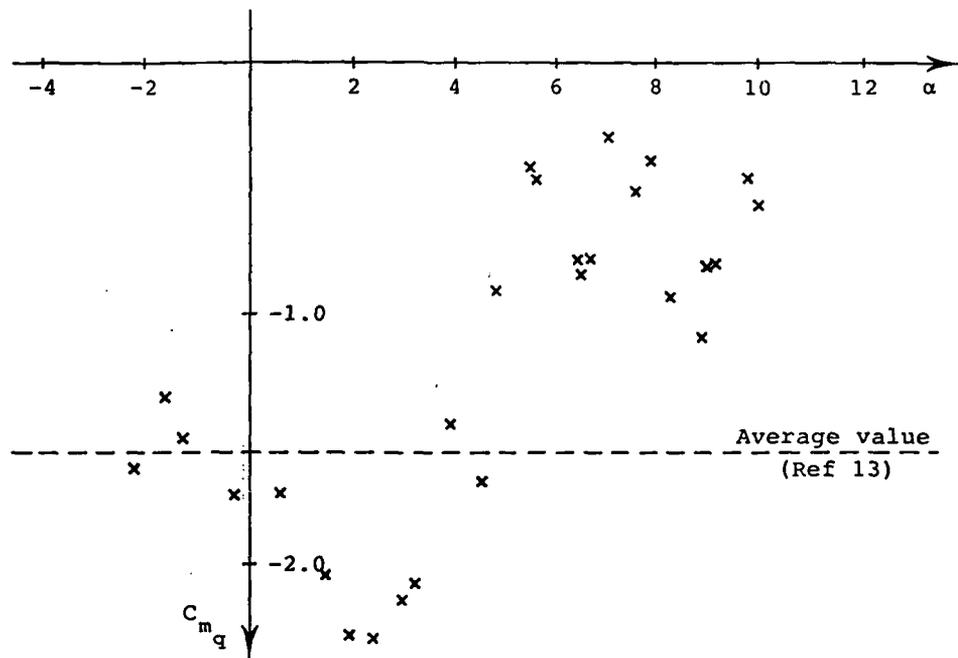


FIG. 15. PITCH DAMPING DERIVATIVE VARIATION WITH ANGLE OF ATTACK

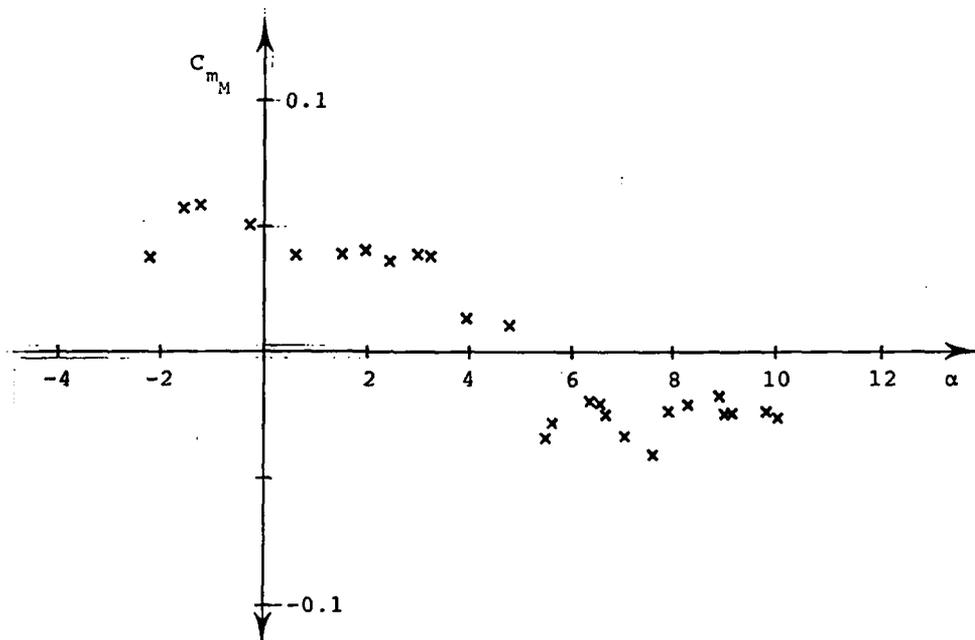


FIG. 16. PITCH MACH NUMBER DERIVATIVE VARIATION WITH ANGLE OF ATTACK

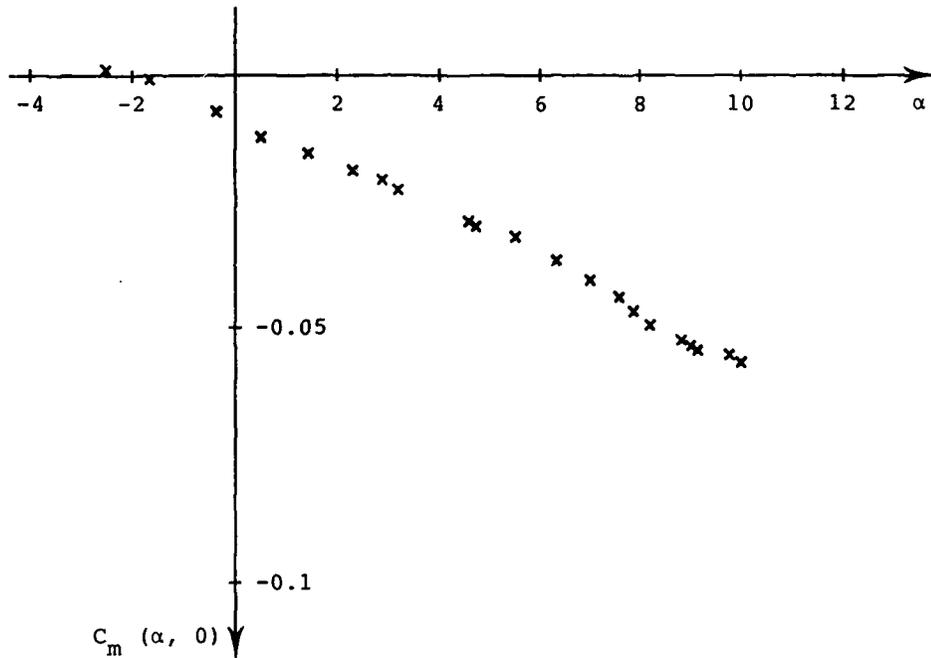


FIG. 17. PITCHING MOMENT VARIATION WITH ANGLE OF ATTACK

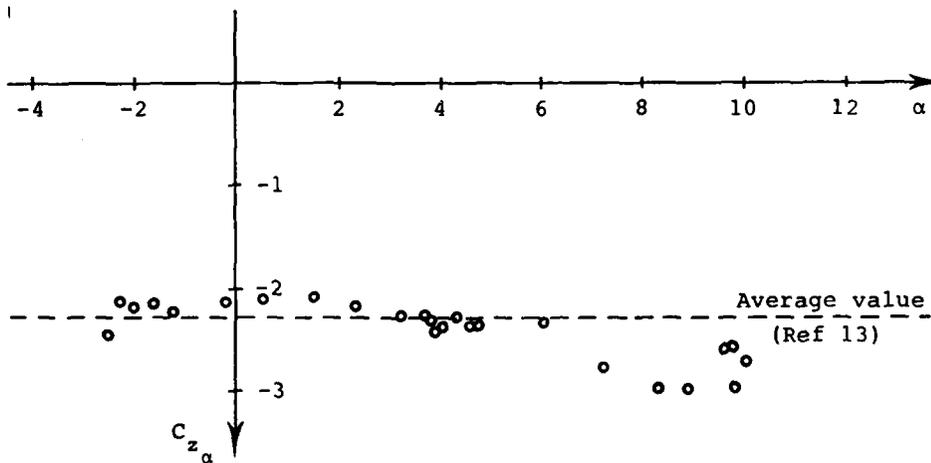


FIG. 18. LIFT CURVE SLOPE VARIATION WITH ANGLE OF ATTACK

53/5.13

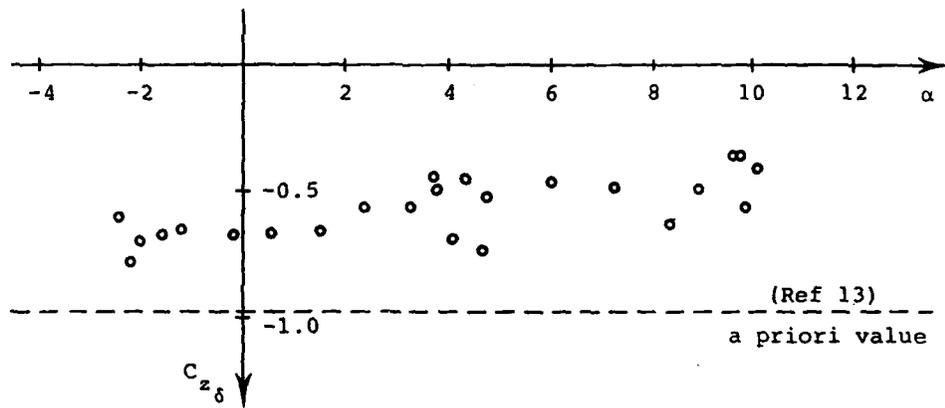


