SPACE SCIENCES LABORATORY

AEROPHYSICS SECTION

THEORETICAL PERFORMANCE FOR MHD GENERATORS
UTILIZING NON-EQUILIBRIUM IONIZATION IN
PURE ALKALI METAL VAPOR SYSTEMS

by

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Magnetohydrodynamic Power Generation

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ABSTRACT

Each member of the alkali metal series has been investigated to determine the best system for a closed cycle MHD generator utilizing magnetically induced non-equilibrium ionization. MHD generator performances have been calculated for one-dimensional steady state flows with constant Mach number in which the dimer concentration is neglected. The calculations are for a generator with segmented electrodes.

Potassium appears to be the best choice among the alkali metal series for a 1 megawatt generator operating at total temperatures near 1600°K and total pressures around 20 psia. This choice is based upon considerations of the calculated generator performance, the vapor pressure, and the equilibrium concentration of dimer in the vapor.

The calculations show that the performance of an MHD generator utilizing a non-equilibrium condition of the electrons is primarily dominated by the elastic electron-neutral collision cross section. If the elastic electron collision cross section in potassium is 4×10^{-14} cm^2, then magnetic fields of 130,000 gauss will be required to obtain the non-equilibrium conditions necessary to obtain power densities near 1 kw/cm^3 when the Mach number is around 2. However, if the cross section in potassium is near 1×10^{-15} cm^2, (as recently reported in the literature) then magnetic fields of 12,000 gauss will produce power densities near 1 kw/cm^3 at a Mach of 2.

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Introduction

In order to operate a closed loop MHD generator with high power density at temperatures compatible with existing "high temperature materials", a non-equilibrium condition of the electrons must be created in the plasma. Based upon the theory of magnetically induced non-equilibrium ionization (Ref. 5), calculations have been performed for each member of the alkali metal series. The calculations are based upon a flow which is one-dimensional, steady state and in which the dimer concentration is negligible.

These calculations together with considerations of the vapor pressure and equilibrium dimer concentration indicate that potassium is the best choice of working fluids among the alkali metal series.

Equations:

For the case of segmented electrodes, the electron temperature is related to the total gas temperature by the following equation, (Ref. 5):

$$\frac{T_e}{T_o} = \frac{1 + \frac{\gamma}{3} (\omega\tau)^2 M_1^2 \delta^{-1} (1-K)^2}{1 + \frac{1}{2}(\gamma - 1) M_1^2} \quad \ldots \quad (1)$$

where

T_e is the electron temperature.

T_o is the total gas temperature.

γ is the ratio of heat capacities, (c_p / c_v).

ω is the electron cyclotron frequency.

τ is the average time between electron-non-electron collisions.

M_1 is Mach number.

δ is the loss factor which is close to unity for vapors with monatomic particles.

K is the loading factor which is the ratio of the load voltage to the open-circuit voltage.

The electron temperatures are calculated for the case where the Mach number is constant, and where $\delta = 1$ and $K = 0$.

The electron density is determined from the Saha equation based upon the electron temperature:

$$\frac{n_e^2}{n_n - n_e} = \frac{(2\pi m_e k T_e)^{3/2}}{h^3} e^{-\left(\frac{E_0}{kT_e}\right)} \qquad \ldots \quad (2)$$

where

n_e is the electron density.

n_n is the neutral particle density.

m_e is the electron mass.

k is the Boltzmann constant.

h is the Plank constant.

E_o is the first ionization potential of the vapor.

T_e is the electron temperature.

The neutral particle density is determined from the ideal gas law:

$$n_n = P_1 / kT_1 \qquad \ldots \quad (3)$$

where

n_n is the neutral particle density.

P_1 is the static pressure.

T_1 is the static temperature.

k is the Boltzmann constant.

The static pressure is calculated from the isentropic flow equation:

$$P_1 = \frac{P_o}{\left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma - 1}}} \qquad \ldots \quad (4)$$

where

P_1 is the static pressure.

P_o is the total pressure.

γ is the ratio of heat capacities, (c_p / c_v).

M_1 is the Mach number.

The static temperature is calculated from the isentropic flow equation:

$$T_1 = T_o \left(\frac{P_1}{P_o}\right)^{\frac{\gamma - 1}{\gamma}} \qquad \ldots \quad (5)$$

where

T_1 is the static gas temperature.

T_o is the total gas temperature.

P_1 is the static gas pressure.

P_o is the total gas pressure.

γ is the ratio of heat capacities, (c_p / c_v).

The conductivity due to the ions is related to the electron temperature, the Debye shielding length, and the average impact parameter by the following equation, (Ref. 10):

$$\sigma_i = \frac{1.51 \times 10^{-2}\, T_e^{3/2}}{\ln \Lambda} \qquad \ldots \qquad (6)$$

where

σ_i is the conductivity due to the ions.

T_e is the electron temperature.

Λ is the ratio of the Debye shielding length to the average impact parameter.

The ratio of the Debye shielding length to the average impact parameter is related to the electron density and electron temperature by the following equation, (Ref. 10):

$$\Lambda = \frac{3}{2 Z Z_1 e^3} \left(\frac{k^3 T_e^3}{\pi n_e} \right)^{1/2} \qquad \ldots \qquad (7)$$

where

Z is the number of electronic charges associated with the ion.

Z_1 is the number of electronic charges associated with the electron.

e is the electron charge.

k is the Boltzmann constant.

T_e is the electron temperature.

n_e is the electron density.

Since the case of singly ionized particles is considered, $Z = Z_I = 1$.

The conductivity due to the neutral particles is determined from the scalar conductivity equation in which the electrons are assumed to have a Maxwellian distribution:

$$\sigma_n = \left(\frac{e^2}{\left(\frac{8 m_e k}{\pi} \right)^{1/2}} \right) \left(\frac{n_e}{T_e^{1/2} (n_n - n_e) Q_{en}} \right) \quad (8)$$

where

- e is the electron charge.
- m_e is the electron mass.
- k is the Boltzmann constant.
- n_e is the electron density.
- T_e is the electron temperature.
- n_n is the neutral particle density.
- n_e is the electron density.
- Q_{en} is the electron-neutral elastic collision cross section.

The calculations are performed with a constant electron-collision cross section, thus neglecting the Ramsauer effect.

The plasma electrical conductivity is determined from the relation:

$$\sigma_p^{-1} = \sigma_i^{-1} + \sigma_n^{-1} \quad (9)$$

The gas velocity is determined from the sonic velocity equation:

$$U_1 = \left(\frac{\gamma R T_1}{m_n}\right)^{1/2} M_1 \qquad \cdots \quad (10)$$

where

U_1 is the gas velocity.

γ is the ratio of heat capacities, (c_p / c_v).

R is the universal gas constant.

T_1 is the static temperature.

m_n is the atomic weight of the vapor.

M_1 is the Mach number.

The relation between the magnetic field, the electron density, the electron cyclotron frequency, the average time between electron-non-electron collisions, and the plasma electrical conductivity, is given by the following equation, (Ref. 1):

$$B(1-K) = \left(\frac{n_e\, e\, c\, \omega \tau}{\sigma_p}\right) \quad \ldots \quad (11)$$

where

B is the magnetic field strength.

K is the loading factor which is the ratio of the load voltage to the open circuit voltage.

n_e is the electron density.

e is the electron charge.

c is the velocity of light.

ω is the electron cyclotron frequency.

τ is the average time between electron-non-electron collisions.

σ_p is the plasma electrical conductivity.

