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# COMBUSTION OF TWO-PHASE MIXTURES

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ABSTRACT: A study was made of the gas-dynamic stability of the propagation of detonation in an aerosol and the combustion of a solid fuel with respect to one-dimensional perturbations of the exponential type. The internal structure of the processes is emphasized; feedback equations are derived for each case. Analytic criteria of instability are found, delimiting the region of the possible existence of steady-state regimes of the processes under study and showing ways of stabilizing the pulsations that arise.



## NOTICE





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## Abstract

A study was made of the gas-dynamic stability of the propagation of detonation in an aerosol and the combustion of a solid fuel with respect to one-dimensional perturbations of the exponential type. The internal structure of the processes studied is centrally considered, and from an analysis of the mechanical interaction of the structure with spontaneous perturbations, feedback equations are derived for each case. The system of boundary conditions controlling the perturbed states of processes is based on the successful application to the zones of their occurrence of fundamental theorems of hydrodynamics concerning change in mass, in momentum, and in energy. As a result, analytic criteria of instability are found, delimiting the region of the possible existence of steady-state regimes of the processes being analyzed and indicating ways of stabilizing the pulsations that arise.

## Principal Symbols

$v$ = velocity

$p$ = pressure

$\rho$ = density

$c$ = speed of sound

$T$ = temperature

$M$ = Mach number

$c_p$ = heat capacity at constant pressure

$k$ = ratio of heat capacities

$m$ = particle mass

$d$ = mean particle size

$a$ = thermal diffusivity

$l$ = width of induction zone of process

Indices of Principal Parameters

0 is medium of starting fuel

1 is induction region ("dark" zone)

2 is region of end reaction products

3 is flame region

T is fuel

2 is gas phase

(stroke) stands for perturbations

## Introduction

As soon as the steady-state model of any physical processes is constructed, at once from the considerations of the mathematical correctness and the possibility of the practical realization there arises the problem of the stability of this working scheme relative to nonsteady perturbations of a given type. Special interest in this respect lies in the analysis of the time buildup of perturbations spontaneously arising in the process itself and thus determining the nature of its internal stability. It is precisely the control of the latter that is most difficult to achieve in engineering practice.

The foregoing can apply fully, in particular, to processes of the combustion of two-stage systems realized during the propagation of detonation in an aerosol and in ordinary (achieved by a thermal mechanism) combustion of solid fuel (SF), since the thermal decomposition of the condensed phase into gaseous reagents preceding the combustion is accompanied by the intensive dispersion of its particles (droplets) from the solid surface. With some common ground from the standpoint of the analysis of stability, the two processes will be examined in the present investigation, consisting of two sections.

The mathematical approach to these problems remains within the frame of reference of the linearized perturbations of the one-dimensional type (that is, with relatively large wavelength along the surface of the process front) and primary attention is concentrated on the gas-dynamic (mechanical) aspect of the stability problem.

## 1. Stability of Combustion of Solid Fuel

Let us use as the model of the steady-state combustion of SF, in accordance with experimental data, a locally plane scheme of process, which in a coordinate system advancing together with its front normal to the x axis is represented as follows. The condensed phase of the initial fuel travelling at the combustion rate $v_0$ (region 0: $x < -L_0$), on thermally decomposing, becomes gasified into intermediate reagents at the solid surface $x = -L_0$. It is essential that the gasification of SF is accompanied by the intensive dispersion of its particles (droplets) from the surface. Further, in the direction of the y axis they follow one after the other, the three zones of the process occurring successively in the gas phase. The vapor-smoke-gas or "dark" zone with width $l = L_0 - L$ occupies the region $1 (-L_0 < x < -L)$ and is analogous to the induction period of chemical processes in gases, differing only by the presence of suspended particles of the starting fuel. In the flame of the gas phase (region 3: $-L < x < 0$) embracing the thermal internal and mean interval of intensive chemical transformations, from the intermediate reagents 1 there are formed the end combustion products, which occupy region 2 ($x > 0$).

