AD-783 941

THE GURNEY FORMULA: VARIATIONS ON A THEME BY LAGRANGE

Sigmund J. Jacobs

Naval Ordnance Laboratory
White Oak, Maryland

21 June 1974

DISTRIBUTED BY:

NTIS
National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NOLTR 74-86	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER AD 783 941
4. TITLE (and Subtitle) THE GURNEY FORMULA: VARIATIONS ON A THEME BY LAGRANGE		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) SIGMUND J. JACOBS		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Ordnance Laboratory White Oak, Silver Spring, Maryland 20910		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS ORD Task # 35D/001/092-1/UR-023-04-01, Energy Transfer Studies
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE 21 June 1974 13. NUMBER OF PAGES 37
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Fragmentation	Hydrodynamics	Missile Warheads
Detonation	Gurney Formula	Ballistics
Bombs	Explosions	
Cylinder Expansion Test	Amunition	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The Gurney formula for predicting the velocity of fragments from bombs and artillery shells is shown to be modelled after the interior ballistic problem of Lagrange. Though the model differs significantly from the real situation for the interaction of a detonating charge with the metal case, the resulting simple equation gives a fairly good prediction of mean fragment velocity. It is shown that the accuracy of the prediction can be improved ...

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. Abstract (continued)
by altering one constant term in the equation. Experimental results on the velocity of fragments and on the velocity of expansion of metal cylinders are analyzed. It is shown that differences in reported Gurney constants are due to differences in experimental conditions under which the experiments were performed and to incorrect data reduction. Appropriate corrections are introduced to bring most of the results into close agreement. A new set of Gurney constants is given for the ten most frequently used explosives for both the original formula and the proposed modified form. Precautions which must be taken in defining the Gurney constant and in applying the equation are discussed.

NOLTR 74-86 21 June 1974

Reported herein is an investigation of the Gurney formula and a restudy of results of experimentally derived Gurney constants. It is shown that differences in reported Gurney constants stem from differences in experimental conditions of the tests and in some instances from incorrect analysis of the data. A modified Gurney equation which improves the range of application in terms of charge/mass ratio is given.

The work was performed under ORD Task No. 35D/001/092-1/UR-023-04-01, "Energy Transfer Studies." It should be of interest to engineers and scientists who are designing, developing, testing, or analyzing fragmenting weapons or other devices or systems in which the velocity of fragments propelled by explosions is of importance.

ROBERT WILLIAMSON II
Captain, USN
Commander

D. C. Hornig

D. C. HORNIG
By direction

CONTENTS

		Page
I.	INTRODUCTION	1
II.	THE LAGRANGE APPROXIMATION IN N DIMENSIONS	3
III.	DETONATION IN A CYLINDER	5
IV.	NOL CYLINDER EXPANSION EXPERIMENTS	12
V.	DISCUSSION	19
VI.	REVISED GURNEY CONSTANTS	23
VII.	CONCLUDING REMARKS	27
REFERENCES		28
APPENDIX A		A-1

ILLUSTRATIONS

Figure	Title	Page
1	Energy Distribution in a 1-D Detonation, $\gamma=3$	6
2	Expansion of a Metal Cylinder in a Steady State Detonation	9
3	Vectors for Cylinder Expansion Conversion	10
4	Expansion Velocity, Steel Cylinders, Composition B	16
5	Expansion Velocity, Aluminum Cylinders, Composition B	17
6	Comparison of Fitted Curves for Steel and Aluminum with the Gurney Formula	18
A-1	Notation for Calculation of Mean Cylinder Velocity	A-2

TABLES

Table	Title	Page
1	Case Expansion Data	13
2	$V_g(A)$ for the Gurney Equation, $V = V_g(A)/\sqrt{M/C+A}$; in Feet/Second	25

THE GURNEY FORMULA: VARIATIONS ON A THEME BY LAGRANGE

I. INTRODUCTION

An approximation to the velocity of fragments expelled from a projectile or bomb was formulated in a very simple expression by Gurney[1,2] during World War II. Since that work, numerous papers have appeared on the subject[3-11]. These include applications to plane (sandwich), cylindrical, and spherical symmetric systems; and also discuss limitations of the formula. It is interesting that two authors, Henry[8] and Defourneaux[10], noted (but without references) that the plane flow formulation is an approximation to the interior ballistic problem of guns, a problem first stated by Lagrange in 1793 (see[12-14] for details). Lagrange's approximation is just the Gurney formula for the motion of a projectile in a uniform diameter tube assuming the propelling gas to expand with uniform but time dependent density. Thus Gurney's formulation and others that follow it are simply variations on the Lagrange theme. A more exact solution to the Lagrange problem, Love and Pidduck[13], is a classic in the literature of hydrodynamics. It is probably the first detailed characteristic solution to a Riemann flow problem. The Love and Pidduck solution shows that gas density must be a function of position as well as time even in a one dimensional flow. More important to the fragment velocity problem is the fact that we are dealing with the acceleration of a metal by a detonating explosive in which the flow behind the detonation is vastly different from the Lagrange model. Yet we find, despite this large deviation from the model, that the Gurney equation with minor modification can serve as a good approximation for predicting fragment velocity as a function of explosive composition. It must be used judiciously, however. In principle the entire problem can be precisely calculated by the use of a two dimensional hydrodynamic code with appropriate thermodynamic data. One finds that here, too, one must use judgement to get the results correctly interpreted. It is therefore of advantage to employ the much simpler Gurney approach in many instances.

D. R. Kennedy[9] cited a practical difficulty regarding the Gurney formula; the Gurney constant has not been pinned down adequately. He also shows that the values given by Army and Navy sources for the Gurney constant differ by about 10%, Navy values being low. It appears that all the Navy values cited may, in fact, be quotes or minor variants of results from one series of experiments performed at NOL in 1953 by Solem, et al[5]. A follow-up paper by Solem and Singleton[6] recognized some of the problems arising in the use of the Gurney formula. This paper based on a cylinder expansion

technique could have cleared up some of Kennedy's findings. It showed that the "constant" in the formula as written for cylindrical symmetry is not a constant. The formula when applied to ultimate fragment velocity is correct to within experimental error only over a limited range of C/M, C being the explosive charge mass per unit length of a cylindrical cased charge and M being the case mass per unit length. Part of this paper is devoted to a review of the work by Solem and Singleton in references [5] and [6]. It will show that the range of application of the Gurney type formulation can be extended by a further variation on the Lagrange theme. In our treatment we will generally use the ratio, M/C, because it simplifies the mathematics.

II. THE LAGRANGE APPROXIMATION IN N DIMENSIONS

Defourneaux[10] summarized the Lagrange-Gurney formulation for plane parallel motion (n = 1), axially symmetric or cylindrical motion (n = 2), and for point symmetric or spherical motion (n = 3) in the single equation

$$V^2 = (2E_g)\left[\frac{1}{M/C + n/(n+2)}\right] \quad ; \tag{1}$$

E_g being the Gurney energy per unit mass of charge and V the fragment velocity. Although Defourneaux refers to the value of n as representing initiation on a plane, an axis, or a point this equation is, in fact, derived by the assumption that the total kinetic energy E_g is distributed between the expanding gas and the moving case with the gas velocity a linear function of the distance from the plane, line, or point origin. A consequence of this assumption is that the gas density is uniform; a function only of the time or distance traveled by the case. This inherent assumption like that of Lagrange implies that a uniform pressure also exists in the gas products at any instant. This permits consideration of a second formula to define the Gurney energy as a function of case displacement. Neglecting changes in gas composition its entropy will be constant; the gas internal energy and pressure will consequently be only a function of volume. We can then write a more general conservation of energy equation

$$E_g = E_o - E(v), \tag{2}$$

where E_o is the initial energy per unit mass in the gas and E(v) is the internal energy retained after expansion to specific volume v. The case displacement is related to the specific volume in the model by the relation

$$(r/r_o)^n = v/v_o,$$

r being the distance from the origin to the gas/case interface, and subscript o referring to the initial position. For a polytropic gas $E = pv/(\gamma-1)$ and equation (2) leads to the Gurney energy[8,10,14]

$$E_g(r) = E_o\,[1 - (r_o/r)^{n(\gamma-1)}]. \tag{3}$$

We should note that Henry[8] used equations (2) and (3) but ignored the kinetic energy of the gas in relating equations (1) and (2). It has been the usual practice to refer to the Gurney energy or to the Gurney velocity $V_g = \sqrt{2E_g}$ as constants. It is apparent from equation (2) that these parameters are a function of the radial expansion which can occur before the case breaks up and ceases to be accelerated by the gas. The Gurney velocity will therefore be a function of the case material, in particular, its ductility under dynamic loading conditions. One can get a rough estimate of the ratio E_g/E_o by using the approximate ratio $r/r_o = 1.5$ which is about the value at which a steel case breaks. Using $\gamma = 3$ and $n = 2$ (for a cylinder) we find the energy ratio to be about 0.80 for a cylindrical expansion. It is usually smaller. Equation (3) could be of use when one wishes to estimate the value of E_g for a radius ratio other than that at which velocity has been measured.