The power density in a segmented electrode generator is given by:

$$W\left(\frac{1-K}{K}\right) = \sigma_p \, U_1^2 \left[B(1-K)\right]^2 \qquad \cdots \quad (12)$$

where

- W is the power density.
- K is the loading factor which is the ratio of the load voltage to the open-circuit voltage.
- σ_p is the plasma electrical conductivity.
- U_1 is the gas velocity.
- B is the magnetic field strength.

The calculations are performed for the case where viscous boundary layer formations are negligible, and U_1 is constant.

Sample Results:

MHD generator performances are presented for pure lithium, sodium, potassium, rubidium, and cesium, under the following conditions:

Total pressure of 10, 20, and 30 psia

Total temperature of 1400, 1600, 1800, and 2000°K

Mach number of 0.8, 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0

Electron temperature range of static gas temperature up to 4000°K.

Sample results are shown in figures 1 - 13.

Discussion:

Theoretical performance calculations are presented for lithium, sodium, potassium, rubidium, and cesium, in figures 1-5 respectively. These results are based upon the largest elastic electron-neutral collision cross sections reported in the literature. In the case of rubidium, a linear variation between potassium and cesium was assumed. A comparison of alkali metal vapor systems is presented in figure 6. A complete comparison at a total pressure of 20 psia and a total temperature of 1400°K was impossible because of the low vapor pressure of lithium. These results show that the decrease in the ionization potential of the heavier elements is greatly overshadowed by the increase in atomic weight and especially by the increase in electron-neutral collision cross section. Lithium is undesirable because of the high operating temperatures required. Rubidium and cesium are undesirable because of their high atomic weights and large electron collision cross sections. Either potassium or sodium would be acceptable working fluids; however, the amount of equilibrium dimer in the vapor is much greater in sodium than in potassium, as shown in figure 17. Consequently, potassium appears to be the best choice among the alkali metals as an MHD working fluid.

Recent data (References (8) and (9)) indicate the collision cross sections for potassium and cesium to be much lower than has usually been assumed. If this is true, the MHD generator performance is greatly enhanced, as shown in figures 7, 8, and 9. The influence of the magnetic field upon the plasma electrical conductivity is shown in figure 10. The influence of the Mach number upon the performance is shown in figure 11. The effect of electron-potassium collision cross section is shown in figure 12. The relationship between the plasma electrical conductivity and the electron temperature is shown in figure 13. Thus, if for potassium $Q_{en} = 10^{-15}$ cm^2, power densities of 1 kw/cm^3 can be expected for potassium with a total gas temperature of 1400°K, a total pressure of 20 psia, a Mach number of 2, and a magnetic field around 64,000 gauss with a loading factor of 0.75.

For convenience, the details of the calculation procedure are presented in appendix A. Vapor pressure curves of the alkali metals are presented in appendix B. The vapor pressure data presented herein were obtained from R. E. Honig (Ref. 3). Honig's vapor pressure data review, the most comprehensive collection currently available, is based primarily upon data reported in References (4), (6), and (11). Curves of the equilibrium dimer concentration in pure alkali metal vapors are presented in appendix C. As expected, the relative equilibrium concentrations of dimer are also reflected by the heats of dimer dissociation at 0°K, Ref. (2).

Alkali Metal	Heat of Dimer Dissociation at 0°K
Li_2	1.02 e.v.
Na_2	.73 e.v.
K_2	.514 e.v.
Rb_2	.49 e.v.
Cs_2	.45 e.v.

The computer routine, used to calculate the MHD generator performance, is presented in appendix D.

APPENDIX A

Calculation Procedure

Calculation Procedure

A. The quantities to be specified apriori are:

T_o, the total temperature of gas, ^oK

P_o, the total pressure of gas, atm

M_1, the Mach number in the MHD generator, dimensionless

T_e, the electron temperature at the entrance of MHD generator ^oK

γ, the ratio of heat capacities, (c_p/c_v), dimensionless

E_o, the first ionization potential of the gas, electron volts

Q_{en}, the electron-neutral collision cross section, cm^2

The quantities γ, E_o and Q_{en} are determined from the particular gas chosen.

B. Calculation of T_1, (the static temperature of neutral particles in the MHD generator, ^oK).

The quantity, T_1, is determined from the steady isentropic flow equation:

$$T_1 = T_o (P_1/P_o)^{\frac{\gamma - 1}{\gamma}} , \ ^oK \qquad \ldots \qquad (13)$$

where:

T_o is in ^oK

P_1 is in atm.

P_o is in atm.

γ is dimensionless

C. Calculation of P_1, (the static pressure of gas in the MHD generator, atm.).

The quantity, P_1, is determined from the steady isentropic flow equation:

$$P_1 = \frac{P_o}{\left(1 + \frac{\gamma - 1}{2} M_1^{\,2}\right)^{\gamma/\gamma - 1}}, \text{ atm.} \qquad \cdots \quad (14)$$

where:

P_o is in atm.

γ is dimensionless

M_1 is dimensionless

D. Calculation of n_n, (the neutral particle density, # of neutrals/cm^3)

The quantity n_n at the entrance of the MHD generator is determined from the ideal gas law:

$$n_n = \frac{P_1}{k\ T_1}$$

since

$k = 1.38042 \times 10^{-16}$ ergs/ °K, (the Boltzmann Constant)

1 erg = 1 dyne-cm

1 atm. = 1.013246×10^{6} dynes/cm^3

Thus

$$n_n = \left(\frac{1 \text{ atm.}}{1.38042 \times 10^{-16} \frac{\text{ergs}}{{}^\circ K} \; {}^\circ K} \right) \left[\frac{\text{erg}}{\text{dyne - cm}} \right]$$

$$\times \left[\frac{1.013246 \times 10^{6} \text{ dynes}}{\text{cm}^2 \text{ - atm.}} \right]$$

$$n_n = \frac{0.7340 \times 10^{22} P_1}{T_1} , \quad \frac{\# \text{ of particles}}{\text{cm}^3} \qquad \ldots \quad (15)$$

where:

P_1 is in atm.

T_1 is in °K

E. Calculation of n_e, (the electron density, # of electrons/cm^3)

The electron density is determined from the Saha equation based upon the electron temperature:

$$\frac{n_e^{\,2}}{n_n - n_c} = \frac{\left(2\pi m_e k T_e\right)^{3/2}}{h^3} \quad e^{-E_o / k T_e}$$

Since

m_e = 9.1085×10^{-28} grams (electron mass)

k = 1.38042×10^{-16} ergs/°K (the Boltzmann Constant)

h = 6.6252×10^{-27} erg-sec (the Plank Constant)

Thus

$$\frac{\left(2\pi m_e k T_e\right)^{3/2}}{h^3} = \frac{\left[(2\pi)(9.1085 \times 10^{-28}\ \text{gr.}) \times 1.38042 \times 10^{-16}\ \frac{\text{ergs}}{^{\circ}\text{K}}\ {^{\circ}\text{K}}\right]^{3/2}}{(6.6252 \times 10^{-27}\ \text{erg-sec.})^3}$$

$$= 2.4146 \times 10^{+15}\ \frac{1}{\text{erg}^{3/2} - \text{sec}^3}$$

$$= 2.4146 \times 10^{15}\ \frac{1}{\text{cm}^3}$$

Also since

$$1\ \text{e.v.} = 1.60207 \times 10^{-19}\ \text{joules}$$

$$1\ \text{joule} = 10^7\ \text{ergs}$$

Thus

$$\frac{E_o}{k T_e} = \frac{(1\ \text{e.v.})\left[1.60207 \times 10^{-19}\ \frac{\text{joules}}{\text{e.v.}}\right]\left[10^7\ \frac{\text{ergs}}{\text{joule}}\right]}{\left(1.38042 \times 10^{-16}\ \frac{\text{ergs}}{^{\circ}\text{K}}\ {^{\circ}\text{K}}\right)}$$

$$= 11{,}606\ \frac{E_o}{T_e}\ \text{, dimensionless}$$

where

E_o is in e.v.