Adopting the above-described course of the media as basic, let us examine its perturbed state of the one-dimensional type engendered by random internal causes. This approach enables us to study locally correlations of the inception and buildup of normal fluctuations during the combustion of SF from the end face of the detonation charge and the radial oscillations during the combustion of cylindrical turbular charges of SF, since it is known experimentally [1] that there is no relationship of these oscillations with the erosional mechanism of combustion. Perturbations of pressure $p'_j$, of velocity $v'_j$, and of entropy $S'_j$ imposed on the main flow of regions j = 1, 2, 3 of the gas phase of the process are represented by the sum of solutions to the linearized system of one-dimensional gas-dynamic equations

$$\begin{aligned} &\frac{\partial v'_j}{\partial t}+v_j\frac{\partial v'_j}{\partial x}+\frac{1}{\rho_j}\frac{\partial p'_j}{\partial x}=0,\\ &\frac{\partial p'_j}{\partial t}+v_j\frac{\partial p'_j}{\partial x}+\rho_j c_j^2\frac{\partial v'_j}{\partial x}=0,\\ &\frac{\partial S'_j}{\partial t}+v_j\frac{\partial S'_j}{\partial x}=0. \end{aligned} \qquad (1)$$

Here the small volumetric concentration of the dispersed particles of the initial fuel in zone 1 permits using the model of a medium in the form of a continuum with several averaged (effective) gas-dynamic characteristics.

With respect to perturbations of the condensed phase (region 0), based on the large difference in the densities of the solid material in this region and the gaseous reagents of region 1 ($\rho_0 \gg \rho_1$), in [2] were showed from the law of the conservation of mass at the boundary of these media $x = -L_0$ that these perturbations can be neglected, just as the shifting of the boundary itself, compared with the perturbations in the gas phase of the process. As a result, the conditions for the reflection of perturbations at the solid surface are formulated by the requirement of rigorous acoust:: reflection and by the linearized law of the conservation of energy

$$v_1'=0, \quad p_1'+\frac{\rho_1 c_1^2}{(\kappa_1-1)c_{p1}}S_1'=0 \quad \text{for } x = -L_0. \tag{2}$$

The momentum transmitted here from the gaseous medium to the solid can be easily found from the corresponding law of conservation.

The boundary conditions for the transition of the perturbations through flame zone 3 are achieved by applying the classical integral theorems of hydrodynamics on the change of mass, of momentum, and of energy [3]

$$\begin{aligned} &\frac{\partial}{\partial t}\int_\tau \rho\, d\tau+\int_\sigma \rho v\, d\sigma=0, \\ &\frac{\partial}{\partial t}\int_\tau \rho v\, d\tau+\int_\sigma \rho v^2\, d\sigma=\int_\sigma p\, d\sigma, \\ &\frac{\partial}{\partial t}\int_\tau \rho\left(\frac{v^2}{2}+W\right)d\tau+\int_\sigma \rho v\left(\frac{v^2}{2}+W\right)d\sigma=\int_\sigma pv\, d\sigma, \end{aligned} \tag{3}$$

which we first drew upon [4] in the theory of the stability of normal gas combustion. Here W stands for enthalpy, $\tau$ and $\sigma$ are the control volume and its delimiting surface. We can adopt as the control volume either the entire flame zone 3, or else, by dividing it into intermediate subregions, apply the theorems (3) for each element of this subdivision separately. Then the selection of a sufficient number and extent of the subregions enables us to achieve as close as possible a matching of the stepwise distribution of the gas-dynamic parameters with their true (experimental) continuous distribution of the intraflame section of the process.

In the simplest first case, linearization of theorems (3) leads to the following equations for the perturbations:

