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DETERMINING THE COMPARATIVE EFFICIENCY OF SCHOOLS THROUGH DATA ENVELOPMENT ANALYSIS

by

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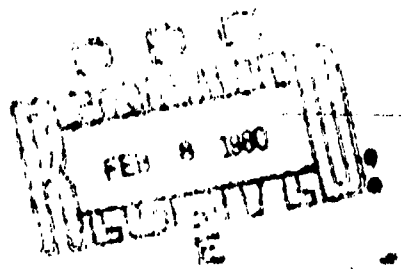

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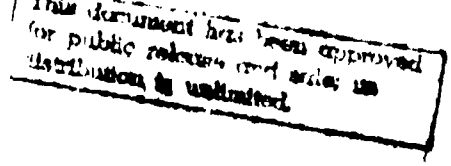

ABSTRACT

Conventional methods for comparing the relative productivity of schools employ least squares regression to find expected achievement of schools with the same input characteristics. The result is that one typically contrasts the relative effects of "predictor" variables on achievement rather than comparing school units with respect to their input/output efficiency. A newly developed input/output method for comparing the efficiency of decision making units is presented and is applied to elementary schools in an urban school district. The method is found to identify efficient and inefficient schools and provides management information relative to input and output measures.

ACKNOWLEDGMENTS

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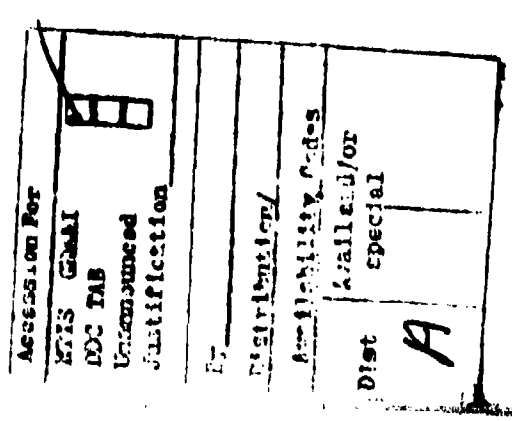

BACKGROUND

Methods for evaluating the relative productivity of decision making units in the public sector have lagged behind similar applications where production functions were more directly obtainable. Charnes and Cooper in [4] recently reviewed relevant development in economic theory from the standpoint of managerial economics. They also described the methodological developments undertaken with E. Rhodes [5,6] to measure the efficiency of "decision making units" with special reference to not-for-profit enterprise and government agencies. This resulted in a technique that they call Data Envelopment Analysis for measuring and distinguishing different kinds of efficiencies such as "program efficiency" and "managerial efficiency." The utility of the theory has been demonstrated in their secondary analysis of Program Follow-Through evaluation data [5,11]--an important federally-funded intervention aimed at improved education for disadvantaged children.

In the works cited, Charnes, Cooper and Rhodes succeeded in quantifying the relative efficiency of decision making units (DMU's) within a set of like units and, further, conceptualized a method for comparing the relative efficiency of two sets of units classified on some a priori basis [5]. Only a limited set of variables were selected for illustrative purposes, the objective being to show how one might compare two sets of schools operating under different programs.

Since the theory and application of Data Envelopment Analysis (DEA) has not been published in the literature ordinarily read by scholars in educational administration, the primary purpose of the present paper is to introduce the theory to the field. Further, since many of those who will be interested in the concepts employed will have had little prior contact with the relevant quantitative methodology, this paper is written in as non-technical a manner as is possible. This intent seems worth the risk that the result may be unsatisfying to both those who want to know more about applications and to those who want better explication of the mathematical models employed in the theory.

The latter group--serious scholars seeking access to the analysis--are directed to the cited papers by Charnes, Cooper and Rhodes. For those primarily interested in applications, the present paper is directed to a study of the possible use of these measures by management at the individual school level. For this purpose, an application is reported which examines the relative efficiency of the elementary schools in a single urban school district.