T_e is in $^{\circ}$K

Finally

$$\frac{n_e^2}{n_n - n_e} = 2.4146 \times 10^{+15}\, T_e^{3/2}\, e^{-11,606\, E_o/T_e}, \quad \frac{\#\text{ of particles}}{cm^3} \tag{16}$$

where

n_e is # of electrons/cm^3

n_n is # of neutrals/cm^3

T_e is in ^oK

E_o is in electron volts

F. <u>Calculation of Λ</u>, (the ratio of the Debye Shielding length to the average impact parameter, dimensionless)

The quantity Λ is determined from the following Equation (Ref. 10):

$$\Lambda = \frac{3}{2ZZ_1 e^3}\left(\frac{k^3\, T_e^3}{\pi\, n_e}\right)^{1/2}$$

Since $Z = Z_1 = 1$ in this case

$e = 4.80288 \times 10^{-10}$ e.s.u. (electron charge)

$k = 1.38042 \times 10^{-16}$ erg/^oK

1 erg $= 1$ gr - cm^2/sec^2 = 1 dyne-cm

1 e.s.u. = (dyne-cm^2)$^{1/2}$

Thus

$$\Lambda = \frac{\frac{3}{2}\left(1.38042 \times 10^{-16} \frac{\text{erg}}{^{\circ}\text{K}} \; ^{\circ}\text{K}\right)^{3/2}}{(4.80288 \times 10^{-10} \text{ e.s.u.})^3 \left[\left(\frac{\text{dyne-cm}^2}{\text{e.s.u.}}\right)^{1/2}\right]^3 \pi^{1/2}} \left(\frac{T_e^{\;3/2}}{n_e^{\;1/2}}\right)$$

$$= 1.2388 \times 10^4 \frac{T_e^{\;3/2}}{n_e^{\;1/2}}, \text{ dimensionless} \qquad \cdots \quad (17)$$

where

T_e is in °K

n_e is in # of electrons/cm^3

G. Calculation of σ_i, (the conductivity due to the ions, mhos/meter)

The quantity σ_i is calculated from the following equation (Ref. 10):

$$\sigma_i = \frac{1.51 \times 10^{-2} \; T_e^{\;3/2}}{\ln \Lambda} \quad \frac{\text{mho}}{\text{meter}} \qquad \cdots \quad (18)$$

where

T_e is in °K

Λ is dimensionless.

H. Calculation of σ_n, (the conductivity due to the neutrals; mhos/meter)

The quantity σ_n is determined from the scalar conductivity equation (Ref. 1):

$$ {}_{n} = \frac{n_e \, e^2 \, \tau}{m_e} $$

where

$$ \tau = \frac{1}{\overline{V}\,(n_n - n_e)\; Q_{en}} $$

and

$$ \overline{V} = \left(\frac{8}{\pi} \; \frac{kT_e}{m_e} \right)^{1/2} $$

Thus

$$ \sigma_n = \left(\frac{e^2}{m_e \left(\frac{8}{\pi} \; \frac{k}{m_e} \right)^{1/2}} \right) \left(\frac{n_e}{T_e^{1/2} \, (n_n - n_e)\; Q_{en}} \right) $$

Since

e	=	4.80288×10^{-10} e.s.u.
m_e	=	9.1085×10^{-28} grams
k	=	$1.38042\mathrm{x}10^{-16}$ ergs/$^{\circ}$K
1 erg	=	gr. cm^2/ sec^2
1 e.s.u.	=	gr$^{1/2}$ cm $^{3/2}$/ sec
cm/sec	=	(1/9 x 10" ohm)

Thus

$$\sigma_n = \left(\frac{(4.80288 \times 10^{-10}\ \text{e.s.u.})^2 \left(\frac{n_e}{T_e}^{1/2} (n_n - n_e)\, Q_{en}\right)}{(9.1085 \times 10^{-28}\ \text{gr}) \left(\frac{8}{\pi} \frac{1.38042 \frac{\text{erg}}{^{\circ}\text{K}}\ ^{\circ}\text{K}}{9.1085 \times 10^{-28}\ \text{gr}}\right)^{1/2}} \right)$$

$$\times \left[\frac{\text{gr}^{1/2}\ \text{cm}^{3/2}/\text{sec}}{\text{e.s.u.}}\right]^2 \left[\frac{1}{9 \times 10^{11}\ \frac{\text{ohm-cm}}{\text{sec}}}\right] \left[\frac{10^2\ \text{cm}}{\text{meter}}\right]$$

$$\sigma_n = \frac{4.525 \times 10^{-8}\ n_e}{T_e^{1/2} (n_n - n_e)\, Q_{en}}, \quad \frac{\text{mho}}{\text{meter}}. \qquad \ldots \quad (19)$$

where

T_e is in $^{\circ}$K

n_n is # of neutrals/cm ^{3}

n_e is # of electrons/cm^3

Q_{en} is in cm^2

I. Calculation of σ_p, (the plasma electrical conductivity, mhos / meter)

The plasma electrical conductivity is determined from the following equation:

$$\sigma_p^{-1} = \sigma_i^{-1} + \sigma_n^{-1}$$

or

$$\sigma_p = \frac{\sigma_i\, \sigma_n}{\sigma_i + \sigma_n}, \quad \frac{\text{mhos}}{\text{meter}} \qquad \ldots \quad (20)$$

where

σ_i is in mhos/meter

σ_n is in mhos/meter

J. Calculation of $\omega\tau$, (dimensionless)

The quantity $\omega\tau$ necessary to sustain the given T_e is determined from the equation as shown below (Ref. 5):

$$\omega\tau = \left(\frac{\frac{T_e}{T_1} - 1}{\frac{\gamma}{3} M_1^{\,2}} \right)^{1/2} \quad \text{, dimensionless}$$

where

T_e is in $^{\circ}$K

T_1 is in $^{\circ}$K

γ is dimensionless

M_1 is dimensionless

K. Calculation of B(1-K) (the magnetic field, gauss)

The quantity B(1-K) (the magnetic field necessary to sustain the desired when the generator is short circuited), is calculated from the equation

$$B(1-K) = \left(\frac{n_e\, ec\, \omega\tau}{\sigma_p} \right)$$

The above equation is derived from the following simple relations (Ref. 1):

$$\omega = \frac{e\, B(1-K)/c}{m_e}$$

and

$$\sigma_p = \frac{n_e\, e^2\, \tau}{m_e}$$

Since

$e = 4.80288 \times 10^{-10}$ e.s.u. (electron charge)

$c = 2.998 \times 10^{10}$ cm/sec (speed of light)