$$
\begin{aligned}
&\alpha\left(v_1' - \frac{d\varepsilon}{dt} + \frac{p_1'}{\rho_1 v_1} M_1^2 - \frac{v_1}{c_{p1}} S_1'\right)_{x=-L} = \left(v_2' - \frac{d\varepsilon}{dt} + \frac{p_2'}{\rho_2 v_2} M_2^2 - \right. \\
&\qquad \left. - \frac{v_2}{c_{p2}} S_2'\right)_{x=0} + \frac{\alpha}{q} \int_{-L}^{0} \frac{1}{v_3} \frac{\partial}{\partial t}\left(\frac{p_3'}{\rho_3 v_3} M_3^2 - \frac{v_3}{c_{p3}} S_3'\right) dx, \\
&\left[2v_1' + (1+M_1^2)\frac{p_1'}{\rho_1 v_1} + \frac{v_1}{c_{p1}} S_1'\right]_{x=-L} = \left[2v_2 + (1+M_2^2)\frac{p_2'}{\rho_2 v_2} + \right. \\
&\qquad \left. + \frac{v_2}{c_{p2}} S_2'\right]_{x=0} + \int_{-L}^{0} \frac{1}{v_3} \frac{\partial}{\partial t}\left(\frac{p_3'}{\rho_3 v_3} M_3^2 + v_3' - \frac{d\varepsilon}{dt} - \frac{v_3}{c_{p3}} S_3'\right) dx, \qquad (4) \\
&\frac{1}{\alpha}\left[v_1' - \frac{d\varepsilon}{dt} + \frac{p_1'}{\rho_1 v_1} + \frac{v_1}{c_{p1}} \frac{S_1'}{(\kappa_1 - 1) M_1^2}\right]_{x=-L} = \left[v_2' - \frac{d\varepsilon}{dt} + \frac{p_2'}{\rho_2 v_2} + \right. \\
&\qquad \left. + \frac{v_2}{c_{p2}} \frac{S_2'}{(\kappa_2 - 1) M_2^2}\right]_{x=0} + \frac{1}{v_3} \int_{-L}^{0} \frac{\partial}{\partial t}\left[v_3' - \frac{d\varepsilon}{dt} + \right. \\
&\qquad \left. + \frac{p_3'}{\rho_3 v_3}\left(\frac{M_3^2}{2} + \frac{1}{\kappa_3 - 1}\right) - \frac{v_3}{c_{p3}} \frac{S_3'}{2}\right] dx,
\end{aligned}
$$

where $\alpha = p_1/p_2$, $q = p_1/p_3 \simeq T_3/T_1$, and $\varepsilon(t)$ is the displacement of the flame zone 3 under the effect of perturbations.

The gas-dynamic parameters of the main flow of region 3 are found by some single-step averaging of their continuous distribution over the extent of the flame section. In particular, this can be achieved by integral averaging with respect to temperature. When theorems (3) are applied for the set of subregions of zone 3, this averaging must be carried out over the extent of each subregion separately, so that the distribution of the principal parameters will be multistep in character, more closely reflecting the true nature of the process.

Let us select, further, as the control volume for the application of theorems (3) a small neighborhood of the steady-state leading boundary of the flame so that its perturbed state does not exceed the limits of this neighborhood. The utility of the proposed approach is obvious, since in general the position of the leading boundary for a steady flame is arbitrary to a large extent. Then applying theorems (3) in this case adds to system (4) the following linearized equations: when x = -L

$$
\begin{aligned}
&q\left(v'_1+\frac{p'_1}{\rho_1 v_1}\mathrm{M}_1^2-\frac{v_1}{c_{p1}}S'_1\right)=v'_3+\frac{p'_3}{\rho_3 v_3}\mathrm{M}_3^2-\frac{v_3}{c_{p3}}S'_3,\\
&2v'_1+\frac{p'_1}{\rho_1 v_1}(1+\mathrm{M}_1^2)-\frac{v_1}{c_{p1}}S'_1=2v'_3+\frac{p'_3}{\rho_3 v_3}(1+\mathrm{M}_3^2)-\frac{v'_3}{c_{p3}}S'_3,\\
&\frac{1}{q}\left[v'_1+\frac{p'_1}{\rho_1 v_1}+\frac{v_1}{c_{p1}}\,\frac{S'_1}{(\kappa-1)\mathrm{M}_1^2}\right]=v'_3+\frac{p'_3}{\rho_3 v_3}+\frac{v_3}{c_{p3}}\,\frac{S'_3}{(\kappa_3-1)\mathrm{M}_3^2}.
\end{aligned}
\qquad (5)
$$

From the standpoint of the gas-dynamic aspect of stability on which we have concentrated our attention, two fundamental mechanisms of the interaction of perturbations with the internal structure of the "dark" zone 1 and the flame region 3 will play a fundamental role in forming the feedback of the combustion process in the gas-phase flame. The first principal mechanism arises from the accumulation of local increments of the rate of the reacting gas phase along the intraflame section of the trajectory and thus pertains entirely to the manifestation of hydrodynamic effects of the gas medium. The increment in the velocity of the rate of propagation of the combustion zone 3 in the gas caused by this mechanism was derived by us [5] in the following integral form:

$$\delta v_1^{(1)} = v_3 \int\limits_{t-t_3}^{t} \frac{\partial v_3'}{\partial x}\bigg|_{x=v_3(t'-t)} dt'; \quad L = v_3 t_3 \tag{6}$$