The above model leads to a simple mathematical formulation which turns out to be useful even when the model is not closely adhered to. When a charge is detonated within a metal cylinder, we find we are dealing with a wave phenomenon. The detonation wave results in a gas product having axial motion and a distribution of velocity and density far from the uniform state envisioned in the model. Despite this difference we find it possible to use the Gurney formula as an acceptable approximation over a limited range of the ratio C/M. This is accomplished by fitting experimental data to the equation to obtain E_g which is now an arbitrary constant. We will show that by a slight modification, replacing the term $n/(n+2)$ in Equation (1) by an arbitrary constant, we can, in fact, extend the range of applicability of the "Gurney equation" to values of M/C as small as may be needed in practice. One must, however, modify the value of V_g if the conditions in the application differ from those in the test used for evaluation of V_g. For example, the results of tests with open ended cylinders will lead to lower speeds for the fragments near the end of the cylinder than would be obtained with closed cylinders. The speeds of fragments near the center of open ended cylinders will, however, be much the same as those for closed cylinders. Some of the differences cited by Kennedy can be traced to such differences in test configuration. The discussion in the next section shows some of the expected features of gas product distribution in detonated cylinders. It also outlines the method used to determine case speed when the explosive is detonated in a long cylinder.

III. DETONATION IN A CYLINDER

To simplify the discussion of this report we assume the explosion product to be a polytropic gas. We will illustrate the discussion with examples in which the value of γ is 3. This is actually a fair approximation for explosives like Composition B loaded at or near the maximum bulk density. To see how a detonation leads to a different situation than the idealized Gurney model, we first illustrate the energy distribution in a detonated cylinder encased in a perfectly rigid tube[15]. Figure (1) shows the distribution of energy per unit volume as a function of position along the tube of explosive at the instant that the detonation front has reached the end opposite the plane of initiation. In this figure, Q is the energy of detonation per unit weight. For a polytropic gas product we can write the following hydrodynamic relations

$$P_j = Du_j\rho_o \ , \qquad D/u_j = \gamma + 1 \ , \qquad P_j = \rho_o D^2/(\gamma + 1) \ ,$$

$$\rho_j/\rho_o = v_o/v_j = (\gamma + 1)/\gamma \ , \tag{4}$$

$$E_j - Q = p_j(v_o - v_j)/2 = u_j^2/2 \ .$$

D is detonation velocity, u is particle velocity, p is pressure, ρ is density, and v is specific volume. The subscripts o and j refers to the initial state and the Chapman-Jouguet detonation product state, respectively. With these equations and the polytropic gas relations it turns out that the value of Q is related to the detonation velocity and the detonation pressure by

$$Q = \frac{p_j}{2(\gamma - 1)\rho_o} = \frac{D^2}{2(\gamma + 1)\ (\gamma - 1)} \ , \text{ per unit mass} \tag{5}$$

A simple illustration of the above equations is given to show the approximate magnitudes for Composition B. This explosive has a detonation velocity of approximately 8 mm/μsec at a loading density ρ_o of 1.7 g/cc and γ is very nearly 3. Using these values and

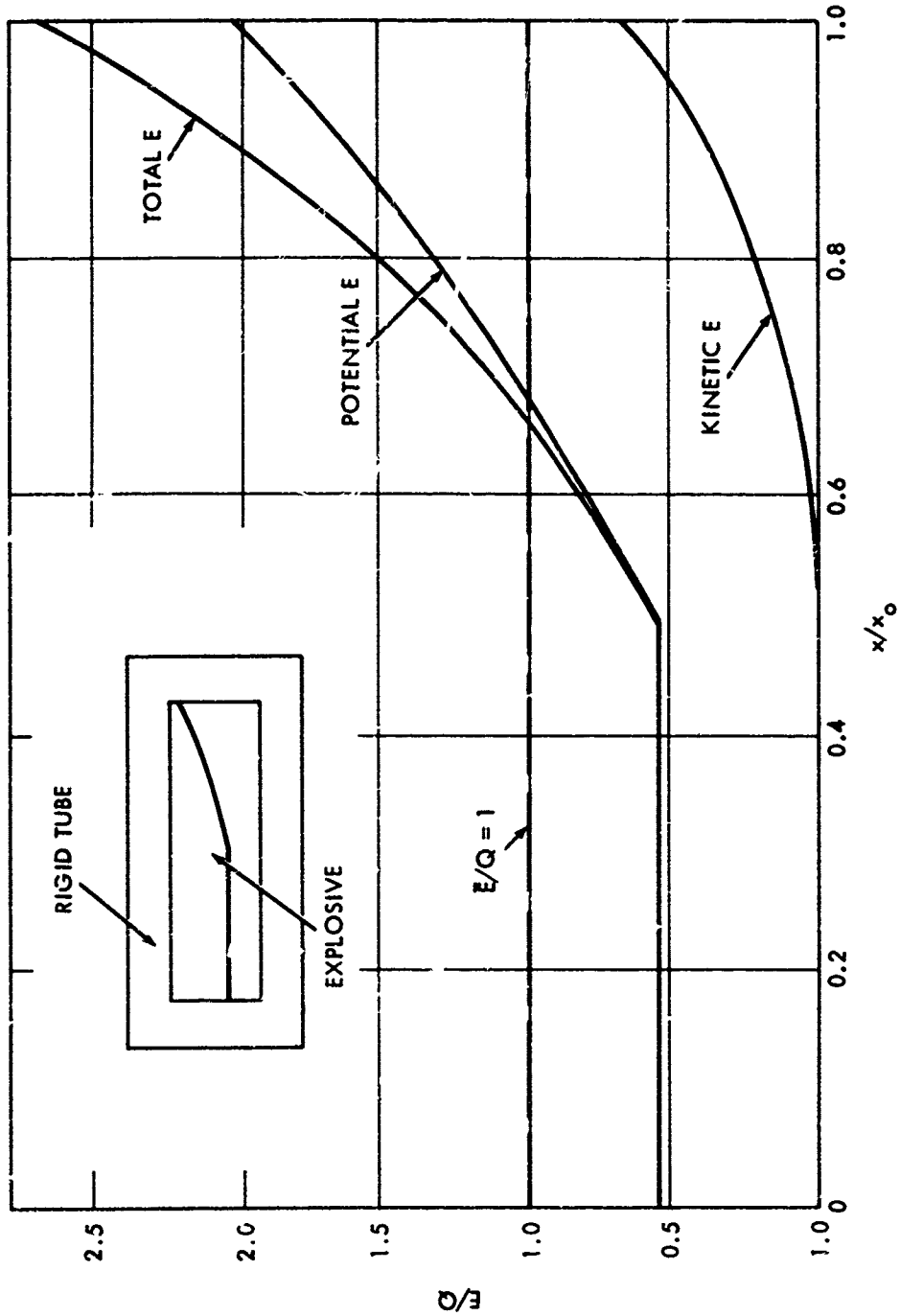


FIG. 1 ENERGY DISTRIBUTION IN A 1-D DETONATION, $\gamma = 3$

applicable conversion factors we get

$$p_j = 10 \times 8^2 \times 1.7/(3 + 1) = 272 \text{ kilobars} = 2.684 \times 10^5 \text{ atm.}$$

$$u_j = 8/4 = 2 \text{ mm/}\mu\text{sec} = 2000 \text{ m/sec}$$

$$\rho_j = 4 \times 1.7/3 = 2.267 \text{ g/cc}$$

$$Q = 8 \times 8/(2 \times 4 \times 2) = 4 \text{ kilojoules/g} = 996 \text{ calories/g.}$$

A γ of 2.8 would bring the detonation pressure and energy closer to the experimental values.