We want to concentrate on the applicability of DEA in the management of an urban public school district with emphasis on (a) the identification of school units which make better use of input resources in terms of measured outputs, and (b) the obtaining of estimates of the extent to which inputs are underutilized in DMU's which are relatively unproductive.

The fundamental model employed in Data Envelopment Analysis will be presented first. Then we will try to establish a non-mathematical grasp of its major concepts by means of a simplified, hypothetical example and finally we will present the results of an exploratory application in a school district.

DATA ENVELOPMENT ANALYSIS

In their work, Charnes, Cooper and Rhodes introduce first a conceptual model which, in the form of input/output ratios, makes it possible to relate efficiency measurement approaches in engineering, economics, etc. to each other. After exhibiting this property of the conceptual model, Charnes, Cooper and Rhodes then show how this model may be replaced by an equivalent ordinary linear programming problem. Thus, we see that there are two equivalent models employed--a conceptual one and a computational one. Let us consider first the conceptual model which is quite straight-forward in expression.

The Conceptual Model: A Fractional Programming Approach

Assume a set of managerial units such as schools which are all engaged at the same level (elementary or secondary) in production of the same outcomes. We will call this unit of analysis the Decision-Making Unit (DMU) and we are interested in comparing the measured outputs of the DMU's relative to the input resources available to them. Thus we must define a multiple set of valued outputs which can be measured for all units and a multiple set of inputs which our knowledge of the operational context, gained either from experience or prior analysis, indicates that they are resources which may contribute to the output productivity of DMU's.

Now, we relate the input set to the output set by means of a fractional expression or ratio. However, we impose two conditions on the relation: first, inputs and outputs are to be weighted in such a way that each unit is compared to all the others in the set and is constrained not to be larger

than the best input/output ratio observed for any DMU. Secondly, we impose the condition that the weights are calculated to give the largest possible ratio value for a given unit without violating the first condition.

We shall presently see that the effect of these conditions is to supply an empirically determined objective measure of efficiency based upon extremal relations rather than average expectations. But first, let us write the model symbolically:

Let y_{rj} = measurement of rth valued output for decision-making unit j.

x_{ij} = measurement of ith input for decision making unit j.

u_r = weight for output r to be calculated from the analysis

v_i = weight for input i to be calculated from the analysis

Objective to guarantee condition 2

$$\text{Maximize } h_0 = \left(\sum_{r=1}^{s} u_r y_{r_0} \Big/ \sum_{i=1}^{m} v_i x_{i_0} \right)$$

Constraints to guarantee condition 1

$$\left(\sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{i=1}^{m} v_i x_{ij} \right) \leq 1 \quad \text{for}$$

$$j = 1, \ldots, n$$

$$u_r, v_i, y_{rj}, x_{ij} > 0$$

The Efficiency Concept Defined by the Model

Note that in the objective function, we are defining a value (h_0) which increases as the weighted combination of outputs increases relative to the weighted combination of inputs. Since the inputs are resources

employed in producing the outputs, then our objective is to observe among like units the ones having the greatest amount of output for the amount of resources used. Further, we will require the weights (by means of constraints) to be set relative to the input/output ratios of all the other units in the comparison set. In simple terms, we compare all units to locate the best ones in the set and use these as our criterion of efficiency.

We may now state this concept more formally and relate it to its use in the literature of economics where it is designated as Pareto Efficiency.* The latter, as given in [4], may be paraphrased here as follows:

> "A DMU (Decision Making Unit) is not efficient in producing its output (from given amounts of input) if it can be shown that some redistribution of resources will result in the same amount of this output with less of some resource and no more of any other resource. Conversely, a firm is efficient if this is not possible."

This is the definition of efficiency we shall employ with a 100% rating being achieved only by an efficient DMU. While we do not here detail the argument from economic theory, it perhaps suffices to say that all of welfare economics rest on this definition of efficiency, i.e. the so-called Pareto optimality condition. In our case, this has the advantage of not requiring us to assign weights on an a priori basis to the various educational inputs and outputs. Instead, as we shall see, these are obtained directly in an objective manner from the data and the models we shall employ.

