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INVESTIGATION OF THE ACCURACY OF LINEAR PISTON THEORY WHEN APPLIED TO CYLINDRICAL SHELLS

by

Hans Krumhaar

California Institute of Technology

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Graduate Aeronautical Laboratories

California Institute of Technology

Pasadena, California

Piston theory was introduced into aeroelasticity in the linearized form by Ashley and Zartarian as a handy tool in 1956, see ref. 1. This theory furnishes an approximation for the aerodynamic pressure acting on a slightly deformed flat plate in a supersonic airstream. The linearized piston theory is widely used in the investigation of the flutter of flat panels. Because of the lack of appropriate approximations for the aerodynamic pressure acting on a vibrating cylindrical shell, linear piston theory is also used in investigations of the flutter of cylindrical shells, see refs. 3 and 4.

There are doubts about the accuracy of using the linear piston theory for cylindrical shells. This opinion was strengthened by recent studies at Caltech, see refs. 4, 5, and 8. In the following, a short survey of an investigation of the accuracy of the linear piston theory when applied to cylindrical shells is given. For further details the reader is referred to ref. 6.

We consider an infinitely long circular cylindrical shell of radius R, which is exposed externally to a uniform airstream parallel to the cylinder axis. The Mach number, the density and the velocity of sound of the undisturbed airstream are denoted by M, ρ_0 and a_0 respectively. Let a cylinder coordinate system x, r, θ be chosen, where the positive direction of the x-axis coincides with the positive direction of the airstream. (I.e., a negative Mach number means that the airstream is moving in the negative x-direction.) The shell is assumed to be slightly deformed by a harmonically oscillating standing wave of the form

$$w = w_0 \cos(n\theta) s(x) e^{i\omega t}, \quad \omega \geq 0 \tag{1a}$$

with

$$s(x) = \sin(\nu x) \quad \text{or} \quad s(x) = \cos(\nu x), \ \nu \gtrless 0 . \tag{1b}$$

The radial displacement of the shell, w, is measured positive in the outside direction. By proper superposition of the formulas, set forth by Leonhard and Hedgepeth in ref. 7, one obtains an exact expression for the aerodynamic pressure Δp, which is originated by the shell vibration (1). Two of the main parameters of these expressions are

$$M_1 = M - \frac{\omega}{\nu a_0} \ ; \ M_2 = M + \frac{\omega}{\nu a_0} \tag{2}$$

These exact expressions involve fractions of cylinder functions and their first derivatives. Applying the well-known asymptotic expansions for cylinder functions (see ref. 2) we obtain for these fractions the following asymptotic expansions for $\xi \to \infty$ (we restrict ξ to real, positive values):

$$\frac{H_n^{(1)}(\xi)}{H_n^{(1)'}(\xi)} = -i - \tfrac{1}{2}\xi^{-1} - i\left(\frac{n^2}{2} - \frac{3}{8}\right)\xi^{-2} - \left(n^2 - \frac{3}{8}\right)\xi^{-3} + O(\xi^{-4}); \tag{3a}$$

$$\frac{H_n^{(2)}(\xi)}{H_n^{(2)'}(\xi)} = \overline{\left(\frac{H_n^{(1)}(\xi)}{H_n^{(1)'}(\xi)}\right)} \tag{3b}$$

$$\frac{K_n(\xi)}{K_n'(\xi)} = -1 + \tfrac{1}{2}\xi^{-1} + \left(\frac{n^2}{2} - \frac{3}{8}\right)\xi^{-2} - \left(n^2 - \frac{3}{8}\right)\xi^{-3} + O(\xi^{-4}) \tag{3c}$$

$$\text{for } \xi \to \infty.$$

Here and in the following O denotes the Landau-symbol. (The barred symbol in (3b) denotes the conjugate complex term.)

The asymptotic expansions (3) lead to asymptotic expansions for the aerodynamic pressure Δp. These expansions allow an investigation of the accuracy of the linear piston theory approximation

$$\Delta p^{**} = a_0 \rho_0 \left\{ a_0 M \frac{\partial w}{\partial x} + \frac{\partial w}{\partial t} \right\} \tag{4}$$

when applied to cylindrical shells. Furthermore improved approximations can be obtained from the asymptotic expansions of Δp.

For

$$|M_1| > 1; \quad |M_2| > 1 \tag{5}$$

one obtains finally, separating the linear piston theory expression (4),

$$\Delta p = a_0 \rho_0 \left\{ a_0 M \frac{\partial w}{\partial x} + \frac{\partial w}{\partial t} \right\} + \sum_{m=0}^{3} \frac{1}{R^m} \left(F_m w + F_m^* \frac{\partial w}{\partial x} \right) +$$

$$+ w \left[O\left(\frac{1}{\nu^3 R^4} \frac{1}{M_1^3 [1 - \frac{1}{M_1^2}]^{5/2}} \right) + O\left(\frac{1}{\nu^3 R^4} \frac{1}{M_2^3 [1 - \frac{1}{M_2^2}]^{5/2}} \right) \right] + \tag{6a}$$

$$+ \frac{\partial w}{\partial x} \left[O\left(\frac{1}{\nu^4 R^4} \frac{1}{M_1^3 [1 - \frac{1}{M_1^2}]^{5/2}} \right) + O\left(\frac{1}{\nu^4 R^4} \frac{1}{M_2^3 [1 - \frac{1}{M_2^2}]^{5/2}} \right) \right]$$