The idealized picture in Figure (1) is a pretty good representation of the energy distribution behind a detonation in the plane wave case (i.e., rigid confinement). Had the left closure been removed, the potential energy to the left of the halfway point would have decreased toward the opening and the kinetic energy would have correspondingly increased; the energy distributions to the right of center would remain unchanged. At the time shown there would be no change in the energy distribution if we removed the right closure. At a slightly later time the closed right end would lead to a reflected shock (less kinetic energy, more potential energy) moving to the left from the closure, whereas an open right end would lead to escapement of gas resulting in a rarefaction wave (more kinetic and less potential energy) which moves to the left from the open end. If, instead of a rigid case we had one which was deformable, there would be a radial expansion of the case and the gases within it after passage of the detonation wave. A velocity near the limit of this expansion is approximated by the Gurney formula. For γ equal to 3 the gas density will vary as the square root of the potential energy in the plane flow shown. To this change there will be the additional density decrease due to radial expansion. We can see therefore that the real hydrodynamic problem will differ quite markedly from the Gurney model. Qualitatively the detonation energy, Q, will be approximately the E_o of Equation (2). This will be closer to the truth if the case is closed at both ends and M/C is relatively large.

To see approximately how the case moves behind a detonation wave, it is convenient to consider a steady state flow problem. This type of problem can be approximated experimentally by using a long cylinder of cased explosive and by looking at the motion in a region far from the plane of initiation. If in the original rest system the detonation is moving to the left, velocity -D, we can transform the motion to a system with the detonation front at rest by imagining the cased cylinder ahead of the detonation to be moving to the right at velocity D. In this new system the velocities and other properties in the detonated explosive and in the case in the vicinity of the detonation front will be only a function of position,

Figure (2). The gas immediately after detonation will be moving at velocity $D - u_j$ (neglecting the reaction zone). This by Chapman-Jouguet theory is the sound speed in the gas. The case will expand laterally in a shock wave and a centered set of Mach lines in the gas will originate at the corner between the detonation front and the case. On these Mach lines the sound speed is the component of the flow velocity normal to a given Mach line. The C-J condition implies that the first Mach line is parallel to the detonation front. As a result, gas expansion starts at all radii as soon as the detonation is complete at the front. This situation contradicts the "detonation head theory" of Cook. (Wilkins has confirmed by detailed 2-D calculations[16] that the axis pressure in a typical detonation in an unconfined cylinder falls to about 30% of the CJ value at a distance of one charge radius behind the detonation front.) Waves and flow lines are qualitatively sketched in Figure (2). One streamline in the gas is shown; the boundary between case and explosive is also a streamline.

The motion of a cylinder of finite thickness proceeds in a series of step jumps. After 2 or 3 transits the shocks and rarefactions in the case settle down to weak waves with small pressure changes; the material density of the case will then return to almost its initial value. Taylor (see reference [3] made the useful observation that this implied a case velocity equal to detonation velocity in the steady state flow. The velocity vector is at a new angle α to the cylinder axis. This idea makes it possible to transform the flow back to the initial rest system and thus obtain the case speed and direction relative to our usual frame of reference. The analysis is shown in Figure (3a). Angle α is obtained from the y displacement, $V_a t$ by the observation that $\tan \alpha = V_a/D$. (V_a is an apparent velocity.) The vector representing the case motion in the steady state is D at angle α to the horizontal. We add vectorially the vector -D at angle zero. The resultant is vector V at an angle $\alpha/2$ to the vertical. This is true case velocity and direction resulting from a detonation moving to the left in a system initially at rest. The case has acquired a velocity component in the direction of the detonation.

The above scheme has a practical application to the determination of case velocity as a function of thickness in the cylinder expansion test. The velocity is seen to be given by the simple relation

$$V = 2D \sin (\alpha/2). \qquad (6)$$

In actual practice the tube's length to H.E. diameter ratio need not be excessively great to approximate steady state. The ℓ/d ratios of 2.5 and 6 used by Solem[6] gave only slight differences in the calculated expansion velocity. Kury[17] has used larger ℓ/d ratios; he states that the difference between observations of V_a disappears when made at 3 times the diameter or greater.

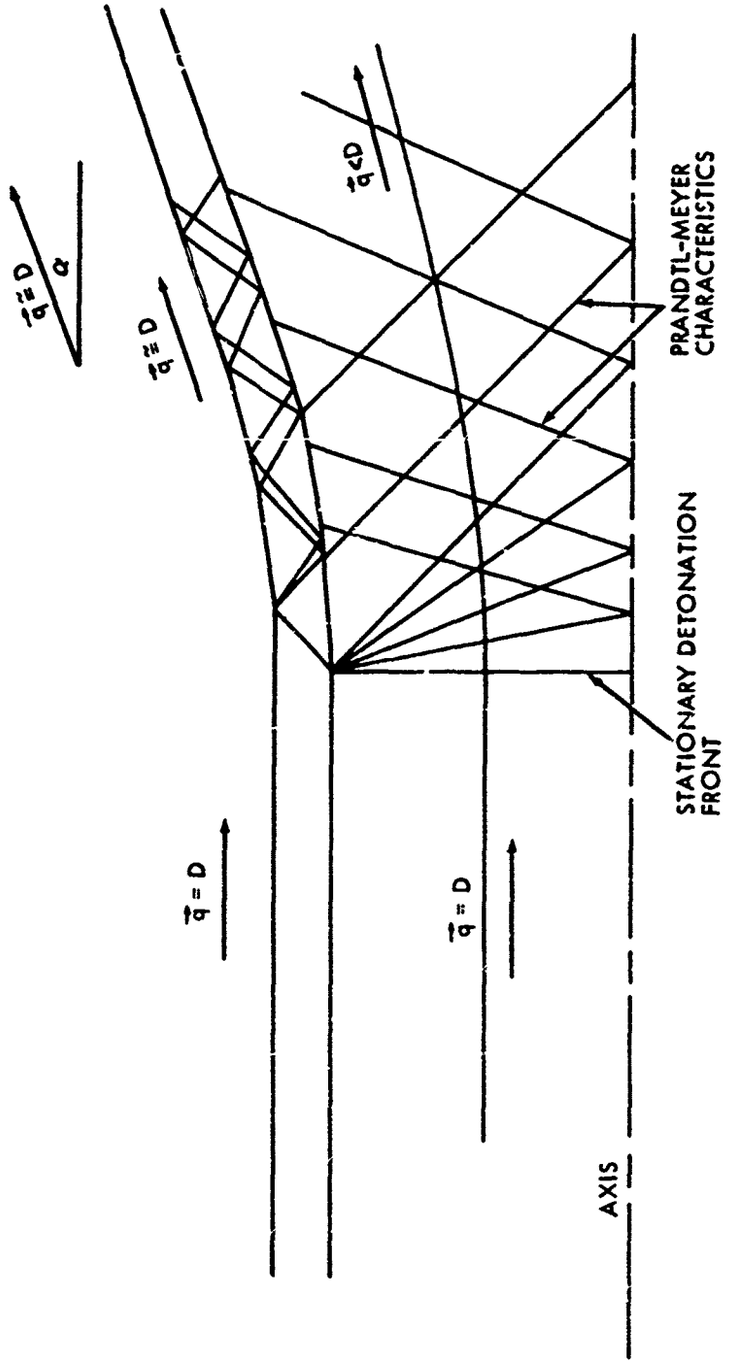
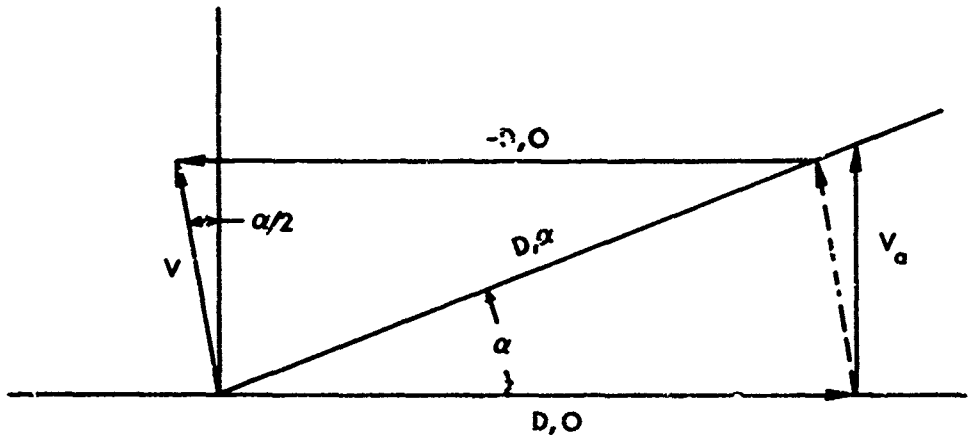
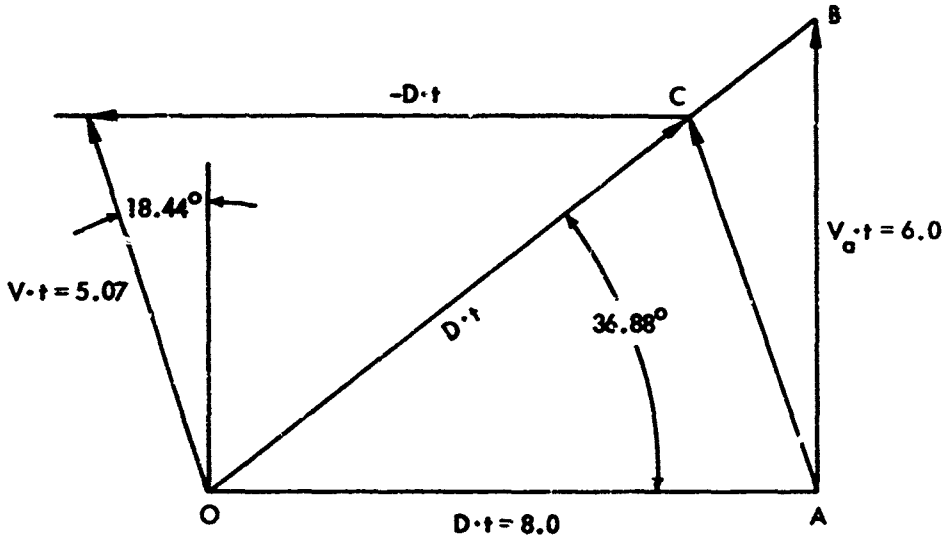