The second mechanism reflects in the formation of the feedback the effect of interaction of the two different phases -- gas and solid (liquid), for it was produced by the inertial effects of the mechanical interference of particles (droplets) of fuel dispersed from the solid surface into the gaseous state in the thermal decomposition of SF, with the perturbed state of these gaseous reagents. These effects show up owing to the massiveness (intertia) of the suspended particles compared with their surrounding gas medium, as a result which the dispersed particles are subject to the inertial forces of acoustic acceleration:

$$F = -m\frac{dv'}{dt} \quad (m \sim \rho_T d^3), \tag{7}$$

when sound waves travel through the gas phase. The work done by the developed perturbations of the transport inertial forces (7) on the suspended particles is expended mostly in advancing the suspended particles into the zone of intensive chemical transformations of the flame region 3. Therefore, the perturbations in some way or other promote the entry into this zone of additional amounts of fuel, and their combustion (or the suppression of the process intensity) entails an increment in the rate of propagation of the entire combustion zone 3. The overall effect of the operation of these forces (7) over the extent of the process studied in the gas phase was quantitatively estimated by us earlier [6] in the integral form and for our case reduces to the following expression for the increment in the combustion rate

$$\delta v_1^2 = A\left[\int_{-L_0}^{-L}\left(-\frac{dv_1'}{dt}\right)dx + \int_{-L}^{0}\left(-\frac{dv_3'}{dt}\right)dx\right] \qquad (8)$$

The cofactor A, placed in front of the specific characteristics of work and acting as the rate of tne given feedback mechanism, is measured by the total mass of the displaced fuel particles additionally entering the flame during perturbations. Since we here are working not with the mass combustion rate, but the linear combustion rate, this mass must bc calculated per unit area of the combustion front, that is, it must be divided by the mass of the colume of gas phase penetrated by the middle of a mean-sized particle, and as a result of which it was found in [5] that

$$B = Ac_1 = D\frac{m}{\rho_2 d^2 L_0} = E\frac{d}{L_0}\frac{c_2}{c_T}\frac{\rho_T}{\rho_2}\frac{a_T}{a_2} \qquad (9)$$

where the ratio of the speeds of sound reflect the dissipative effect of the blurring of the acoustic front by the dispersion of particles, and the ratio of the thermal diffusivities reflects the effect of the thermal characteristics of the particles on their combustion compared with the surrounding gaseous reagents. The quantity E is the number of particles dispersed on the average per unit burning surface $L_0^2$ in the characteristic time of the processes in the gas phase $t_*$ and it is determined experimentally, naturally taking on the least value for the steady-state combustion regime. Actually, high-frequency vibrations of the solid surface can only intensify the dispersion of particles by separating them by inertial forces.

The array of the two above-described mechanisms leads to the final expression of the feedback in the form of the following equation for the increment in the rate of propagation of the gas-phase flame

$$\delta v_1 = v_1'\Big|_{x=-L} - \frac{d\varepsilon}{dt} = \delta v_1^{(1)} + \delta v_1^{(2)} \tag{10}$$

Now specifying the exponential form of the dependence of the shift in the combustion front $\varepsilon \sim \exp \omega t$, let us represent the solutions to system (1) for perturbations in the regions j = 1, 2, 3, of the type: $\varepsilon(t) \exp(\gamma_{jk} x)$, where

$$\begin{aligned} \gamma_{jk} &= -\frac{\omega}{v_j} \frac{M_j}{M_j + (-1)^k}, \quad (k=1,2) \\ \gamma_{jk} &= -\frac{\omega}{v_j}, \quad (k=3) \end{aligned} \tag{11}$$

respectively for the incident and reflected sound waves and the entropy waves. In the combustion products (region 2), we need take into account only of the acoustic wave exiting into them along the flow, since we are dealing with an investigation of internal stability.