The fractional programming model presented in the foregoing section provides us with the conceptual definition of efficiency. For its operational expression, however, we shall transform it into an ordinary linear programming model, which has all the power and convenience of readily available solution methods. As we shall see, we gain in interpretability of results as well since the solution provides us with measures of slack and opportunity

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*Also called Pareto-Koopmans Efficiency. See [4].

cost in terms of the measured inputs and outputs. The terms slack and opportunity cost will be discussed at a later point.

The Linear Programming Equivalent

It is beyond the scope of the present paper to present the proof of the derivation. This is presented by Charnes, Cooper and Rhodes [6]. Suffice it to say that ordinary algebraic transformations are employed to linearize the fractional program to give the following model. It should be noted that all inputs and outputs are defined in the same way as given above. In addition, we define z_o which is the reciprocal of h_o in the conceptual model, and we have defined slack variables for outputs and inputs.

Let y_{rj} = measurement of rth valued output for decision making unit j

x_{ij} = measurement of ith input for decision making unit j

λ_j = weight for jth decision making unit calculated from the analysis

s_r^+ = slack for rth valued output

s_i^- = slack for ith input

Objective:

Maximize z_o

Constraints:

$$-\left[\sum_{j=1}^{n} (y_{rj}\ \lambda_j)\right] + y_{ro}\ z_o + s_r^+ = 0 \quad \text{for } r = 1,\ldots,s$$

$$\sum_{j=1}^{n} (x_{ij}\ \lambda_j) + s_i^- = x_{io} \quad \text{for } i = 1,\ldots,m$$

$$y_{rj},\ x_{ij} \geq 0\ ;\ \lambda_j,\ s_r^+,\ s_i^- \geq 0$$

The linear programming model is solved for each DMU*, providing an inefficiency value (z_o). As mentioned earlier, from the measure of inefficiency, a measure of efficiency can be obtained $\left(h_o = \frac{1}{z_o}\right)$. Both z_o and h_o will be 1.0 for all units having the best combination of inputs and outputs. And h_o will be less than 1.0 for less efficient units with the value of h_o indicating the degree of relative efficiency. Thus, a DMU with $h_o = .90$ will be only 90% as efficient as the most efficient unit.

In addition, for inefficient units, some inputs will not be fully utilized in the solution when the constraints have been satisfied. The interpretation of this condition is that efficient units are getting more output per unit of input for these resources. We shall call this slack, employing the usual terminology for an excess resource.

Another term, opportunity cost, will be used to indicate how much the value of the objective function (efficiency) could be improved if inputs could be reduced by one unit. (Note that this is opposite to the usual interpretation of opportunity cost--the reason being that we are solving for z_o, the reciprocal of h_o.)

Finally, at a later point, we will consider the range of inputs over which our interpretation of opportunity cost is valid.

A Simple Example to Illustrate the Terms Employed

The foregoing discussion has been necessarily abstract. Let us now consider an illustrative interpretation based upon a simple example.

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*We will not here attempt to discuss the solution or interpretation of linear programming models which, in the present instance, requires also duality theory. See [5,6] for such discussion. Our purpose is to indicate the kind of information provided so that an uninformed reader may still follow the terms employed in the examples which follow.

What we shall attempt to do is (1) use the conceptual model to clearly describe what comparisons among the decision making units are being made. We also wish to explain how the solution to the linear programming equivalent to the conceptual model could be used to improve the productivity of individual decision making units.

To reduce the example to the simplest case, we are taking a transportation example with one output (miles travelled) and one input (gallons of gasoline) where efficiency, slack and opportunity cost have common intuitive meanings. It is assumed that an invidious comparison will not be made to the example later given in which schools are the DMUs. We are not saying that schools must be like automobiles for the analysis to apply.