FIG. 2 EXPANSION OF A METAL CYLINDER IN A STEADY STATE DETONATION



(a)



(b)

FIG. 3 VECTORS FOR CYLINDER EXPANSION CONVERSION

The experimental approach to obtain V is to use a smear camera to observe the transverse motion of the case. When α is small, it is permissible to consider the transverse velocity to be the true case speed. This is not valid when α is large. To show this more clearly we have drawn in Figure (3b) a situation in which the transverse speed V_a of the case is 3/4 of the detonation velocity; i.e., $\tan \alpha = 0.75$. Vector velocities have been changed to displacements by considering the case to have traveled a distance Dt at angle α after constant angle α has been established. Note that the element of the case which is initially at A moves to C rather than B. Analyzed as in Figure (3a), we see AC to have the same direction and magnitude as the vector at 0. In the example shown the result is V equal to 5.07 instead of the 6 units given by V_a when the detonation speed is 8 units. V_a is in significant error. The error in magnitude is, however, trivial when V_a/D is less than 0.2. The experiments to be described included several instances for which V_a/D exceeded 0.2.

IV. NOL CYLINDER EXPANSION EXPERIMENTS

Solem and Singleton[6] studied the radial expansion of steel and aluminum cases accelerated by cast Composition B at a loading density of 1.68 ± .01 g/cm^3. The aluminum cylinders were of 2S type having a nominal density of 2.70 g/cm^3; steel cases were of Shelby tubing (untreated) at a nominal density of 7.85 g/cm^3. Wall thicknesses were varied over a wide range as shown in the experimental results given in Table 1. The explosive diameter was kept constant at 2".0. Open ended tubes of length 5".0 and 12".0 were employed. The explosive was initiated at one end by a plane wave booster. Detonation velocity in the HE was 7850 meters/second. Observations with the smear camera were made with the slit at about 3".75 from the booster end for the 5-inch tubes and at about 8 inches for the longer tubes*. The position for smear camera observation was chosen to optimize the compromise between getting as close an approximation as possible to steady state without interference from rarefaction from the open end. The smear camera observations give radial displacement as a function of time up to the time that the cases fractured and vented. The steel cases were thus observed up to about 2 cm of radial motion and the quoted values of V_a are at 2 cm. The aluminum cases did not vent until they had expanded about 4 to 5 cm in radius. The quoted velocities for these tubes is at about 4 cm of radial expansion. The slightly higher observed velocity for aluminum cases reflects mainly the result of the increased travel distance for the observation. Very little additional velocity is expected after the observation is made so that the data obtained should represent terminal fragment velocity to within mean experimental error of 2 to 3%.

The raw data of Table 1 was reduced to terminal velocities in this report by a more exact calculation than in reference [6]. First a Taylor angle correction was made. Then we noted that the smear camera records the apparent velocity of the outer boundary of the case. Thus, the Taylor angle result must be corrected for the fact

---

* The choice of 5-inch tubes is unfortunate in that the approach to steady state is marginal. Nevertheless, the results obtained agreed with the NOL values for 12-inch cylinders. The results show several important features of case expansion which had not been studied elsewhere. The velocity of the case at the position chosen represents the value for the fastest fragments expelled. We note in Section 5 that reference [17] reports a velocity for Composition B (Grade A) about 7% higher than that obtained here.

TABLE 1 (cont.) - CASE EXPANSION DATA

CASE THICKNESS (in.)	M/C	V_a m/sec	V_o m/sec	V_o ft/sec	V_m ft/sec	$\bar{V}_m$ ft/sec
Aluminum Case, ρ = 2.70, 5 inches long (in Helium)						
0	0	9580	6718	22041	22041	22041
0.002	0.00646	9200 8890	5518	21385	21385	21385
0.005	0.01618	7660 6730	5694	18680	18680	18680
0.125	0.4284	2990	2841	9321	9406	9406
0.250	0.9074	2280	2212	7256	7386	7386
Aluminum Case, ρ = 2.70, 12 inches long (in air, except t=0)						
0.0	0	9880	6825	22391	22391	22391
0.125	0.4284	3280	3087	10127	10219	10219
0.500	2.016	1640 1610	1614 1585	5295 5201	5479 5392	5436

V_a = apparent velocity

V_o = velocity of outer surface

V_m = mean velocity (Appendix A)

$\bar{V}_m$ = averaged V_m

that the case becomes thinner as it expands. Because of the thinning the mean case speed is faster than the surface speed. The mean velocity can be determined with acceptable accuracy if we assume that the case expands as an incompressible fluid. For a measurement at any given radius we can then calculate a total radial momentum which, when divided by the case mass gives the desired mean velocity. The analysis is given in Appendix A. Calculated results are listed in the table.

It was evident from the data that the original Gurney formula equation (1) with the appropriate value n = 2 would not fit the experimental data. We, therefore, rewrite the formula as an empirical two constant relation in the form*

$$V = V_g(M/C+A)^{-1} \tag{7}$$

with A being an arbitrary dimensionless constant. One can argue that the effective kinetic energy of the gas is ACV^2; e.g., A = (constant) $\cdot n/(n+2)$, the constant being less than one due to non-uniform density distribution. The data for the steel and the aluminum cases were reduced separately. Trial fits showed that the steel case data could be best fit with A = 0.3, V_g = 7900 ft/sec. The same value for V_g could be used for the aluminum case data provided that A was changed to 0.2. Figures (4) and (5) are plots of the data and calculated velocities ±3%. In fitting the data we have ignored the very thin case experiments. These experiments were carried out to demonstrate the extreme error that can occur in the original Gurney formula. From a practical point of view the modified formulas give good fit down to M/C ratios as low as 0.05.

The equations for steel and aluminum cases are compared in Figure (6). Note that the change in A with constant V_g results in a shift of the steel curve 0.1 unit in M/C to the left of the aluminum case data. The difference in velocities, steel about 3 to 5% lower than aluminum at M/C greater than 1.0, is mainly due to the smaller radius at which the steel case velocity had to be measured. The larger difference at low values of M/C may, in part be due to differences in tensile strength. A third curve, dashed line, is a plot of the Gurney formula, A = 0.5, fit to the mean value of V at M/C equal to 1.5. In this case V_g turns out to be 8485 ft/sec. If the curve had been fit to the velocity at the value, M/C = 2.5, typical of other experiments, V_g (0.5) would have been more like 8200 ft/sec.

---

* Henceforth, equation (7) will be referred to as the Gurney formula with the understanding that two constants must be specified. When we mention "Gurney constant" without specifying A its value is implied to be 0.5.