(1)

with

$$F_0 = \tfrac{1}{2} i \nu a_0^2 \rho_0 \left(- \frac{M_1}{[1 - \frac{1}{M_1^2}]^{\frac{1}{2}}} + \frac{M_2}{[1 - \frac{1}{M_2^2}]^{\frac{1}{2}}} - \frac{2\omega}{\nu a_0} \right) ; \tag{6b}$$

$$F_0^* = \tfrac{1}{2} a_0^2 \rho_0 \left(\frac{M_1}{[1 - \frac{1}{M_1^2}]^{\frac{1}{2}}} + \frac{M_2}{[1 - \frac{1}{M_2^2}]^{\frac{1}{2}}} - 2M \right) ; \tag{6c}$$

(1) Here and in the following we always choose the positive square-root.

$$F_1 = -\tfrac{1}{4}\, a_0^2 \rho_0 \left(\frac{1}{1 - \frac{1}{M_1^2}} + \frac{1}{1 - \frac{1}{M_2^2}} \right) ; \tag{6d}$$

$$F_1^* = \frac{i}{4\nu}\, a_0^2 \rho_0 \left(- \frac{1}{1 - \frac{1}{M_1^2}} + \frac{1}{1 - \frac{1}{M_2^2}} \right) ; \tag{6e}$$

$$F_2 = \frac{i}{4\nu} \left(n^2 - \tfrac{3}{4}\right) a_0^2 \rho_0 \left(- \frac{1}{M_1 \left[1 - \frac{1}{M_1^2}\right]^{3/2}} + \frac{1}{M_2 \left[1 - \frac{1}{M_2^2}\right]^{3/2}} \right) ; \tag{6f}$$

$$F_2^* = \frac{1}{4\nu^2} \left(n^2 - \tfrac{3}{4}\right) a_0^2 \rho_0 \left(\frac{1}{M_1 \left[1 - \frac{1}{M_1^2}\right]^{3/2}} + \frac{1}{M_2 \left[1 - \frac{1}{M_2^2}\right]^{3/2}} \right) ; \tag{6g}$$

$$F_3 = - \frac{1}{2\nu^2} \left(n^2 - \tfrac{3}{8}\right) a_0^2 \rho_0 \left(\frac{1}{M_1^2 \left[1 - \frac{1}{M_1^2}\right]^2} + \frac{1}{M_2^2 \left[1 - \frac{1}{M_2^2}\right]^2} \right) ; \tag{6h}$$

$$F_3^* = \frac{i}{2\nu^3} \left(n^2 - \tfrac{3}{8}\right) a_0^2 \rho_0 \left(- \frac{1}{M_1^2 \left[1 - \frac{1}{M_1^2}\right]^2} + \frac{1}{M_2^2 \left[1 - \frac{1}{M_2^2}\right]^2} \right) . \tag{6i}$$

The terms $\frac{1}{R^m} F_m w$ and $\frac{1}{R^m} F_m^* \frac{\partial w}{\partial x}$, $m = 0, \ldots, 3$, in (6a), we call "correction terms", the following terms are referred to as "remainder." It is obvious that the remainder tends to zero as soon as one of the parameters M, R, ω, and $|\nu|$ tends to infinity.

An inspection of eqs. (6) leads to the following conclusions:

$$\Delta p - \left\{ \Delta p^{**} - \frac{a_0^2 \rho_0}{2R} w \right\} \to 0 \quad \text{for} \quad |M| \to \infty. \tag{7}$$

$$\Delta p - \left[\Delta p^{**} - \tfrac{1}{2} a_0 \rho_0 \left\{ \frac{1}{M_1 M_2} + \left| \binom{-\frac{1}{2}}{2} \right| \frac{\frac{\nu a_0}{\omega} (M_2^3 - M_1^3)}{M_1^3 M_2^3} + \right. \right.$$

$$\left. \left. + \left| \binom{-\frac{1}{2}}{3} \right| \frac{\frac{\nu a_0}{\omega} (M_2^5 - M_1^5)}{M_1^5 M_2^5} + \ldots \right\} \frac{\partial w}{\partial t} + \right. \tag{8}$$

$$+ \tfrac{1}{2} a_0^2 \rho_0 M \left\{ \frac{1}{M_1 M_2} + \left|\binom{-\frac{1}{2}}{2}\right| \frac{M_2^3 + M_1^3}{M M_1^3 M_2^3} + \left|\binom{-\frac{1}{2}}{3}\right| \frac{M_2^5 + M_1^5}{M M_1^5 M_2^5} + \ldots \right\} \frac{\partial w}{\partial x} \Big] \to 0$$

$$\text{for } R \to \infty \qquad \begin{matrix}(8)\\(\text{Cont'd})\end{matrix}$$