Furthermore, it is hoped that the concepts illustrated will not be confused with measurement issues. In our view, the analytical technique under review and the problems of measurement and selection of inputs and outputs are two separate issues--both highly important. At this point, we are trying to assist the reader in understanding the new analytical technique; not to specify input/output requirements. Measurement concerns are discussed in a later section.

In our example, let us consider four drivers with automobiles of the same make, model, and vintage. Over a one month period, the following measurements for the four automobiles were obtained:

Driver (DMU)	A	B	C	D
Gallons of Gasoline	40	45	50	25
Miles Travelled	1000	900	1400	700
Miles/Gallon	25	20	28	28

It can be seen that drivers of cars C and D are equally efficient while drivers of cars A and B are relatively inefficient in that each uses more gasoline

per mile travelled than the drivers of the cars getting the best gas mileage in the set of cars being compared.

As a measure of the relative inefficiency of car A, we see that if that driver got 28 miles to the gallon as did the drivers of cars C and D, he could have gone 1120 miles on the 40 gallons of input consumed. Thus, driver A's car is only 89% as efficient as either car C or car D ($1000/1120 \doteq 0.89$).

Applying the 0.89 efficiency rating to the actual inputs consumed, we can get a measure of the slack gasoline: 89% of 40 = 35.6 and 40-35.6 = 4.4 gallons. Thus, we define slack in the usual manner as the amount by which an input could be reduced with no associated reduction in output if the unit being evaluated were as efficient as the most efficient unit or units in the set of units being compared.

Finally, we can examine the opportunity cost for the driver of car A by noting that if he were efficient, he would get three miles per gallon more than the 25 he now obtains. Thus, his relative inefficiency is $3/25 = .12$* and for the 4.4 gallons of slack resource, his amount of inefficiency is $.12/4.4 \doteq .028$/gallon. The driver's opportunity cost is .028 which informs us that for each gallon less than the 40 used, the driver of car A becomes .028 more efficient. This will be true for up to 4.4 gallons; i.e., until no less than 35.6 gallons are used to travel the 1000 miles. It should be clear that if 35.6 gallons are used to travel 1000 miles, then miles per gallon is $\frac{1000}{35.6} \doteq 28$ and driver A's car is as efficient as any other car in the set. Further, if less than 35.6 gallons were used by driver A to travel 1000 miles, then driver A's car would become the only

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*We note that the 89% efficiency results in $\frac{1}{.89} \doteq 1.12$ inefficiency. Thus, a unit 89% efficient needs to become .12 more efficient.

efficient car in the set. Thus, the opportunity cost can be interpreted for a maximum reduction of 4.4 gallons.

If we consider opportunity cost in terms of output, we see that for each additional mile travelled for the gasoline consumed, driver A's car gets 0.001 more efficient; for the 120 additional miles required to be efficient, this is .12 more efficient.

The opportunity cost then is:

(1) the increase in efficiency per unit decrease in an input provided all outputs and other inputs remain the same, or

(2) the increase in efficiency per unit increase in an output provided all inputs and other outputs remain the same.

Further, this interpretation can be made only within the range of values that does not change the inefficiency status of the DMU. As was noted above, the opportunity cost for gasoline for the driver of car A was valid only up to 4.4 gallons. Further reductions in gasoline would change the efficiency/inefficiency status of all the drivers in the set and a new solution would need to be obtained.

Note that in our example, the driver of an inefficient car has no information from the analysis about why he is inefficient compared to the others. The reason may be related to the characteristics of his car or the driving conditions and skill of the driver. Nonetheless, our claim is that the knowledge of his relative inefficiency, the amount of slack, and his opportunity cost is useful. Furthermore, if he wishes to include such variables as mentioned above in his analysis, he may do so with a more complex (and satisfactory) model.

Let us now turn to an application of greater interest--an examination of the efficiency ratings for the elementary schools in an urban school district.

AN URBAN SCHOOL DISTRICT APPLICATION

An urban school district with 60,000 pupils in attendance was chosen for the application because a recent study by Jennings [10] provided measures of input from school, community and pupils along with output measures of achievement. Further, a close working relationship with the upper level administration of the school district provided a means for pursuing administrative evaluation of the kind of information provided by the analysis.