Inserting the solutions (11) into the conditions (2), (4), (5), and (10) leads to a secular equation in the form of a ninth-order determinant for finding the eigenvalues $\omega$ controlling the character of process stability. This equation, to a precision of small values of $M^2$ is of the form

$$\begin{aligned} &\left[r_2 + q r_1 \left(\frac{1}{M_3} - 1\right)\right]\left[n(Q_1 + Q_2) + Z(\Delta_2 - \Delta_1) - Z(\Delta_1 + \Delta_2)\frac{\alpha}{q}\left(1 + \frac{1}{M_2}\right)M_3\right] - \\ &-\frac{2}{M_3}\left\{r_2 - \alpha r_1\left(\frac{1}{M_2} + 1\right) + n(b - q r_1 Q_1) - r_1 Z \Delta_1 \left[q + \alpha M_3 \left(1 + \frac{1}{M_2}\right)\right]\right\} = 0, \end{aligned} \tag{12}$$

where

$$Z = \frac{\omega}{v_3} L_0, \quad l = L_0 - L, \quad \beta = \frac{c_3}{c_1}, \quad \Delta = \exp(\gamma_{12} - \gamma_{11}) l,$$

$$r_1 = 1 - M_1 - (1 + M_1)\Delta, \quad r_2 = 2 - \frac{1}{M_1} - \left(2 + \frac{1}{M_1}\right)\Delta,$$

$$f_{3k}=M_3[\exp\gamma_{3k}L-\exp(-Z)],\quad Q_k=\frac{B\beta(\exp\gamma_{3k}L-1)-f_{3k}}{1+(-1)^k M_3},$$

$$b_{1k}=1-(-1)^k B[1-\exp(-\gamma_{1k}l)],\quad \Delta_k\gamma_{3k}L_0=\exp\gamma_{3k}L-1,$$

$$b=b_{11}-b_{12}\Delta,\quad n=Z+(\alpha-1)\left(1+\frac{1}{M_2}\right).$$

The mathematical analysis of the characteristic equation (12) in the case when the size of the "dark" zone l and flame L are of identical order enables us to obtain a sufficient criterion of instability

$$B\alpha>M_3. \tag{13}$$

By virtue of $M_3 \gg 1$ inequality (13) is always satisfied, except for the particular case E = 0 describing the situation in which none of the dispersed particles is able to reach the combustion zones, in some manner being transformed into the gaseous products of the "dark" zone 1. In this case the inertial feedback mechanism simply does not operate.

Therefore, at moderate pressure when the values of l and L are commensurable, inequality (13) ensures as a rule the mechanical instability of the model of the combustion process of SF we have considered. The frequency of the process fluctuations arising due to instability can be estimated as $\nu = 1/t_* = v_*/L$, where $v_*$ is the characteristic (mean) flowrate of the gas phase. Adopting as $L_0$ a value of the order of a millimeter and as $v_*$ a value of the order several M/sec, for $\nu$ we will have a value not exceeding 10 kilohertz, which corresponds to the measurements characteristic for experiment [1].

In the limiting case of a disappearing "dark" zone ($l/L_0 \gg 1$), which corresponds to combustion regimes at high pressures, the instability criterion is expressed according to (12) and by an inequality that is the opposite of (13). Since in this case the flame zone "3" begins just at the solid surface, the mandatory penetration of dispersed fuel particles directly into this zone is ensured, that is, $E \neq 0$. And the latter, in turn, entails the nonfulfillment of the instability criterion. Hence follows the conclusion that a rise in pressure accompanied by a decrease in the extent of the "dark" zone 1 has an essentially stabilizing effect on the SF combustion process. As a result, the combustion regime that is unstable at mean pressures passes into a stable regime at high pressures. This conclusion agrees with practical results [1].

In the limiting case with respect to a large "dark" zone ($1/L_0 \gg 1$), which describes combustion regimes at sufficiently low pressures, the instability criterion deriving from (12) is of the form

$$E\frac{d}{L_0}\,\frac{c_z}{c_T}\,\frac{\rho_T}{\rho_z}\,\frac{a_T}{a_z}\,\frac{\alpha-1}{1+M_2/M_1}>1. \qquad (14)$$

Thus, the nature of the behavior of time perturbations depends now on the ratio of parameters, and as a function of this ratio, either stable or unstable forms of SF combustion at sufficiently low pressures can be admitted as inequality (14), which again is in agreement with practical findings [1].