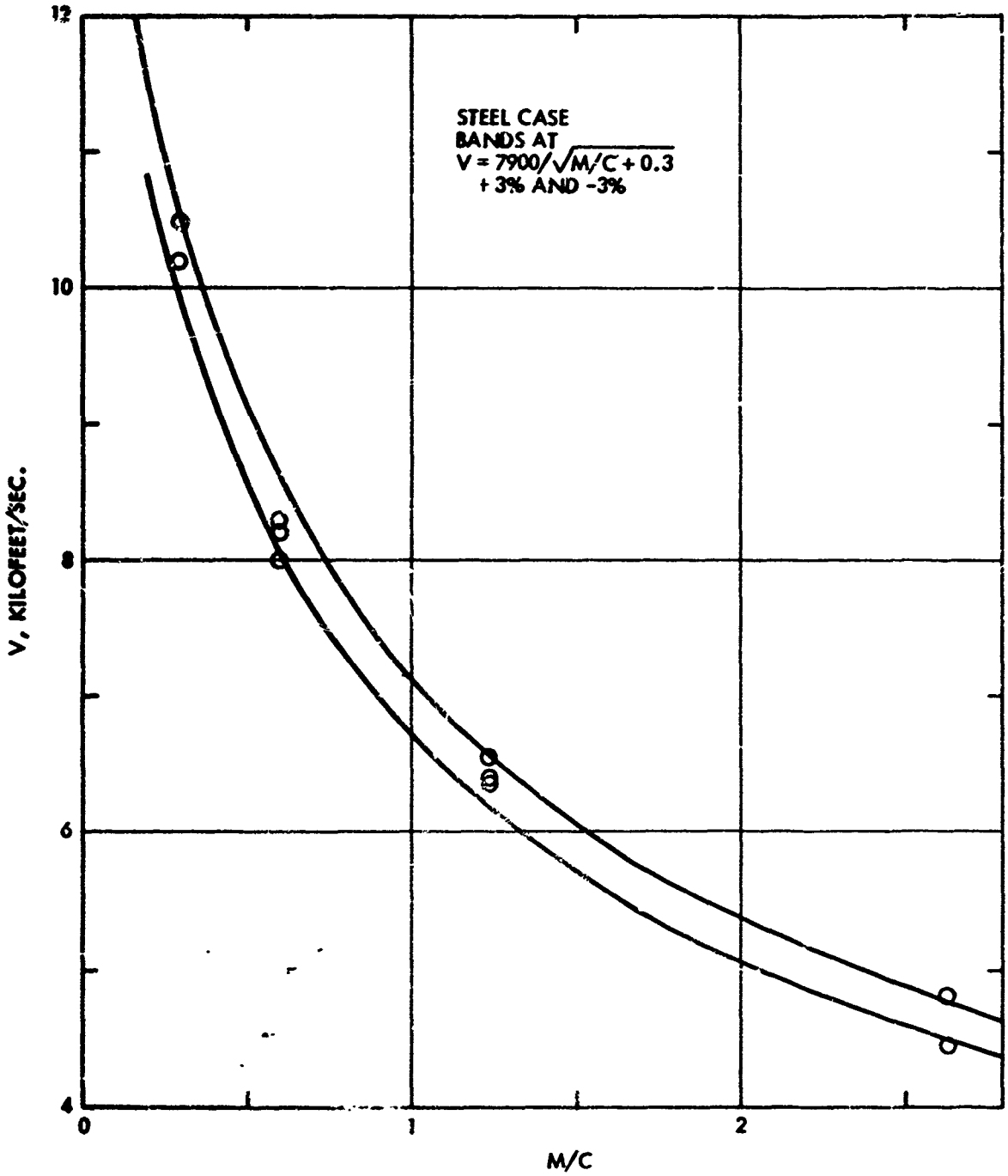


FIG. 4 EXPANSION VELOCITY, STEEL CYLINDERS, COMPOSITION B

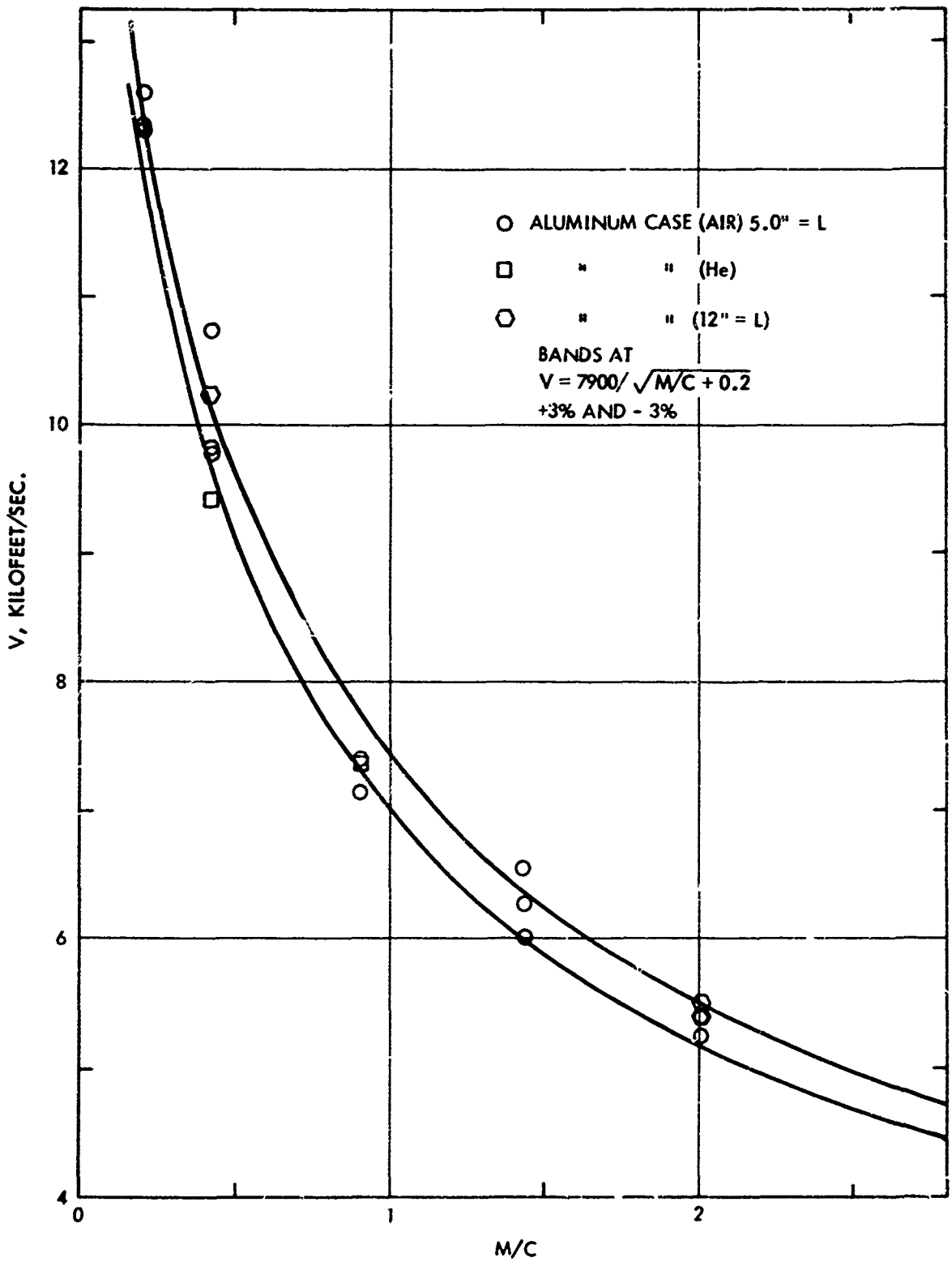


FIG. 5 EXPANSION VELOCITY, ALUMINUM CYLINDERS, COMPOSITION B

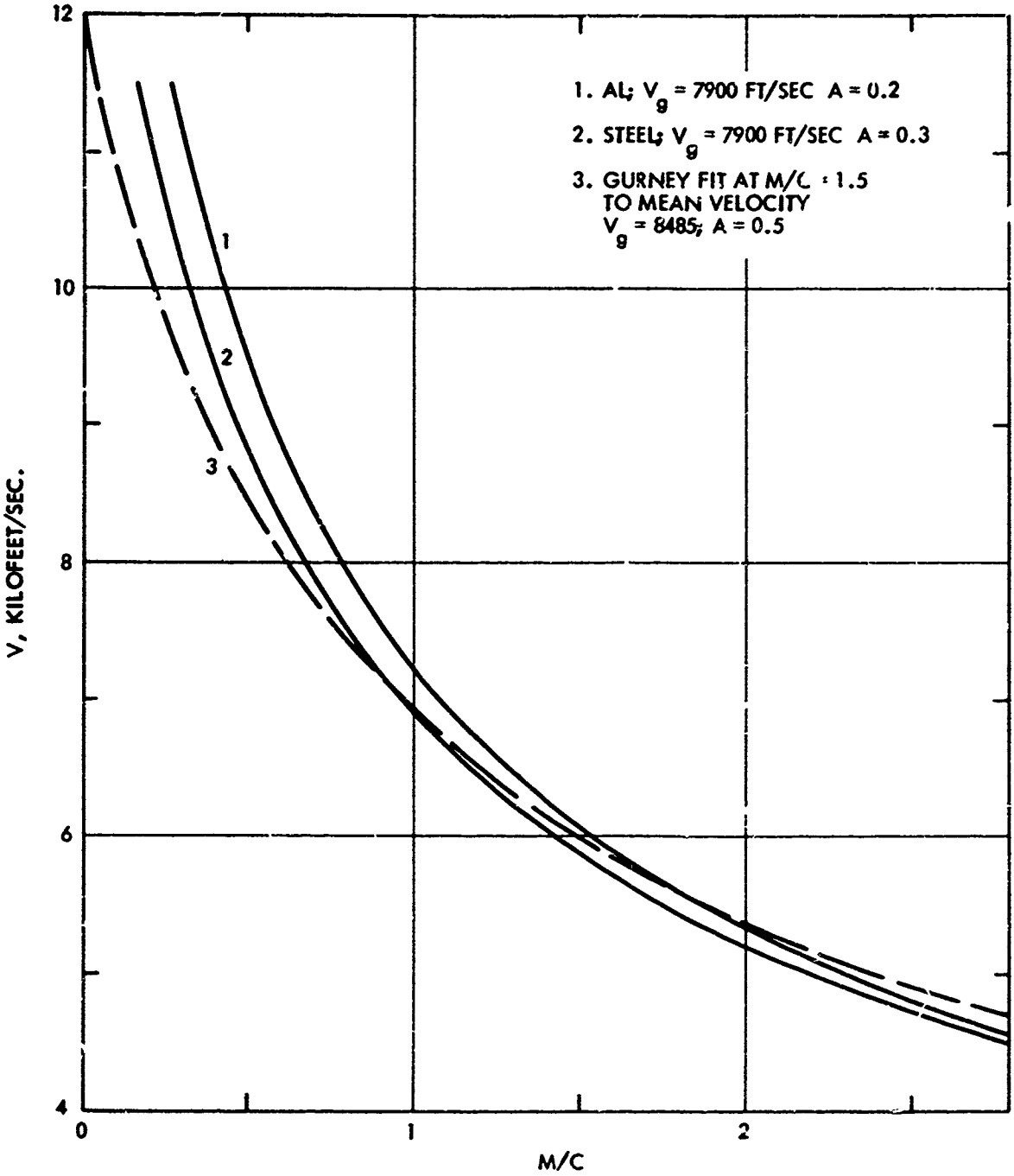


FIG. 6 COMPARISON OF FITTED CURVES FOR STEEL AND ALUMINUM WITH THE GURNEY FORMULA

V. DISCUSSION

5.1 Comparison with Other Case Expansion Work

Kury, et al.[17] at the Lawrence Livermore Laboratory, LLL, have developed a standard cylinder test in which the H. E. diameter is 1.00 inch (25.4 mm), length equal to 12 inches (305 mm). The cylinder is a copper tube 1.00 inch I. D. and 1.2044 O. D. (30.592 mm). Using the handbook value for the density of copper, 8.96 g/cc the M/C ratio for their experiment on Composition B (Grade A), 64% RDX, ρ_c = 1.717 g/cc is 2.341. The velocities V_a they quote at 5 and 19 mm of radial expansion are 1.39 and 1.63 mm/μsec respectively. Corrections for the Taylor angle and wall thickness give velocities of 1.449 and 1.631. We note that the net correction for a measurement at 19 mm is nil; the correction at 5 mm is about 4%. Using A = 0.5 the values for V_g are 8010 and 9020 ft/sec. The NOL result for the aluminum tubes measured at 40 mm should agree with the 19 mm value from LLL except for a small composition correction which we estimate to be 1.0%. Thus our corrected value for V_g falls below the LLL value. This may best be attributed to the short open ended cylinders in the NOL experiments. The LLL result has been confirmed within experimental error by both Hoskins, et al.[18] and Plausen and Mitchell[19].

5.2 Comparison with Fragment Range Results

One of the earliest experiments on fragment velocity was carried out at the Explosives Research Laboratory at Bruceton, Pa. in the early 1940's. The basic experiment used steel cylinders of 2.00 inch I. D. and 3.00 inch O. D. of length, 10 1/8 inches. A base plate was welded onto the tube. The velocity of a large group of fragments was measured at distances of 9 and 25 1/2 ft from the charge. Averages from these measurements have been reported in the Army Materiel Command's Engineer's Design Handbook Series, one source being the volume on Explosives Properties[20]. The data on page 48 of reference [20] for Composition B has been converted to velocity at the charge and a Gurney constant by a linear extrapolation to zero distance of the quoted velocities. The extrapolated velocity, 3074 ft/sec, and the M/C ratio, 5.84, give a Gurney constant of 7740 ft/sec.

The NOL work employs the same range technique as Bruceton but only a nine-foot observation post. In that work, described in reference [5] the tube is open at the ends and the thickness is 0.25 inches. Fifty-two fragments in five experiments were measured for Composition B

(M/C = 2.646) and the velocity averaged to obtain a value which, corrected for drag, gave a Gurney constant of 7880 ft/sec.* On re-examination of the data, we note that the observed speeds varied over a band of ±11%. Taking note of the fact that five experiments were run for each explosive it is appropriate to average the five or perhaps ten fastest fragment velocities to obtain a representative mean for maximum speed. The necessary data is obtained from histograms of fragment speed vs. number of fragments reported in reference [5]. The average for the five fastest fragments lead to V_g = 8540 ft/sec. The ten fastest fragments give V_g = 8410. Thus the five to ten fast fragment average comes into agreement with the cylinder expansion result 8485 ft/sec. (NOL range results are treated more fully in Section 6.)