1 e.s.u. $= gr^{1/2}\ cm^{3/2} / sec$

1 gauss $= gr^{1/2} / cm^{1/2} - sec$

1 sec $= 9 \times 10''$ ohm-cm

Thus

$$B(1-K) = \frac{\left(\frac{\#\text{ of electrons}}{cm^3}\right)\left(\frac{4.80288 \times 10^{-10}\text{e.s.u.}}{\text{electron}}\right)\left(2.998 \times 10^{10}\frac{cm}{sec}\right)\frac{n_e\,\omega\tau}{\sigma_p}}{\frac{\text{mho}}{\text{meter}}}$$

$$\times \frac{\left[\frac{\frac{gr^{1/2}cm^{3/2}}{sec}}{\text{e.s.u.}}\right]\left[\frac{\text{gauss}}{gr^{1/2}/cm^{1/2}-sec}\right]}{\left[\frac{1\text{ meter}}{10^2\text{ cm}}\right]\left[\frac{1/\text{ohm}}{\text{mho}}\right]\left[9 \times 10^{11}\ \frac{\text{ohm-cm}}{sec}\right]}, \text{ gauss}$$

Thus

$$B(1-K) = 1.600 \times 10^{-9}\ \frac{n_e\,\omega\tau}{\sigma_p} \qquad \dots \qquad (22)$$

where

K is dimensionless

n_e is # of electrons/cm^3

$\omega\tau$ is dimensionless

σ_p is in mhos/meter

L. Calculation of U_1, (the gas velocity at the entrance of the MHD generator, meters/second)

The quantity U_1 is determined from the sonic velocity equation:

$$U_1 = \left(\frac{\gamma R T_1}{m_n}\right)^{1/2} M_1$$

Since

$R = 1.987$ cal/gr. mole (the gas constant)

1 calorie = 4.186 joules

1 joule = 10^7 ergs

1 erg = 1 gr-cm^2/sec^2

Thus

$$U_1 = \left(1.987 \frac{\text{calories}}{\text{gr. moles } ^{o}\text{K}} \quad \frac{\text{gr. moles}}{\text{gr}} \quad {}^{o}\text{K}\right)^{1/2}$$

$$\times \left(\left[\frac{4.186 \text{ joules}}{\text{calorie}}\right]\left[\frac{10^7 \text{ ergs}}{\text{joule}}\right]\left[\frac{1 \text{ meter}^2}{10^4 \text{ cm}^2}\right]\left[\frac{\frac{\text{gr-cm}^2}{\text{sec}^2}}{\text{erg}}\right]\right)^{1/2}$$

$$U_1 = 91.20 \left(\frac{\gamma T_1}{m_n}\right)^{1/2} M_1, \quad \frac{\text{meters}}{\text{sec}} \qquad \ldots \qquad (23)$$

where

γ is dimensionless

T_1 is in $^{\circ}K$

m_n is at. wt., gr./gr. atom

M_1 is dimensionless

M. Calculation of W, (the MHD generator power density, kilowatts/cm^3)

$$W = \underline{E} \cdot \underline{j} = -E_y\, i_y = -\sigma_p (E_y - U_1 B) E_y$$

Thus

$$W = \sigma_p (1-K)\, K\; U_1 B^2$$

or

$$W \frac{(1-K)}{K} = \sigma_p\, U_1 \left[B(1-K)\right]^2$$

Since

$$10^4 \text{ gauss} = 1 \text{ weber/meter}^2$$

$$1 \text{ weber} = 1 \text{ volt second}$$

Thus

$$W \frac{(1-K)}{K} = \sigma_p\, U_1^{\,2} \left[B(1-K)\right]^2 \quad \frac{\text{mho-meter}^2\ \text{gauss}^2}{\text{meter - sec}^2}$$

$$\left[\frac{\text{weber/meter}^2}{10^4 \text{gauss}}\right]\left[\frac{\text{volt-sec.}}{\text{weber}}\right]\left[\frac{1/\text{ohm}}{\text{mho}}\right]\left[\frac{\text{watt-ohm}}{\text{volt}^2}\right]$$

$$\times \left[\frac{1 \text{ meter}^3}{10^6 \text{ cm}^3}\right]\left[\frac{\text{KW}}{10^3 \text{ watts}}\right]$$

$$W \frac{(1-K)}{K} = 10^{-17}\, \sigma_p\, U_1^{\,2} \left[B(1-K)\right]^2 \quad \frac{\text{KW}}{\text{cm}^3} \qquad \ldots \quad (24)$$

where

K is dimensionless

σ_p is in mhos/meter

U_i is in meters/second

B is in gauss

APPENDIX B

Vapor Pressure Curves for Alkali Metals

Vapor Pressure of Alkali Metals

The vapor pressure data presented herein were obtained from R E. Honig (Ref. 3). Vapor pressure curves are shown in figures 14-15.

APPENDIX C

Equilibrium Dimer Concentration in Pure

Alkali Metal Vapors

Equilibrium Dimer Concentration in Saturated Pure Alkali Metal Vapors:

Consider the following reaction:

$$M_2 \rightleftharpoons 2M \quad \cdots \quad (25)$$

where M is an alkali metal. As usual, the equilibrium constant for the homogeneous chemical reaction is defined as

$$K_p = \frac{(P_M)^2}{(P_{M_2})} \quad \cdots \quad (26)$$

where P_M is the partial pressure of monomer in the vapor

P_{M_2} is the partial pressure of dimer in the vapor

The mole fraction of dimer is given by the ratio

$$X_2 = \frac{P_{M_2}}{P_o} \quad \cdots \quad (27)$$

where P_o is the static pressure of the vapor

Consequently,

$$X_2 = \frac{(2P_o + K) - \sqrt{K_p^{\,2} + 4K_p\,P_o}}{2\,P_o} \quad \cdots \quad (28)$$

The equilibrium constant is obtained from the well-known equation

$$K_p = e^{\left[2\left(\frac{-F^o}{RT_o}\right)_{X_1} - \left(\frac{F^o}{RT_o}\right)_{X_2}\right]} \quad \cdots \quad (29)$$

where

F^o	is the absolute free energy
R	is the universal gas constant
T	is the static temperature
X_1	refers to the monomer
X_2	refers to the dimer