2. Stability of Detonation in Aerosols

Let us consider a plane non-dimensional scheme of the process as the model of steady-state detonation in an aerosol induced by a shock wave propagating along the normal to the surface of its front (along the x axis). The effect of the two-phase status of the exposed medium can be taken into account according to the method of F. Williams [7], whose calculations showed that the dispersion of the medium leads to a small (within the limits of 10 percent) rise in the parameters of velocity, pressure, and temperature for a self-sustaining detonation regime compared with the rise in these parameters in a gas mixture with identical heat release and initial conditions. In the following [7], we will also assume that the volumetric concentration of droplets (particles) suspended in the gaseous oxidizer small, which is valid in particular for hydrocarbon fuels close to their combustion.

Here, in the F. Williams' theory the identity of the Rankine-Hugoniot equations for steady detonation of an aerosol with the case of gaseous detonation, if the determination of the temperature and heat of reaction in the aerosol are suitably modified. The latter permits simulating the medium of the detonation zone as a continuum with effective (averaged) gas-dynamic acoustics. The shock wave initiating the detonation will in some way or other be blurred at the suspended aerosol particles. However, from the standpoint of the above-introduced assumption that their volumetric concentration is small it is admissible to use the scheme of a discontinuous front.

It is quite understandable that the shock front, on traversing the aerosol, will entrain the initial two-phase system fuel-oxidizer. The massiveness of the fuel particles (droplets) compared with their surrounding gas makes them inertial owing to the large difference in the densities of the solid (liquid) and gaseous phases. As a result of this inertia, the aerosol elements exhibiting it lag behind the surrounding gas and the mass concentration of the fuel directly behind the initiating shock front is reduced.

Let us arrive at the following model of the detonation process in a coordinate system travelling with the leading front along the x axis. The initial aerosol medium advancing at the detonation rate $z_0$ (region 0: $x < -L_0$) exposes an intense shock transformation in the initiating wave front, $x = -L_0$ being transformed into the shock-compressed state of region 1 ($-L_0 < x < -L$), which is analogous to the induction period of the detonation process in gaseous mixtures. In this region, the thermal gas-dynamic conditions engendered by the initiating shock wave ensure the buildup of processes necessarily preceding the intense chemical transformations of the explosive system. The latter, and with it also the effective release of the heat energy of the reactions, must be anticipated after the concentration of fuel reaches a high enough level and the preparatory physical processes will be fundamentally completed. Next follows the region of the active buildup of intense chemical transformations 3 ($-L < x < 0$), as a result of which most of the energy sustaining the steady-state propagation of the leading shock wave is released. For the plane $x = 0$ where the intensive chemical transformations can be regarded as completed, in the case of a self-sustaining detonation regime the Jouget condition is satisfied. The latter ensures the speed of sound of the flow of final reaction products occupying region 2 ($x > 0$) and in the general case exhibiting a subsonic velocity.

The perturbed state of the one-dimensional type of the above-described steady flow is controlled by system (1) obtained by linearizing the gas-dynamic equations about this main flow. Here, the essentially supersonic nature of the rate of detonation propagation ($v_0 > c_0$) excludes the presence of any perturbations in the region 0 of the initial aerosol, since the only source of these is the detonation process itself, that is, we are talking about investigating its internal stability.

The boundary conditions of the principal shock front $x = -L_0$, whose shift due to perturbations is denoted by $\mathcal{E}_0(t)$, can be obtained by linearization of the fundamental laws of the continuity of the relative flows of mass, of momentum, and of energy at this discontinuity. Linearization yields the following equations: when $x = -L_0$

$$v_1' + (\alpha_0 - 1)\frac{d\varepsilon_0}{dt} + \frac{p_1'}{\rho_1 v_1} M_1^2 - \frac{v_1}{c_{p1}} S_1' = 0, \quad \text{where} \quad \alpha_0 = \frac{\rho_0}{\rho_1},$$

$$2v_1' + \frac{p_1'}{\rho_1 v_1}(1 + M_1)^2 + \frac{v_1}{c_{p1}} S_1' = 0, \qquad (15)$$

$$v_1' + \left(\frac{1}{\alpha_0} - 1\right)\frac{d\varepsilon_0}{dt} + \frac{p_1'}{\rho_1 v_1} + \frac{v_1}{c_{p1}} \frac{S_1'}{(\kappa_1 - 1)M_1^2} = 0.$$

The boundary conditions for the transition of the perturbations through the zone of intensive chemical transformations 3 are found from the above-mentioned classical theorems of hydrodynamics (3), that is, they can be represented directly in the form (4) and (5).