It is evident from the results of the comparisons of experiments to determine Gurney constants that one cannot expect to use any given set of constants blindly. Consideration must be given to both the conditions under which the data is obtained and the conditions under which the data is to be employed. Some aspects of this problem will be discussed in the next section.

5.3 Problems Regarding Selection of Data

We define for cylinder expansion experiments the following: r_{io} and r_{oo}, initial inner and outer cylinder radii; and r_o, the outer radius after expansion. The LLL copper tube experiment[17,21] uses r_{io} = 12.7 mm. Data at $r_o - r_{oo}$ = 19 mm is usually employed to determine relative Gurney constants. For this value the expansion ratio, $(r_o - r_{oo})/r_{io}$ is about 1.5. The NOL fragment range experiments (see reference [5]) employed steel tubes with r_{io} = 25.4 mm. The cylinder wall probably reached an expansion ratio of about 0.8 when it fractured and ceased to be accelerated. Both experiments employed M/C ratios in the vicinity of 2 to 2.5.

We have the following comments to make regarding the use of the cylinder expansion data:

1. Relative radial velocities or relative kinetic energies of the metal tube at any r are insufficient for comparison of V_g. It is better to compute the V_g values

---

* It may be of interest to note that the low Bruceton V_g which is an average for 0.5 inch thick cases is reconciled with the NOL average in reference [5] for 0.25 inch wall if A is taken to be 0.3 rather than 0.5. This is additional evidence that the lower value of A is more appropriate in the Gurney formula.

from the data with a given value of A selected for the application. This will correct for the explosive kinetic energy due to density differences. The selected value of r should be scaled to the expected effective r value for metal fracture in the application.

2. If V_g is computed with A = 0.5 from data for which M/C is greater than 2.0, the resulting predictions at M/C less than 1.4 will be low. It is better to use a value of A of about 0.3 for steel cylinders as found in this work.

3. When Taylor angle and wall thickness corrections are made, the effect of tube density on tube velocity is of second order and can be ignored.

The following comments apply to range experiments, in particular, the NOL work (reference [5]).

1. The average velocities of fragments from open ended cylinders give Gurney constants which are too low for most applications. There is a bias because of the low speed of the fragments near the ends. It is also likely that some of the low speeds observed are a result of greater than average air drag on small fragments. More consistent constants can be obtained by selecting the fastest 20 to 50% of the fragments for data reduction. This will be shown in the next section.

2. As suggested for cylinder expansion data, a lower value of A is desirable when the data is to be applied at an M/C ratio much smaller than that at which the test data has been obtained.

3. Explosives with delayed reaction, e.g., aluminized, may not deliver their full effect in 25 to 50 mm inside diameters. Resulting Gurney constants can be too small.