The 55 elementary schools in the district were taken as the decision making units (DMU's) and the level of aggregation of all data was the school unit--outputs expressed as median percentile achievement scores for the school and inputs expressed as school totals, ratios, or percents as was appropriate.

In presenting a secondary analysis of existing data we are limited in our choice of variables but we also gain the advantage of a prior analysis in which a multiple regression analysis had determined a strong relationship ($R^2 > .90$) for both reading and math as criterion variables. Thus, we have evidence that the desired relationship of inputs to outputs exists. Furthermore, previous studies in school effects give credibility to our selection of inputs which are surrogates for home socio-economic level and ethnic group, school resources, and faculty-principal climate [9]. Thus, the variables selected satisfy the following conditions:

1. There is a conceptual basis for the relationship of inputs to outputs.

2. There is an empirically inferred relationship of measured inputs to outputs.
3. The relationship is such that increases in inputs are associated with increases in outputs. For example, we will use percent school attendance rather than percent absence.
4. The measurements have no zero elements. This is a formal requirement of the model and is satisfied by adding a small value (.01) to measurements which have legitimate zero values.

The measures obtained for the analysis were as follows:

Output Measures

Two outputs measured by the California Achievement Test in May, 1977.

y_1 median percentile reading achievement for only those pupils in attendance at the school for a full year

y_2 median percentile mathematics achievement test score for only those pupils in attendance for a full year.

Input Measures

Pupil inputs measured by the California Achievement Test in May, 1976.

x_1 median percentile reading achievement for only those pupils in attendance at the school for a full year

x_2 median percentile mathematics achievement test score for only those pupils in attendance for a full year.

Proxy measures for neighborhood and home conditions (obtained from school district records)

x_3 percent Anglo-American students

x_4 percent students not from low income families

x_5 percent in average daily attendance

x_6 mobility index: (total enrollment - number entered late or withdrawn)/total enrollment

Proxy measures for within school conditions (obtained from school district records)

x_7 number of professional staff per 100 pupils

x_8 total per pupil expenditure for instruction

School organizational climate indicators obtained from Organizational Climate Description Questionnaire [10]; a high score on each dimension indicates the following:

x_9 esprit--an indicator of job satisfaction

x_{10} intimacy--an indicator of how much social interaction exists among teachers

x_{11} thrust--principal motivates teachers by personal example of work orientation

x_{12} consideration--measure of the principal's friendliness and cooperativeness with teachers

Measure of classroom instructional processes (obtained from Individualization of Instruction Inventory [10]; higher score indicates greater degree of individual rather than group oriented teaching methods)

x_{13} total individualized instruction index

Fifty-five linear programming models* were solved--one for each school--and the comparative efficiency rating, h_o, was obtained. An h_o value < 1 indicates that the associated school is inefficient with respect to the measures employed in that a combination of schools has been found which can produce the same amount of output with less input. An h_o value = 1 means that no such combination could be found and thus the associated

*Note that all fifty-five models have the same constraint set; only the objective function is different. The constraint set insures that all DMU's are compared to each other and the objective determines the relative efficiency for each unit.

school is Pareto efficient. Slack values* and opportunity costs were also obtained in order to (a) inspect the extent to which inputs were being under-utilized by DMU's and (b) inspect the opportunity for gaining efficiency by increasing resources fully utilized. The obtained h_o values can be found in Table 1, and slack values and opportunity costs for inefficient schools are given in Table 2.

In Table 1, we see that there are 31 efficient schools among the 55. It might seem that this is an unexpectedly large number of DMU's to be equally efficient. We must remember, however, that the ratio of 2 outputs to 16 inputs is being assessed with weights calculated to maximize the efficiency for the DMU being compared to the remaining set. In addition, we may speculate that administrators of schools in an urban district are being pressed to equalize education opportunities for pupils in the district with the result that schools may be levelled to some extent.