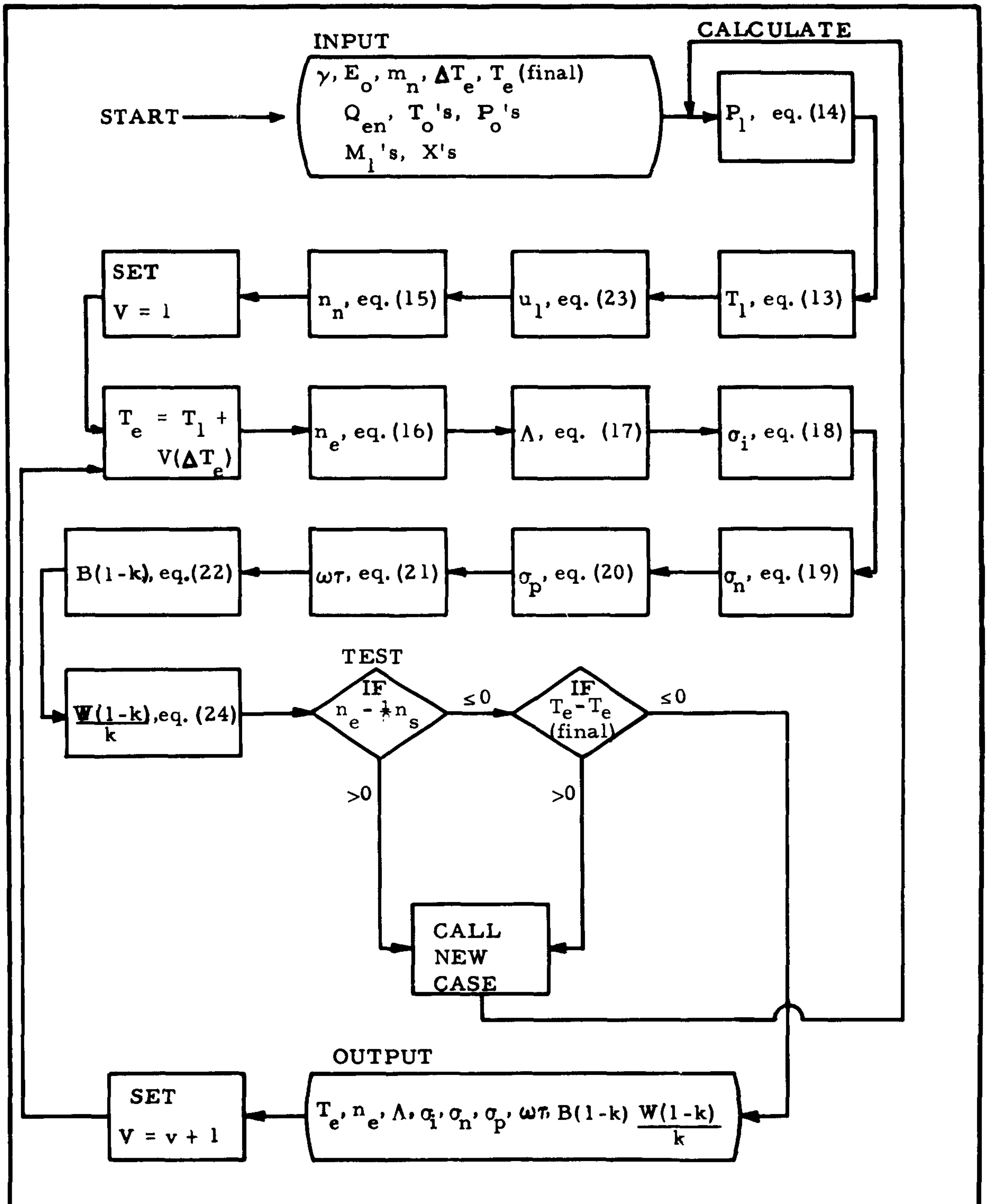
Thus a knowledge of the static pressure, static temperature and free energy permits the dimer concentration to be calculated from equation (28).

Such calculations have been performed by Meisl and Shapiro (Ref. 7). It should be noted that the free energies of the alkali metals were obtained from Reference 1. Dimer curves are shown in figures 16-18.

APPENDIX D

Program Flow Diagram

PROGRAM FLOW DIAGRAM*



*The numbers in parentheses refer to the equations as numbered in Appendix A.

APPENDIX E

Physical Properties

Physical Properties Used for MHD

Performance Calculations

Element	γ	E_o e.v.	Q_{en} (1) cm^2	m_n gr/gr. atom
Lithium	5/3	5.363	2×10^{-14}	6.94
Sodium	5/3	5.12	3×10^{-14}	22.997
Potassium	5/3	4.318	4×10^{-14} 3×10^{-15} 1×10^{-15}	39.1
Rubidium	5/3	4.159	4.7×10^{-14}	85.48
Cesium	5/3	3.87	5.3×10^{-14} 3.6×10^{-15}	132.91

(1) Calculations were performed for old literature values of Q_{en} and also for the latest literature values, (see references 4 and 5).

APPENDIX F

Table of Nomenclature

(Listed in the Order of Appearance)

T_e	Electron Temperature, °K
T_o	Total gas temperature, °K
γ	Ratio of heat capacities, ($\frac{C_p}{c_v}$), dimensionless
ω	Electron cyclotron frequency, 1 / sec.
τ	Average time between electron-nonelectron collisions, sec.
M_1	Mach number in MHD generator, dimensionless
δ	Loss Factor, to account for inelastic collisions, dimensionless
K	Loading Factor, ratio of load voltage to open circuit voltage, dimensionless
n_e	Electron density, # of electrons / cm^3
n_n	Neutral particle density, # of neutrals / cm^3
E_o	First ionization potential of vapor, e.v.
P_1	Static pressure of gas in MHD generator, atm.
T_1	Static temperature of neutral particles in MHD generator, °K
P_o	Total gas pressure, atm.
T_o	Total gas temperature, °K
σ_i	Electrical conductivity due to ions, mhos/meter
Λ	Ratio of Debye shielding length to the average impact parameter, dimensionless
σ_n	Electrical conductivity due to neutrals, mhos/meter
σ_p	Scalar electrical conductivity of plasma, mhos/meter

U_1	Velocity of plasma in MHD generator, meters/second
m_n	Atomic weight of neutrals, gr./gr. atom
W	Power density in MHD generator, kw/cm^3
Q_{en}	Electron-neutral collision cross section, cm^2
V	Mean electron speed, (Maxwellian distribution assumed), cm/sec.
$\underline{E}$	Electric field vector $= \underline{i}\, E_x + \underline{j} E_y + \underline{k}\, E_z$
$\underline{J}$	Current vector $= \underline{i}\, J_x + \underline{j}\, J_y + \underline{K}\, J_z$
M	Any member of the alkali metal series
K_p	Equilibrium constant defined by equation 26, atm
P_M	Partial pressure of monomer, atm
P_{M_2}	Partial pressure of dimer, atm
F^o	Absolute free energy, cal./gr. mole
k =	1.38042×10^{-16} ergs/°K (Boltzmann's constant)
m_e =	9.1085×10^{-28} grams (electron mass)
h =	6.6252×10^{-27} erg-sec. (Plank's constant)
e =	4.80288×10^{-10} e.s.u. (electron charge)
c =	2.998×10^{10} cm/sec. (speed of light)
R =	1.987 cal./gr. mole - °K (gas constant)

Conversion Units

1 .tm	=	1.013246×10^{6} dynes/cm^{2}
1 erg	=	1 dyne - cm
1 e.v.	=	1.60207×10^{-19} joules
1 joule	=	10^{7} ergs
1 e.s.u.	=	$(\text{dyne - cm}^{2})^{1/2}$
1 dyne	=	gr.-cm/sec^{2}
1 ohm	=	$\left(\frac{1}{9 \times 10^{11}}\right)$ sec/cm
1 gauss	=	gr.$^{1/2}$ / cm$^{1/2}$ - sec.
1 calorie	=	4.186 joules
1 weber/meter^{2}	=	10^{4} gauss
1 weber	=	1 volt-sec.