Comments on Applications:

1. Consideration must be given to the nature of the case material and make-up. If we consider a seamless steel case of constant thickness as the norm, then the Gurney value for a more ductile case would be increased because it will fracture at a larger expansion ratio. If the case is made up of discrete preformed fragments the constant V_g will have to be decreased. If the case is of variable thickness one could get a first order prediction by computing velocities as a function of position by assuming the Gurney equation with a given value of A to apply to each section independently.

2. Some of the published Gurney constants are in need of revision. For example, the value for HMX in reference [9] is given as 10230 vice 8800 for Composition B. The ratio is 1.163. From LLL data we find the ratio of V_g's to be 1.097 or 5.5% lower. The values cited in reference [9] appear to be the result of; (a) applying the

Gurney formula for munitions where M/C is 1.0 or small while retaining the normal value 0.5 for A, and (b) neglect of explosive density in comparing HMX with Composition B. We were able to fit the data given in Figure (2) of reference [9] with the constants $A = 0.3$, $V_g = 7900$ at least as well as the author's fit. Our result has the advantage of fitting the experiments at large M/C equally well.

VI. REVISED GURNEY CONSTANTS

In this section we re-examine selected results for some of the explosive compositions which are of military interest. We convert the LLL results at 19 mm of radial expansion to Gurney constants with A = 0.5. We give the results of recalculation of Gurney constants from the NOL range data of Solem, et al.[5] also with A = 0.5. In this calculation only the fastest 20 or 50% of the fragments are used; the latter being considered as most representative of real munitions. We find this selection brings some of the anomalous results of reference [5] into better agreement with expected values. For example, the V_g calculated for pressed and cast TNT and also for Pentolite when based on the total sample differ significantly. This difference is reduced to the measuring error when the fastest 20 or 50% of fragments are used in the calculation. In addition we employ a scheme to be described for interpolating V_g from LLL data which permits one to obtain rather good estimates for compositions that have not been experimentally measured. Numerical results are carried to three or four significant figures to aid the user in making further calculations. The actual accuracy of any result is, at best, 2%.

6.1 Data Reduction Procedure

The LLL data at 19 mm is taken as the standard reference for comparison. The velocity data is reduced without correction for Taylor angle and mean momentum since this correction has been shown to be negligible at 19 mm. The NOL range data has been obtained from the histograms in the report by averaging the velocity for the fastest 20 and 50% of the fragments observed. A ratio of 1.043 has been used to convert the mean velocity at nine feet of travel to a mean value at the charge.

All data has been converted to Gurney constants for the usual Gurney value of A, 0.5. V_g can be converted to the value appropriate to any other A by the equation

$$V_g(A) = V_g(0.5)\sqrt{\frac{M/C + A}{M/C + 0.5}} \quad . \tag{8}$$

The effect of explosive loading density on V_g should be second order so that the proper value of C for the given loading density will take

care of the first order density effect on the velocity constant.

A first order correction for composition can be made by assuming the Gurney energy, $E_g = V_g^2/2$, is a linear function of the E_{gk} by the relation

$$E_g = \Sigma\, n_k E_{gk} \,, \tag{9}$$

which gives

$$V_g = \sqrt{\Sigma\, n_k V_{gk}^{\,2}} \,, \tag{10}$$

where n_k is the weight fraction of the kth constituent, E_{gk} is the corresponding Gurney energy. These formulae also permit an estimation of V_g for a mixture of an explosive and an inert binder. A comparison of LLL results for RDX and HMX with TNT suggests a slight modification for explosive mixtures given by the rule

$$V_g = \Sigma\, n_k V_{gk} \,. \tag{11}$$

We have applied equation (11) to obtain a V_g for RDX (by extrapolation) and other cyclotols from the TNT and RDX/TNT data in reference [17].

6.2 Results

Table 2 lists the comparative values of Gurney constants based on A = 0.5. The LLL data including interpolated constants is given for the 19 mm radial expansion. The NOL range data is then given in three columns for comparison. Based on the evidence that the NOL values computed for the fastest 50% of the fragments in the test are representative of fully cased munitions we have normalized the LLL data to Composition B taken as 8210 feet per second. The last two columns then give the best estimates of V_g for A = 0.5 and A = 0.3. The latter set is preferred for general application since it should apply about equally well at both low and high M/C ratios.

The present values of the Gurney constants are generally about 6% lower than those given by Kennedy. The difference in fragment velocity is not great, however, when the A = 0.3 value is used and the M/C ratio is small. We note that the present results for HMX vice RDX is more consistent with the fact that the explosives are homologues and have very nearly the same heat of formation per unit mass. Kennedy's value differed significantly because no correction was made for the higher density of HMX when the LLL data was normalized to a Gurney constant. The same reason applies to Kennedy's low value for Nitromethane. One unusual result of this work is the high constant found for H-6 when the fastest fragments in a NOL range test

TABLE 2 - $V_g(A)$ FOR THE GURNEY EQUATION, $V = V_g(A)/\sqrt{M/C+A}$; V, V_g IN FEET/SECOND

EXPLOSIVE	Source: LLL [17] @ 19 mm $V_g(0.5)$	NOL [5] ALL FRAGS $V_g(0.5)$	NOL [5] TOP 20% $V_g(0.5)$	NOL [5] TOP 50% $V_g(0.5)$	LLL NORMALIZED $V_g(0.5)$	BEST ESTIMATES A = 0.5 $V_g(0.5)$	BEST ESTIMATES A = 0.3 $V_g(0.3)$	TYPICAL H.E. DENSITY g/cc
HMX	9890				9080	9030	8760	1.89
PETN	9787				8990	8990	8670	1.76
RDX	9739 [a]				8940	8940	8620	1.79
TNT (Cast)	7910	6730	7450	7260	7260	7260	7010	1.60
TNT (Pressed)		6900	7390	7280	7260	7260	7010	
COMP. B (Grade A); 64/36	9014				8280	8280	7990	1.71
COMP. B; 60/40	8940 [a]	7880	8450	8210	[8210]*	8210	7920	1.70
CYCLOTOL; 77/23	9318				9560	9550	9210	1.75
CYCLOTOL; 75/25	9280 [a]	7850	8895	8490	8520	8500	8200	1.72
OCTOL; 78/22	9449				8680	8680	8370	1.82
PENTOLITE; (50/50) (Cast)	8849 [a]	7140	8185	7960	8130	8100	7820	1.69
(Pressed)		7610	8200	7980	8130	8100	7820	
NITROMETHANE	8031				7380	7380	7120	1.14
H-6; RDX/TNT/Al/Wax (47/31/22/5)		7710	8840	8380		8380	8080	1.71

a Interpolated results

* Reference value (see text)

are used as the basis for analysis. The Gurney value is 2% higher than for Composition B. In an LLL type copper case expansion experiment, Plausen and Mitchell[19] found a value 11% lower. The discrepancy between these two results may be due to a diameter effect. Aluminum is known to contribute to case expansion in the LLL tests when in combination with some explosives (see reference [21]) and the effect of the aluminum reaction appears to come late in the expansion of the one-inch charges. The two-inch charge may see the reaction of aluminum at an earlier time after detonation. It would be of value to perform additional cylinder expansion tests on H-6 and other aluminized explosives using two and four-inch scaled experiments to verify or disprove our present results.

The parameters listed in Table 2 are considered to be applicable to solid, unscored cylindric steel cases. Other cased munitions will require a correction factor as previously discussed.

VII. CONCLUDING REMARKS

We have shown that the Gurney equation in its original form is derived as an application of an earlier idea of Lagrange to an explosive case system having cylindrical symmetry. Although the actual situation for a detonating explosive differs markedly from the model employed, the equation with an additional minor modification is found to be a useful approximation for predicting fragment velocity. Though we have not discussed other symmetries it is likely that a similar modification of the basic equation would also lead to a better approximation to experimental results.

A more complete analysis of fragmentation is possible by using 2-D computation. In an attempt to reduce the computer time for a 2-D calculation, Sternberg at this laboratory has generated a code in which the metal case is treated as an incompressible fluid. This code retains the inertial effects of the case but avoids the need for calculation of the wave motions within it. Since the waves in the case are likely to have little effect on the final velocity the results would be of significant utility. A series of basic boundary value problems have been run with this code in connection with this task. The results are to be reported in a follow-up report by Sternberg.