There are 24 inefficient DMU's shown in Table 1. The h_o value given for each indicates the proportion of total inputs that would be required to obtain the observed output if the DMU were as efficient as those with an h_o value of 1.00. As we see, this ranges from a low of 0.78 to nearly efficient (0.98).

The reason for a low efficiency value could be low achievement or high inputs or both. To fully interpret the h_o value for each school, we must look at slack values, opportunity costs and the value of outputs and inputs relative to schools in the set. This is shown in Tables 2 and 3.

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*Note that for efficient schools, all slack variable values = 0.

TABLE 1

COMPARATIVE EFFICIENCY VALUES FOR SCHOOLS

DMU Number	h_o	DMU Number	h_o	DMU Number	h_o
1	1.00	20	1.00	38	1.00
2	1.00	21	1.00	39	1.00
3	1.00	22	0.98*	40	0.98*
4	0.89*	23	0.87*	41	1.00
5	1.00	24	0.87*	42	0.96*
6	0.97*	25	0.97*	43	0.97*
7	0.87*	26	0.91*	44	1.00
8	1.00	27	0.88*	45	1.00
9	1.00	28	1.00	46	1.00
10	1.00	29	0.86*	47	1.00
11	0.97*	30	0.95*	48	0.97*
12	0.78*	31	0.92*	49	0.83*
13	1.00	32	0.93*	50	1.00
14	0.96*	33	1.00	51	1.00
15	0.84*	34	0.94*	52	1.00
16	1.00	35	1.00	53	1.00
17	1.00	36	1.00	54	1.00
18	1.00	37	1.00	55	0.98*
19	1.00				

*Denotes an inefficient school.

TABLE 2

SLACK VALUES AND OPPORTUNITY COSTS FOR THREE SCHOOLS WITH LOWEST COMPARATIVE EFFICIENCY INDEX

	Slack Variable Values			Opportunity Costs		
DMU	12	15	29	12	15	29
z_o	1.279	1.19	1.16			
VARIABLES						
Output Variables						
Reading Posttest	1.35	1.06	0.	0.	0.	0.014
Math Posttest	0.	0.	4.24	0.028	0.030	0.
Uncontrollable Input Variables						
Reading Pretest	0.	0.	0.	0.004	0.034	0.008
Math Pretest	0.	1.75	1.04	0.023	0.	0.
Percent Anglo	0.	34.35	0.	0.006	0.	0.
Percent Not Low Income	0.	33.06	31.81	0.005	0.	0.
Percent Attendance	5.85	43.38	18.29	0.	0.	0.000
Neighborhood Stability	2.16	0.	13.22	0.	0.002	0.001
Controllable Input Variables						
Professionals per 100 Pupils	0.53	1.44	0.71	0.	0.	0.
Instructional Expenditure per Pupil	$50.63	$320.80	$140.63	0.	0.	0.
Faculty Esprit	32.16	16.79	9.41	0.	0.	0.
Faculty Intimacy	0.48	10.54	13.87	0.	0.	0.
Principal Thrust	35.79	19.36	0.	0.	0.	0.
Principal Consideration	29.57	5.43	3.76	0.	0.	0.
Individualized Instruction	10.10	30.34	0.00	0.	0.	0.007

Slack Values and Opportunity Costs for Inefficient Schools

In Table 2 are outcomes for the three most inefficient units. These were arbitrarily chosen as an example since we are not seeking a comparative interpretation of all 55 units.

To discuss Table 2, we need to define one additional concept: efficiency adjustment.* That is, we can determine what the outputs and inputs for a DMU would be if it were efficient. To do so, we must adjust all of them rather than considering them one at a time.

Outputs are adjusted by adding the slack value to the product of z_o and the observed output. Inputs are adjusted by subtracting the slack value from the observed input. Thus, we see that slack has a different interpretation for outputs and inputs: slack is the amount of additional output that would be expected if the DMU were efficient and it is how much less of the input efficient units have for the adjusted output.