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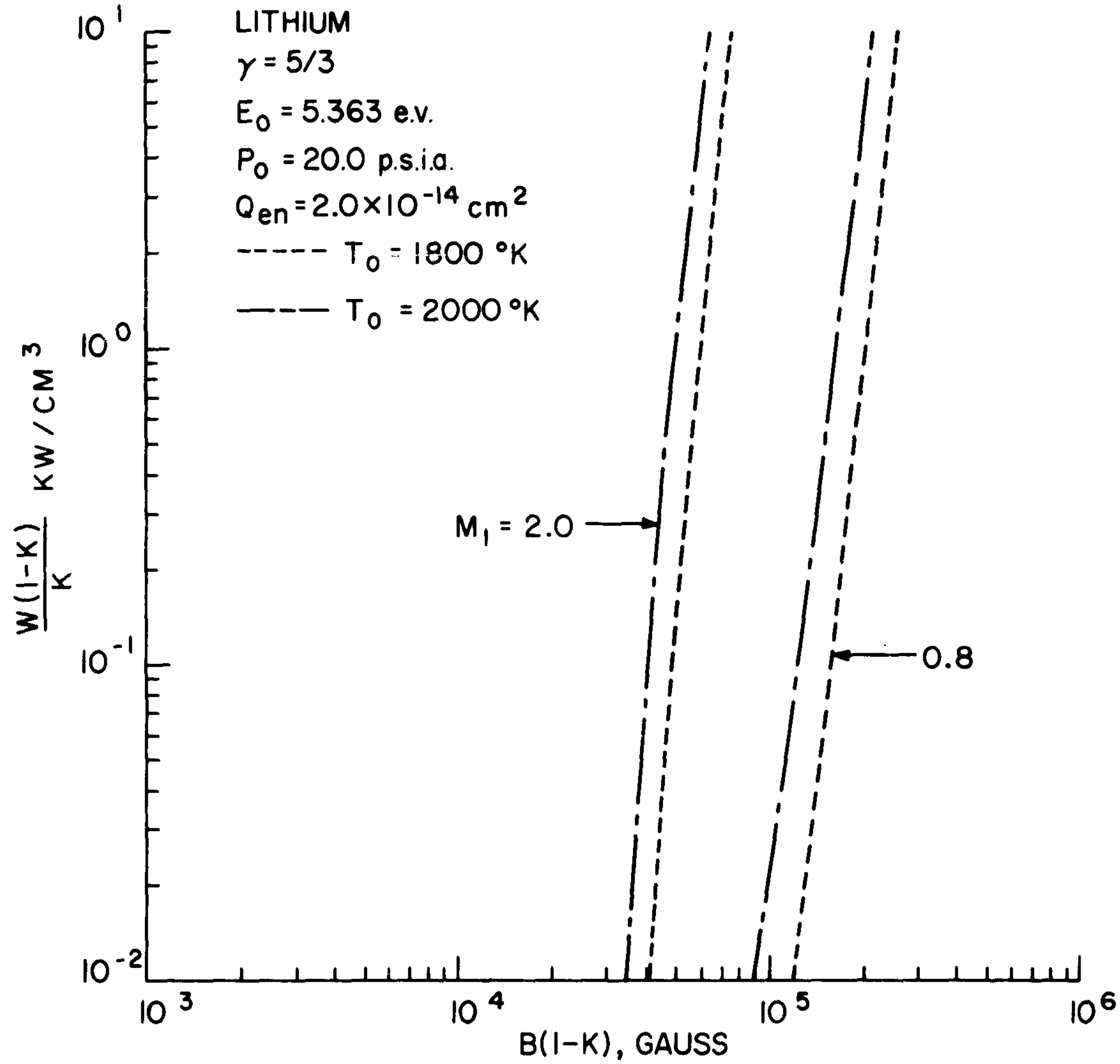