REFERENCES

1. R. W. Gurney, "The Initial Velocity of Fragments from Bombs, Shells, and Grenades", BRL Report No. 405, (Sep 1943)

2. R. W. Gurney, (same title), BRL Report No. 635, (Mar 1947)

3. L. H. Thomas, "Theory of the Explosion of Cased Charges of Simple Shape", BRL Report No. 475, (Jul 1944)

4. T. E. Sterne, "A Note on the Initial Velocities of Fragments from Warheads", BRL Report No. 648, (Sep 1947)

5. A. D. Solem, N. Shapiro, and B. N. Singleton, Jr., "Explosives Comparison for Fragmentation Effectiveness", NAVORD Report 2933, (Aug 1953)

6. A. D. Solem and B. N. Singleton, Jr., "Rapid Expansion of Metal Cylinders Under Explosive Loading I.", NAVORD Report 2768, (Feb 1953)

7. A. K. Aziz, H. Hurwitz, and H. M. Sternberg, "Energy Transfer to a Rigid Piston Under Detonation Loading", Physics of Fluids, 4,380, (1961)

8. I. G. Henry, "The Gurney Formula and Related Approximations," for High Explosive Deployment of Fragments", Report No. Pub-189, Hughes Aircraft Corp. (Apr 1967)

9. D. R. Kennedy, "The Elusive $\sqrt{2E}$", Presented at 21st Annual Bomb and Warhead Section Meeting, Am. Ordnance Assn., Picatinny Arsenal, 22 Apr 1969, (Defense Technology Labs., FMC Corp., Santa Clara, CA.

10. M. Defourneaux, "Transport of Energy in Combustions and Detonations with Confinement", Astronautica Acta, 17, 609, (1972)

11. J. Roth, "Correlation of the Gurney Formula for the Velocity of Explosively-Driven Fragments with the Detonation Parameters of the Driver Explosive", Laboratory Technical Report 001.71, Stanford Research Institute, (8 Apr 1971)

12. Courant and Friedrichs, "Supersonic Flow and Shock Waves"; Interscience Publ., Inc., New York (1948)

13. Love and Pidduck, "Lagrange Ballistic Problem", Phil. Trans. Royal Soc. (London), 22, 167, (1922)

14. J. Corner, "Theory of Interior Ballistics of Gases", Chapter 9, John Wiley and Sons, N. Y. (1950)

15. S. J. Jacobs, "The Energy of Detonation", NAVORD Report 4366, (Sep 1956)

16. M. Wilkins, "Use of One and Two Dimensional Hydrodynamic Machine Calculations in High Explosives Research", 4th Symposium on Detonation, NOL (1965), ACR-126, ONR, U.S. Gov't. Printing Office, (p. 519, Fig.4)

17. J. Kury, H. Hornig, E. Lee, L. McDonnel, D. Ornellas, M. Finger, F. Strange, and M. Wilkins, "Metal Acceleration by Chemical Explosives, Ibid. p. 3

18. N. Hoskins, J. Allen, W. Bailey, J. Lethaby, and I. Skidmore, "The Motion of Plates and Cylinders Driven by Detonation Waves at Tangential Incidence", Ibid. p. 14

19. R. Plausen and C. Mitchell, "Cylinder Expansion (Gurney Constant) and Warhead Fragmentation", NWC TP 5240, Naval Weapons Center, (Sep 1972)

20. Engineering Design Handbook, Explosives Series, Properties of Explosives, AMCP 706-177, Army Mat'l Command (Jan 1971)

21. M. Finger, H. Hornig, E. Lee, and J. Kury, "Metal Acceleration by Composite Explosives", Fifth Symposium on Detonation, ACR-184, Office of Naval Research, U. S. Gov't. Printing Office, p. 137, (Aug 1970)

APPENDIX A: CALCULATION OF MEAN CYLINDER VELOCITY

A logical mean in cylinder expansion velocity should be based on the total momentum divided by the mass. Then we obtain the correct fragment velocity after a case fractures since the total momentum must be conserved. This method of getting a mean velocity implies that some of the kinetic energy appears as a vibration within a fragment after breakup of the cylinder. We adopt the notation, Figure (A-1):

r_o = outer radius of cylinder at time, t.

r_{oo} = outer radius of cylinder at time, zero.

r_i = inner radius of cylinder at time, t.

r_{io} = inner radius of cylinder at time, zero.

x = radius of an element of cylinder at time, t.

x_o = radius of an element of cylinder at time, zero.

We make the following assumptions:

1. The cylindrical case expands radially as an incompressible fluid, density ρ = constant.

2. Mean velocity is given by the integrated momentum divided by the case mass.

3. Case mass per unit length of case = M.

4. Velocity, V_a, of outer case surface = dr_o/dt.

5. $V = V(x)$ is the velocity of a case element initially located at x_o.

By conservation of mass

$$M = \pi\rho(r_{oo}^2 - r_{io}^2),$$

$$= \pi\rho(r_o^2 - r_i^2) \; ; \qquad \text{(A-1)}$$

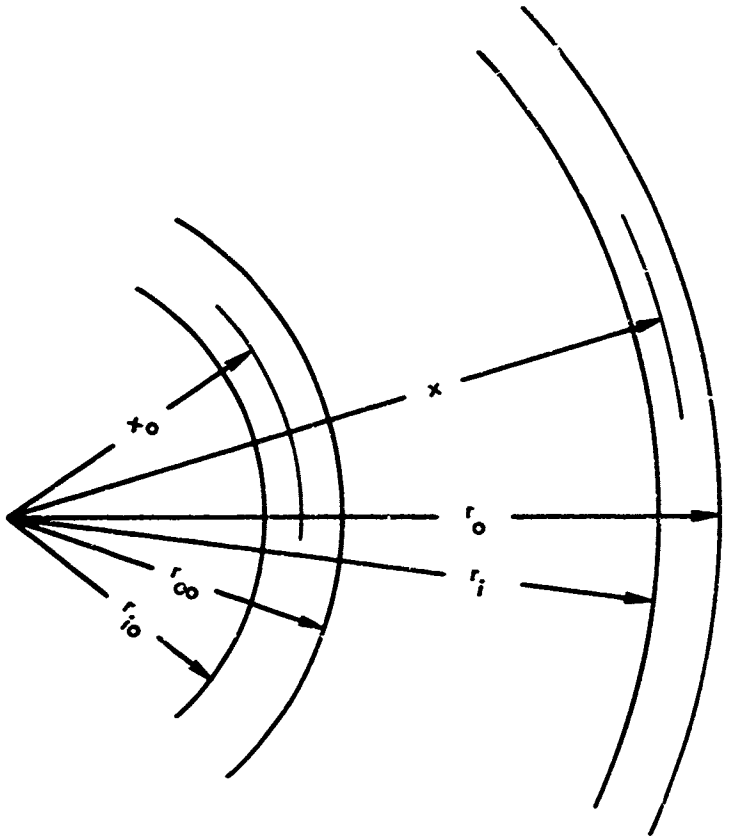


FIG. A-1 NOTATION FOR CALCULATION OF MEAN CYLINDER VELOCITY

then

$$r_i^2 = r_o^2 - (r_{oo}^2 - r_{io}^2) \ . \qquad \text{(A-2)}$$

The mass between any element of the case and the outer surface is also a constant so we may write

$$x^2 = r_o^2 - (r_{oo}^2 - x_o^2) = r_o^2 - \text{constant}. \qquad \text{(A-3)}$$

Differentiating equation (A-3) gives the velocity at radius x,

$$dx/dt = (r_o/x)dr_o/dt. \qquad \text{(A-4)}$$

The momentum at radius x is

$$d(MV) = 2\pi\rho(xdx)(r_o/x)dr_o/dt \ . \qquad \text{(A-5)}$$

Integration of equation (A-5) from r_i to r_o gives

$$MV = 2\pi\rho r_o(r_o - r_i)dr_o/dt \ . \qquad \text{(A-6)}$$

The mean velocity is then obtained from equations (A-6) and (A-1)

$$\bar{V} = \frac{2r_o}{r_o + r_i}\frac{dr_o}{dt} \ . \qquad \text{(A-7)}$$

Calling $dr_o/dt = V_a$ we write

$$\bar{V} = (1 + \varepsilon)\ V_a \ ,$$

where

$$\varepsilon = (r_o - r_i)/(r_o + r_i) \ . \qquad \text{(A-8)}$$

r_i is given as a function of r_o by equation (A-2).

Equation (A-8) shows very simply that the mean velocity must be greater than the apparent velocity of the outer surface and that the correction is given by half the metal thickness divided by the mean radius. In the LLL standard cylinder expansion experiment the correction amounts to 1.6% for measurements at 19 mm and 5.5% for measurements at 5 mm. The correction is greater for several of the NOL cylinder expansion experiments when aluminum cases were employed.