From the mechanical point of view, interest lies above all in the effect of interaction of two different phases forming the aerosol -- the fuel and the gaseous oxidizer. In this respect, this effect plays a main role in forming the feedback mechanism of the perturbations of the medium with the internal structure of the detonation process. The large difference in the densities of the suspended fuel particles (droplets) and the surrounding gas is responsible for their inertia, as a result of which -- as the acoustic waves pass through the gas phase -- the aerosol particles are subject to inertial forces generated by these perturbations. The work done by these forces over the extent of the zone of the detonation process is expended mainly in advancing additional portions of the aerosol to the region of intense chemical transformations. Their combustion (or their suppresion of the intensity of the chemical transformations) produces an increment in the combustion rate $\delta v_1$, which we have already quantitatively estimated above by expression (8). The additional release (or reduction) of heat produced here ultimately intensifies (or weakens) the initiating shock wave. The latter actually determines the change in the rate of propagation of the detonation process as a whole due to perturbations.

As a result, we have the feedback equation on analogy with (10) in the form

$$\delta v_1 = v_1'\Big|_{x=-L} \frac{d\varepsilon}{dt}, \tag{16}$$

where $\varepsilon(t)$ stands for the perturbed shift of the zone of intensive chemical transformations 3 culminated by the Jouget plane, and $\delta v_1$ is taken directly unchanged from (8).

As for the coefficient of the intensity of this feedback mechanism A, all the considerations given above remain valid, with the exception of the effect of thermal diffusivity. The latter plays a fundamental role against the background of other fast-occurring processes (in particular, evaporation for the liquid aerosol). Therefore on analogy with (9), the quantity A becomes

$$B = Ac_1 = E\frac{d}{L_0}\frac{c_z}{c_T}\frac{\rho_T}{\rho_z}, \tag{17}$$

where the quantity E estimated from experiment has the significance of the number of aerosol particles averaged per unit surface in the detonation wave.

Note that we can refine this method, as already stated above, by an additional subdivision of the region intensive chemical transformations into subregions, to each of which the theorems in the form of (5) can be applied separately.

Like section 1 of this study, let us specify $\mathcal{E}_0$, $\mathcal{E} \sim \exp \omega t$ and let us seek the solutions to system (1) in the form of (11). Substituting these solutions into conditions (15), (16), (4), and (5) yields a secular equation for determining the eigennumber $\omega$, which is expressed by a tenth-order determinant.

Its analysis in the limiting case $1 = L_0 - L \ll L_0$ shows that there is no process instability However, this case is physically of little significance for the size of the induction zone cannot be assumed small. Actually, the more intense the initiating shock front, the more intense will be concentration of the burning aerosol behind it fall off and the farther the zone of the effective chemical transformation will be advanced.

Another limiting approach when $1 \gg L_0$ is physically more realistic and leads, for self-sustaining detonation, to the following characteristic equation:

$$\sum_{k=1}^{2} \exp(Z - \gamma_{1k} L_0) \left\{ \alpha - \frac{(-1)^k}{M_1} - r_1 \left[ 1 - \frac{(-1)^k}{M_1} \right] + r_2 \left[ (\alpha_0 - 1)\left( 1 - \frac{b_{3-k}}{1-(-1)^k M_1} \right) + \frac{(-1)^k}{M_1} \right] \right\} \left\{ \left[ 1 + \frac{(-1)^k}{M_1} \right] \frac{1}{(\kappa_1 - 1) M_1^2} + \frac{(-1)^k}{M_1} \left( 1 - \frac{1}{\alpha_0} \right) \right\} + \frac{2}{M_1}\left( 1 - \frac{1}{\alpha_0} \right) \left[ r_1 - \alpha + \frac{1}{(\kappa_1 - 1) M_1^2} - r_2(\alpha - 1) \right] \cdot \exp[-(\gamma_{11} + \gamma_{12}) L_0] = 0 \qquad (18)$$

where

$$b_k = 1 - (-1)^k B[1 - \exp(-\gamma_{1k} L_0)]$$

$$r_1 = 1 - \frac{\alpha}{\kappa_2 - 1}, \quad r_2 = \alpha - 1 + \frac{2\alpha}{\kappa_2 - 1}, \quad Z = \frac{\omega}{v_1} L_0$$