Considering only one output and one input for DMU 12 in Table 2, for example. This school has reading achievement of 34%ile and 4.55 teachers per 100 students (neither value shown in table). If it were as efficient as other schools, it would have (34*1.279) + 1.35 $\doteq$ 45%ile reading achievement and 4.55 - .53 = 4.02 teachers per 100 students.

The opportunity costs in Table 2 have similar interpretations for both inputs and outputs, but may have different implications for management response. In addition, they may be considered singly rather than all together as was the case with slack adjustments.

Thus, we see that DMU 15 could increase its efficiency .03 by one %ile increase in mathematics achievement and DMU 29 could increase its efficiency by improving reading achievement.

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*Formally derived as adjusting to the efficiency surface in Charnes, Cooper and Rhodes [6].

For uncontrollable inputs, a school may not be able to improve efficiency but may find which variables contribute to their less efficient status and by how much. For example, DMU 12 is effectively restricted from improving its efficiency by the amount indicated for reading prestest score, math pretest score, percent Anglo student population, and percent of families that are not low income.

For controllable inputs, an opportunity exists for improving efficiency, but in Table 2, we find there are zero opportunity costs for schools 12 and 15 and only one for school 29. There, the school could increase its efficiency by .007 for each unit of decrease in individualized instruction.

This discussion has been necessarily general. Let us now seek to be more specific in the interpretation by focusing on a single DMU and considering the management information provided by the Data Envelopment Analysis. We will do this in the discussion section which follows.

DISCUSSION

We have shown that we can identify individual school units that are less efficient than other comparable units in terms of measured achievement scores relative to input factors representing entering achievement, school neighborhood characteristics, expenditures for instruction, type of instruction, and attributes of faculty and principal. It is beyond the scope of the present analysis to determine administrative reallocation of resources in order to achieve greater overall efficiency. What we can do, nowever, is a significant improvement in presently available management information in the administration of schools--we can identify inefficient units and show what output and input variables contribute to their less efficient status. We can also determine what their inputs and ouputs would

be if they were efficient. The administrative response to such information should not necessarily be to reduce the slack resource. In some cases, the resource is mandated by law or is allocated on a formula basis. Even when local leeway is present, the absence of causal evidence would suggest the need for a cautious management response.

What is suggested, then, is a three-step procedure for the use of DEA results in school districts: (a) Identification of inefficient units by top-level administration and reporting of results to individual schools, (b) consideration of slack variable values and opportunity costs by school unit administrators and their interpretation in terms of changes in the school unit operation targeted to the improvement of efficiency, and (c) reanalysis at the time of the next achievement testing to determine if adjustments indicate expected improvements.

Let us illustrate these steps by a discussion of a possible scenario for school 12.

As was shown in Table 1, the h_0 value for school 12 is 0.78, the most inefficient school in the set. Part of the reason for this may be observed in Table 3 where we see that the school has a low achievement score on both reading and math. This low output is exacerbated by the higher reading and math achievement in the previous year. We see from the adjusted value column that achievement would need to be 10%ile higher than observed in order for this school to be efficient.

The uncontrollable variables indicate a school that will likely have difficulty in increasing achievement--there is a large minority student enrollment, many low income families and a large mobility of population in the school's attendance zone. Of these uncontrollable factors, we see by the opportunity cost column that the minority enrollment and low income factors contribute to the inefficiency of the school.

TABLE 3

OBSERVED VALUES, ADJUSTED VALUES, AND OPPORTUNITY COSTS FOR AN INEFFICIENT UNIT (DMU 12)