FIG. 19. ELEVATOR FORCE DERIVATIVE VARIATION WITH ANGLE OF ATTACK

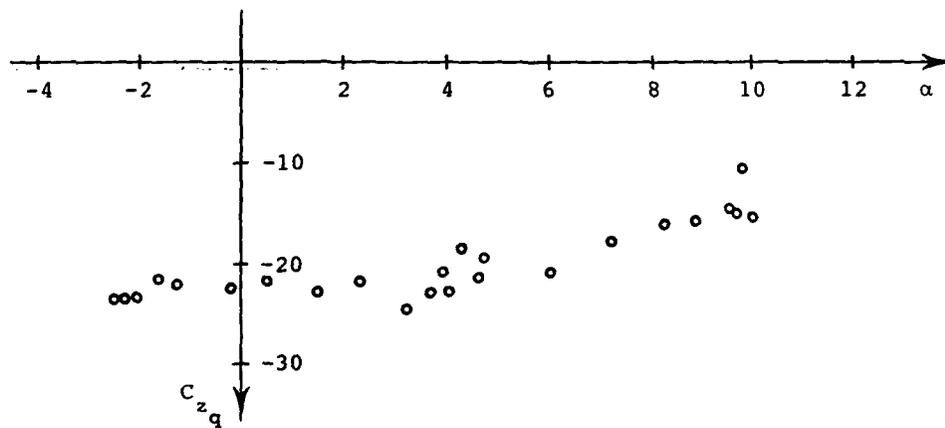


FIG. 20. FORCE DAMPING DERIVATIVE VARIATION WITH ANGLE OF ATTACK

5/1/60

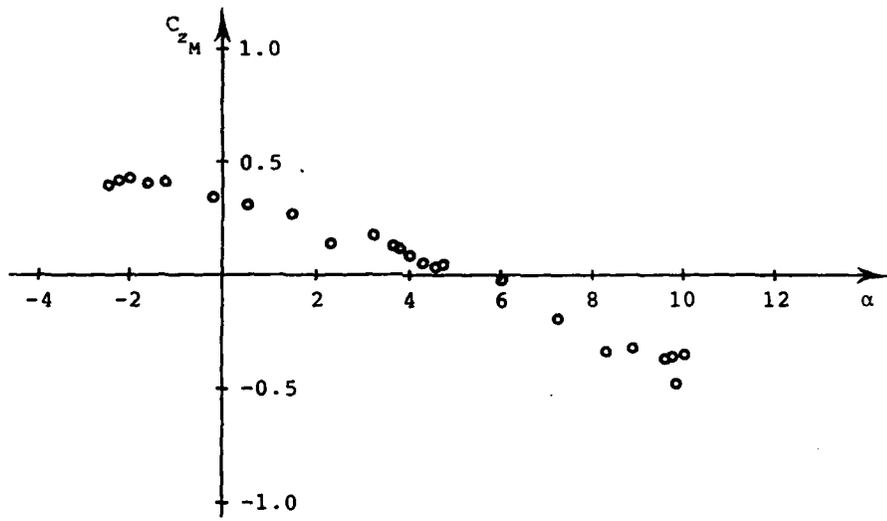


FIG. 21. FORCE MACH NUMBER DERIVATIVE VARIATION WITH ANGLE OF ATTACK

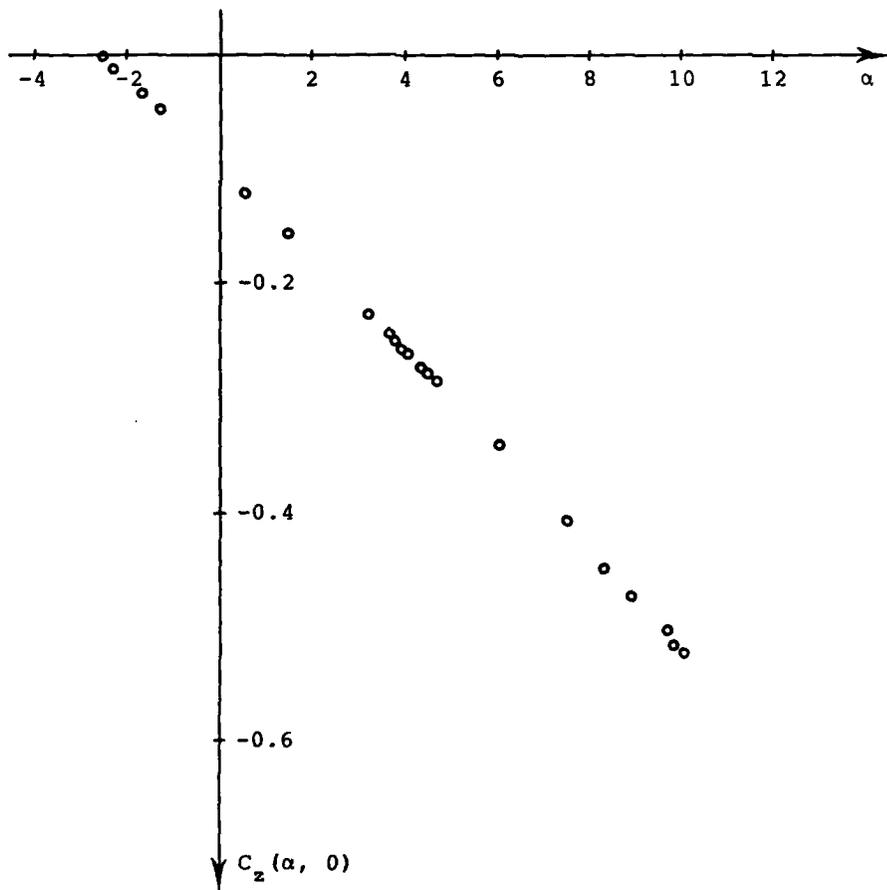


FIG. 22. Z-FORCE VARIATION WITH ANGLE OF ATTACK

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16. ABSTRACT A least squares regression analysis program has been documented and its advantages and shortcomings when used for analysing flight data have been summarised. It has been shown that the shortcomings can be largely overcome by pre-processing flight measurements via compatibility checking. A particular advantage of the least squares approach is the ability to partition data into angle of attack subsets. Application to flight data from a delta wing aircraft at M=0.65 has been successful in extracting non-linear features, including a sharp drop in pitch damping at around 4 degrees angle of attack, possibly associated with the development of the leading edge vortex. Comparison with previous results, internal consistency, and small scatter all confirm the			

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16. ABSTRACT (CONT.)

effectiveness of this approach even with moderate quality instrumentation. The methodology described has considerable potential for application to highly non-linear flight regimes.

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