Figure 1. Power Density for Lithium Vapor, $Q_{en} = 2.0 \times 10^{-14}$ cm^2

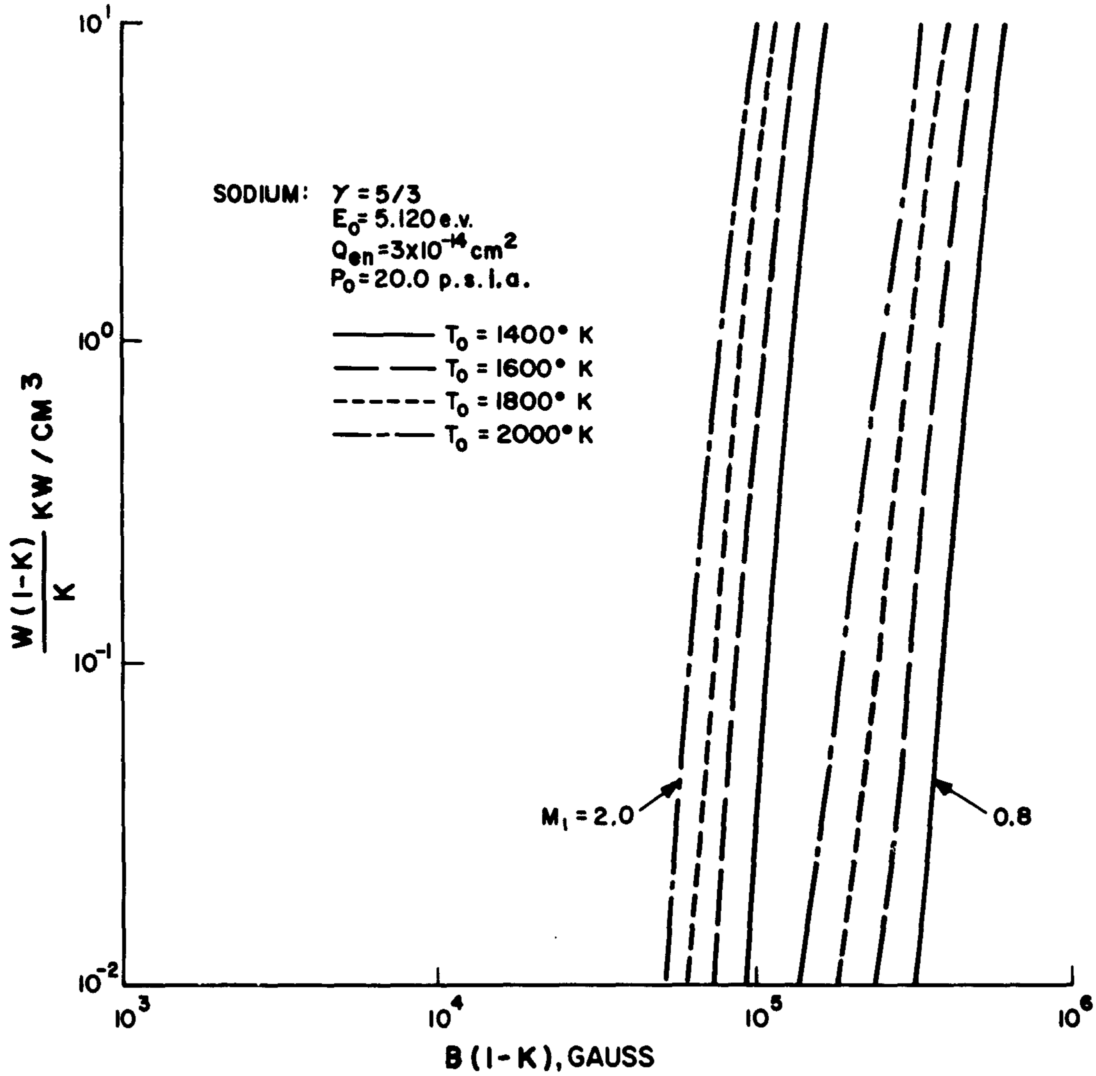

Figure 2. Power Density for Sodium, $Q_{en} = 3 \times 10^{-14}$ cm^2

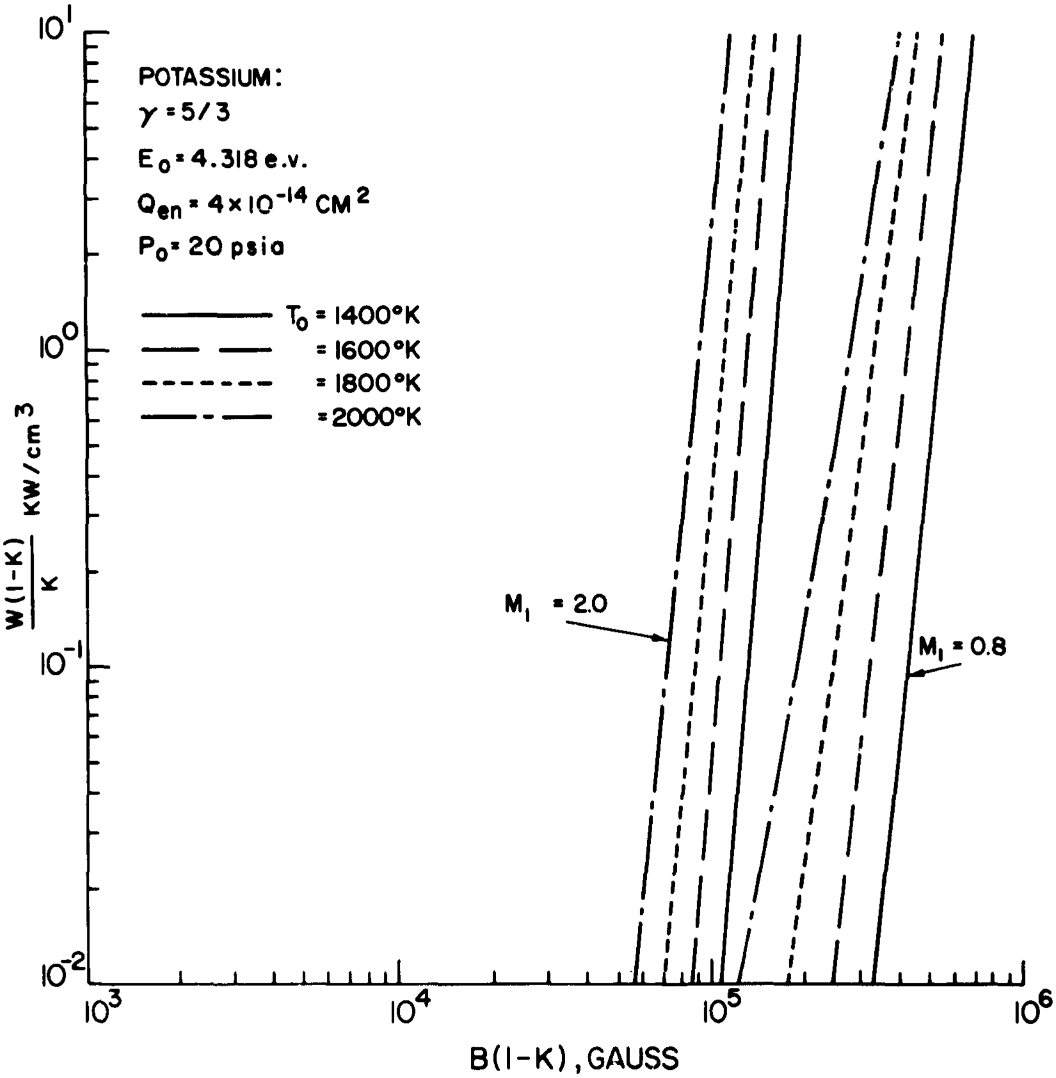

Figure 3. Power Density for Potassium Vapor, $Q_{en} = 4 \times 10^{-14}$ cm^2