	Observed Value	Adjusted* Value	Opportunity Cost	Slack
Reading Posttest	34%ile	44.8%ile		1.35
Math Posttest	35%ile	44.8%ile	.0286	
Reading Pretest	47%ile	47 %ile	.0036	
Math Pretest	41%ile	41 %ile	.0226	
Percent Anglo	19%	19 %	.0056	
Percent Not Low Income	16%	16%	.0046	
Percent Attendance	92%	86.1%		5.86
Neighborhood Stability	58.2%	55.8%		2.16
Professionals per 100 Pupils	4.55	4.02		0.53
Instructional Expenditure per Pupil	$834.00	$783.00		50.63
Faculty Esprit	78	45.8		32.16
Faculty Intimacy	54	53.5		0.48
Principal Thrust	89	53.2		35.79
Principal Consideration	79	49.4		29.57
Individualized Instruction	60	49.9		10.10

*$z_o = 1.279$

Looking at controllable factors, a somewhat dismal picture emerges: other schools are getting more output than this school with less staff (.53/100), fewer dollars per pupil for instruction ($50.63), lower attendance (5.86), and even slightly less population stability.

The faculty of this school will need to consider these data and consider their recommendations. Their consideration will be conditioned by the finding that the difficulty appears to be on the output side--they are simply not getting the achievement expected for the resources committed. At least,other schools appear to be accomplishing more under equally difficult conditions.

If we consider a different circumstance--an inefficient school with high achievement, we might consider reallocating slack resources to an efficient unit with low outputs. For example, DMU 40 has 85%ile scores in reading and mathematics and a slack of $16 per pupil in instructional expenditures. Perhaps some of that money could go to DMU 3 which is efficient but has only 20%ile scores in reading and mathematics. Further, DMU 3 spends less money per pupil in instructional expenditures than some other efficient schools with higher achievement.

As we have just illustrated, efficient DMU's in some cases have lower achievement than desired. In this case, the usefulness of the information provided is limited. In other words, the opportunity costs do not provide any information as to whether DMU 3, for example, could utilize additional instructional money effectively. If there is a range of interpretability--and in many instances there is not--the opportunity cost gives how much more efficient the unit would become if the input were reduced and the outputs remained the same or how inefficient the unit would become if the input were increased and outputs remained the same. If DMU 3 received additional money and did not also attain additional output, then this unit could simply become

inefficient. Thus, this uninvestigated limitation of the solution would seem to be an opportunity for school personnel and researchers to work together toward additional exploitation of the technique.

To return to a discussion of DMU 12, if some analysis such as the one suggested were made by consultants to the staff of the school, then they could be assisted in proposing a modified operating plan for the subsequent year. This, along with district-level modifications, would become the goal-setting vehicle for the DMU.*

After a year of operation under the modified plan, measures would again be obtained and DEA computed for all schools. If the efficiency-increasing steps have been effective, it would be detectable by an increase in h_o or perhaps, in the best outcome, the DMU would be classed as efficient. Of course, in the unlikely event that all schools in the district improved their effectiveness, no change or even a drop in h_o could be observed, since it is a comparative measure. However, in this case, an increase in outputs for all the schools should be observed.

In this fashion, the analysis provides the basis for needs identification, program planning and evaluation. These are all needed management tools for school district administration.

SUMMARY

Management of schools has been hindered by lack of appropriate analytical tools. A technique called Data Envelopment Analysis (DEA) has been employed to measure the productivity of individual schools in an urban

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*A similar working relationship is currently being tested by including an elementary school principal in the project team. He is, in turn, working with his staff to determine the feasibility of the DEA application for school unit planning.

school district and to identify those that are less efficient than others with respect to the Pareto-Koopmans Optimality Criterion. Concepts of efficiency, slack, opportunity cost, and efficiency adjustment were defined and discussed. An urban school district was used for an application and, finally, a discussion of results presented the outline of a procedure for using DEA results as management information for the improved efficiency of schools.

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13. ABSTRACT

Conventional methods for comparing the relative productivity of schools employ least square regression to find expected achievement of schools with the same input characteristics. The result is that one typically contrasts the relative effects of "predictor" variables on achievement rather than comparing school units with respect to their input/output efficiency. A newly developed input/output method for comparing the efficiency of decision making units is presented and is applied to elementary schools in an urban school district. The method is found to identify efficient and inefficient schools and provides management information relative to input and output measures.

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