Considering that the shock wave must be intense for the possibility of initiating the detonation process, we can assume $\alpha_0$ to be close to the limiting compression $(k_1 - 1)(k_1 + 1)$. Then the mathematical analysis of equation (18) leads to the instability criterion:

$$\frac{B(\alpha-1)M_1\varphi}{\alpha\frac{\kappa_2}{\kappa_2-1}-1-(\alpha-1)M_1^2\varphi}>1 \qquad (19)$$

where

$$\varphi=\frac{\alpha}{1-M_1^2}\,\frac{\kappa_2+1}{\kappa_2-1}$$

which ensures the exponential buildup of perturbations with time. In particular, this is true for the extent of the detonation process initially occupying the interval $-L_0 < x < 0$. The rapid disintegration of the detonation wave dimensions greatly increases the effectiveness of losses that begin to play a basic role in the possibility that the process exists in general. The dissipative effects as a result can rise to the extent that the detonation process is simply quenched. In this case the instability criterion (19) can serve simultaneously as the mechanical condition for estimating the explosion safety of a flammable aerosol.

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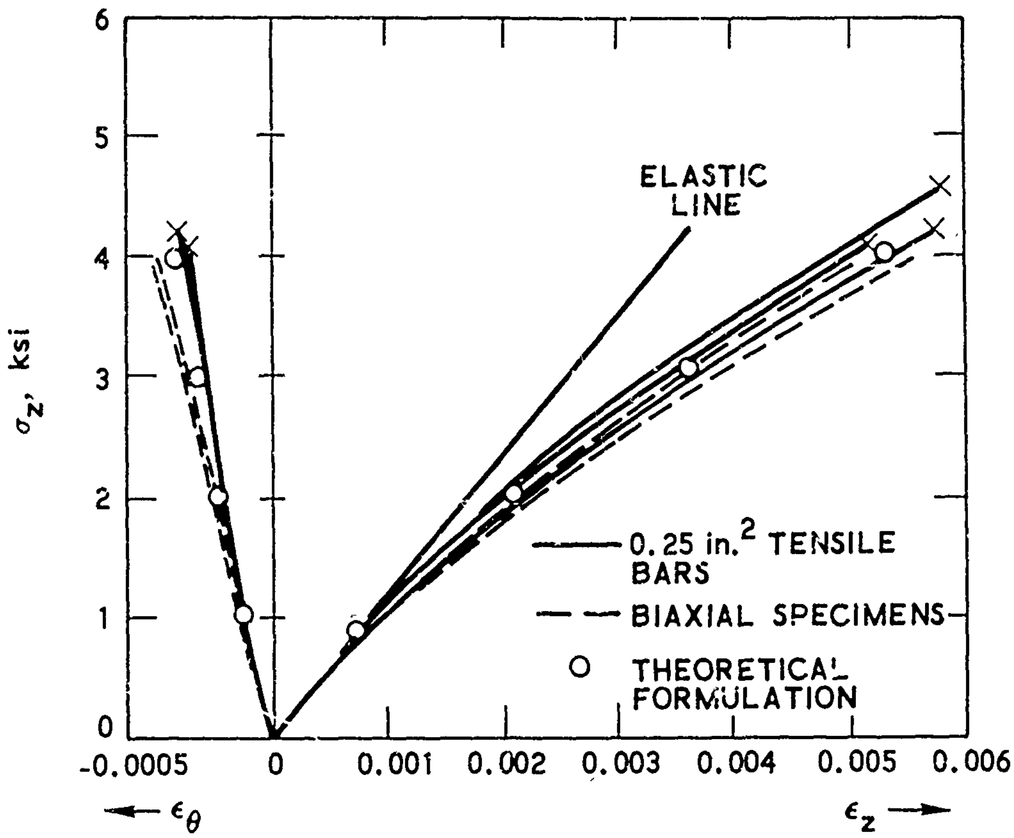


Fig. 1. With-Grain Tension S'rain Response of ATJS at 70°F (adapted from reference cited in footnote 5)