The latter result is obtained after F_0 and F_0^* have been expanded in power series of $\frac{1}{M_1}$ and $\frac{1}{M_2}$ and after some regrouping. Obviously the terms $M_2^\mu - M_1^\mu$, $\mu = 3, 5, \ldots$, contain the factor $\frac{\omega}{\nu a_0}$. The terms $M_2^\mu + M_1^\mu$, $\mu = 3, 5, \ldots$, contain the factor M. The relations (7) and (8) lead to

$$\Delta p - \Delta p^{**} \to 0 \text{ for } |M| \to \infty;\ R \to \infty . \tag{9}$$

Further one obtains

$$\Delta p - \left\{ \Delta p^{**} - \frac{a_0^2 \rho_0}{2R} w \right\} \to 0 \text{ for } \omega \to \infty. \tag{10}$$

Hence

$$\Delta p - \Delta p^{**} \to 0 \text{ for } R \to \infty;\ \omega \to \infty. \tag{11}$$

In order to investigate the limiting process $|\nu| \to \infty$ we restrict M by $|M| > 1$. Thanks to this inequality the inequalities (5) are satisfied for sufficiently large values of $|\nu|$. Then we learn

$$\Delta p - \Big[\Delta p^{**} + a_0 \rho_0 \left(\frac{|M|}{[M^2-1]^{\frac{1}{2}}} \frac{M^2-2}{M^2-1} - 1 \right) \frac{\partial w}{\partial t} + a_0^2 \rho_0 \left(\frac{M|M|}{[M^2-1]^{\frac{1}{2}}} - M \right) \frac{\partial w}{\partial x} - \frac{a_0^2 \rho_0}{2R} \frac{M^2}{M^2-1} w \Big] \to 0 \text{ for } |\nu| \to \infty. \tag{12}$$

From (12) we arrive at

$$\Delta p - \left\{ \Delta p^{**} - \frac{a_0^2 \rho_0}{2R} w \right\} \to 0 \quad \text{for} \quad |M| \to \infty, \; |\nu| \to \infty, \tag{13}$$

$$\Delta p - \Delta p^{**} \to 0 \quad \text{for} \quad |M| \to \infty, \; |\nu| \to \infty, \; R \to \infty. \tag{14}$$

These investigations demonstrate that for the case $|M_1| > 1$; $|M_2| > 1$ the linear piston theory expression Δp^{**} can be considered as a first order approximation of the aerodynamic pressure Δp. Nevertheless the replacement of Δp^{**} by the approximation

$$\Delta p^{***} = \Delta p^{**} - \frac{a_0^2 \rho_0}{2R} w = a_0 \rho_0 \left\{ a_0 M \frac{\partial w}{\partial x} + \frac{\partial w}{\partial t} - \frac{a_0}{2R} w \right\} \tag{15}$$

is suggested as a first order step of improvement for the application to cylindrical shells. Improved approximations can be obtained from eqs. (6). For large values of n especially the correction terms of the order $(1/R)^2$ and $(1/R)^3$ in eqs. (6) can become quite significant. For flutter investigations it is often necessary to consider values of n up to the order of 20.

Further investigations along this line (see ref. 6) disclose that in the cases a.) $|M_1| < 1$; $|M_1| < 1$, b.) $|M_1| < 1$; $|M_2| > 1$; c.) $|M_1| > 1$; $|M_2| < 1$ the linear piston theory expression Δp^{**} can no longer be considered as a first order approximation for the aerodynamic pressure Δp.

References:

1. Ashley, H., — Piston Theory - A New Aerodynamic Tool for the Aeroelastician. Journal of the Aeronautical Sciences, Vol. 23, pp. 1109-1118, Dec. 1956.

2. Erdelyi, A., Magnus, W., Oberhettinger, F., Tricomi, G. T. — Higher Transcendental Functions. (Bateman Manuscript Project) Vol. 2. McGraw-Hill Book Company, Inc., New York-Toronto-London, 1953.

3. Fung, Y. C. — A Summary of the Theories and Experiments on Panel Flutter. AFOSR TN60-224, California Institute of Technology, Guggenheim Aeronautical Laboratory, May 1960.

4. Fung, Y. C. — Some Recent Contributions to Panel Flutter Research. GALCIT Structural Dynamics Report No. SM 62-53, December 1962.

5. Krumhaar, H. — Supersonic Flutter of a Cylindrical Shell of Finite Length in an Axisymmetrical Mode. AFOSR 1574, California Institute of Technology, Guggenheim Aeronautical Laboratory, Oct. 1961.

6. Krumhaar, H. — The Accuracy of Applying Linear Piston Theory to Cylindrical Shells. GALCIT Structural Dynamics Report No. SM 62-50, California Institute of Technology, February 1963.

7. Leonhard, R. W., Hedgepeth, J. M. — On Panel Flutter and Divergence of Infinitely Long Unstiffened and Ring-Stiffened Thin-Walled Circular Cylinders. NACA Rep. 1302, 1957.

8. Stearman, R. Lock, M. Fung, Y. C. — Ames Tests on the Flutter of Cylindrical Shells. GALCIT Structural Dynamics Report No. SM 62-37, California Institute of Technology, December 1962.