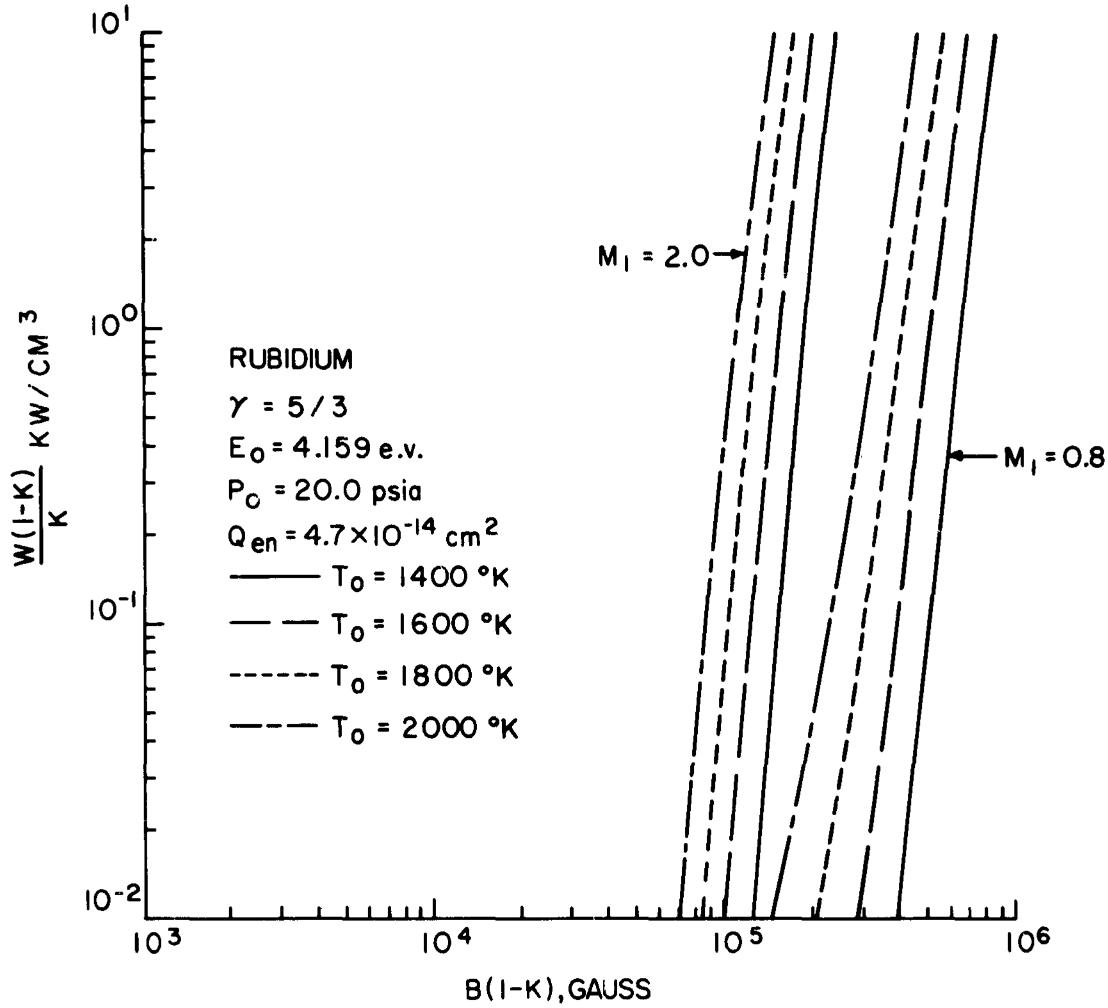

Figure 4. Power Density for Rubidium, $Q_{en} = 4.7 \times 10^{-14}$ cm^2

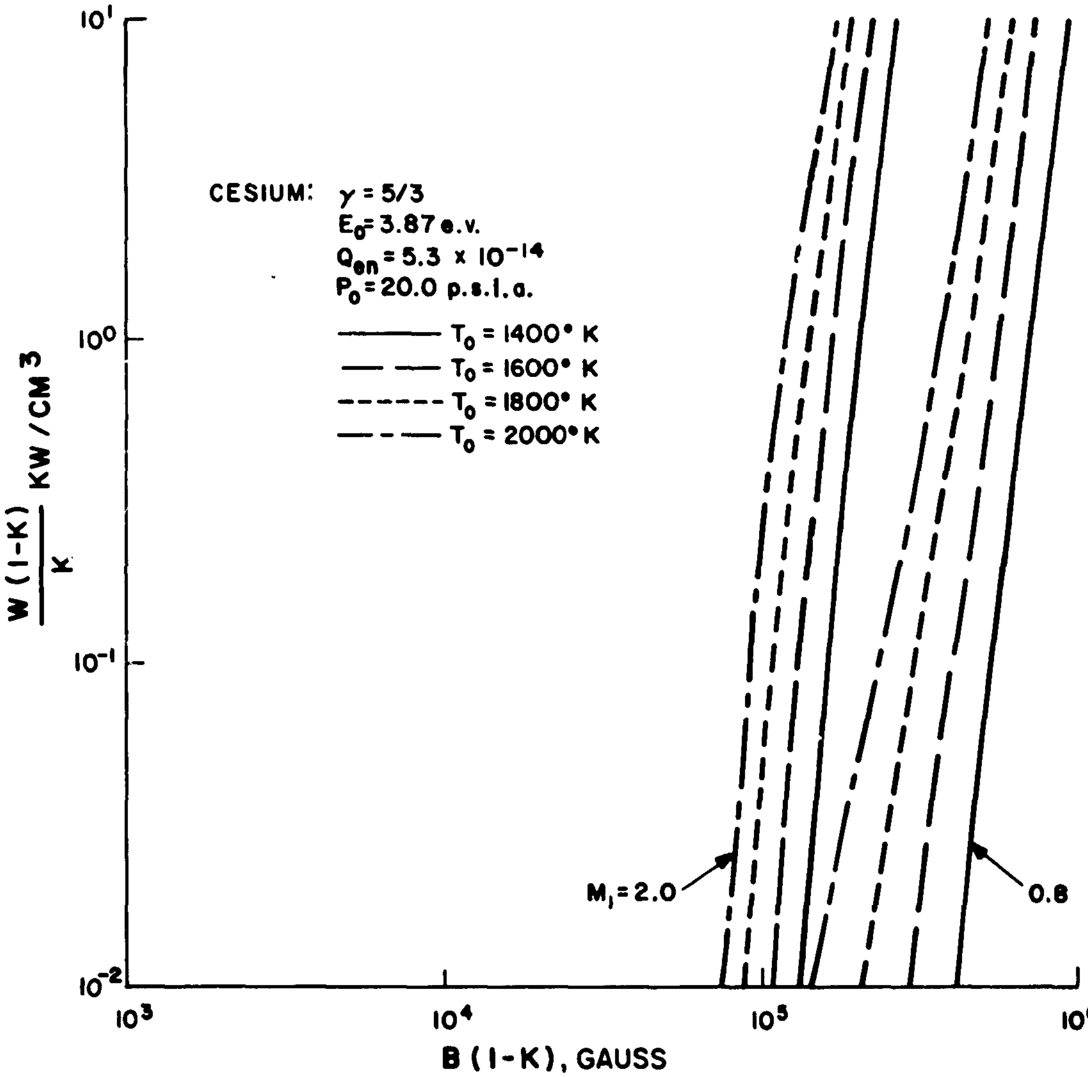

Figure 5. Power Density for Cesium, $Q_{en} = 5.3 \times 10^{-14}$ cm^2

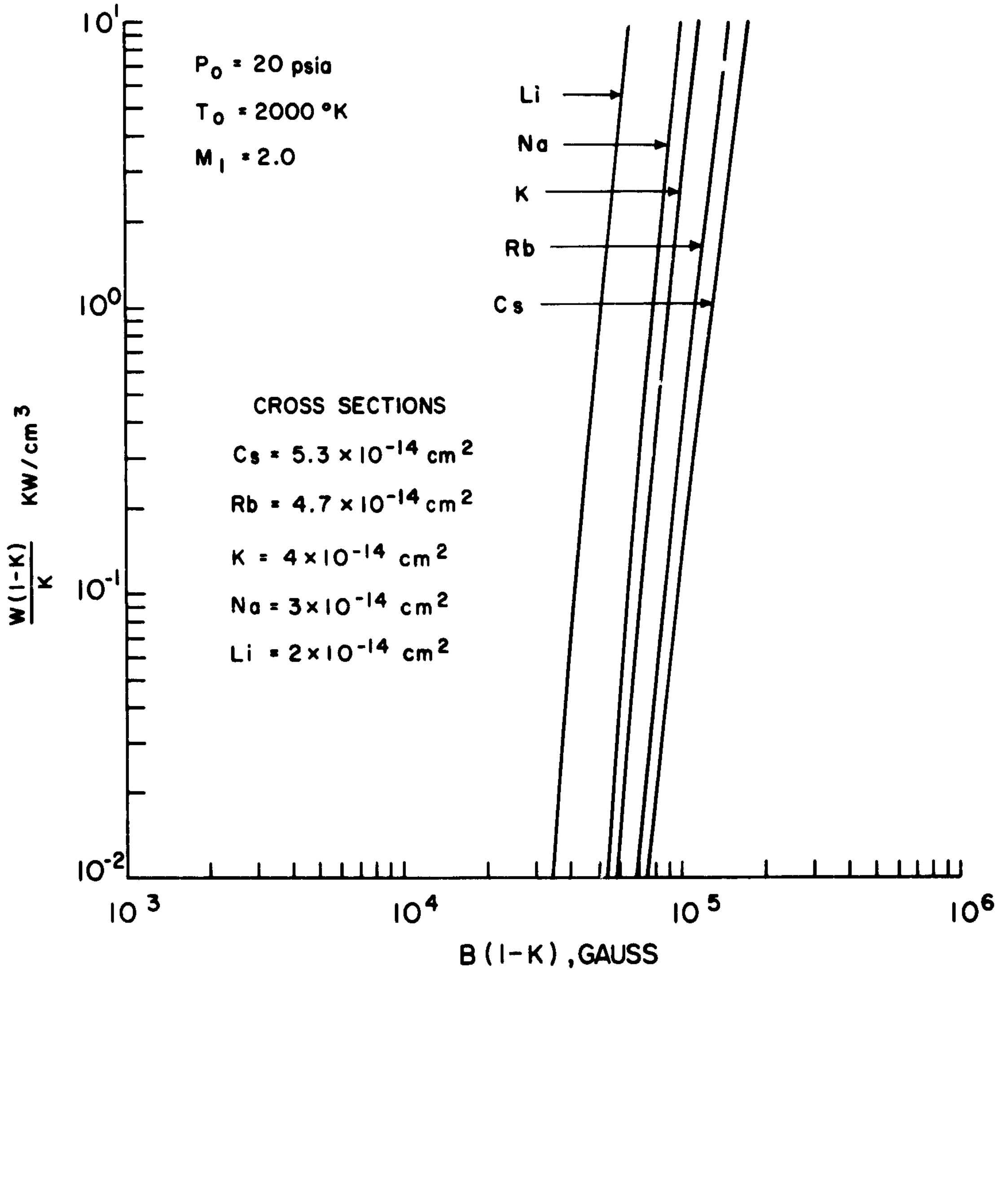

Figure 6. Comparison of Power Densities for Pure Alkali Metal Vapor Systems

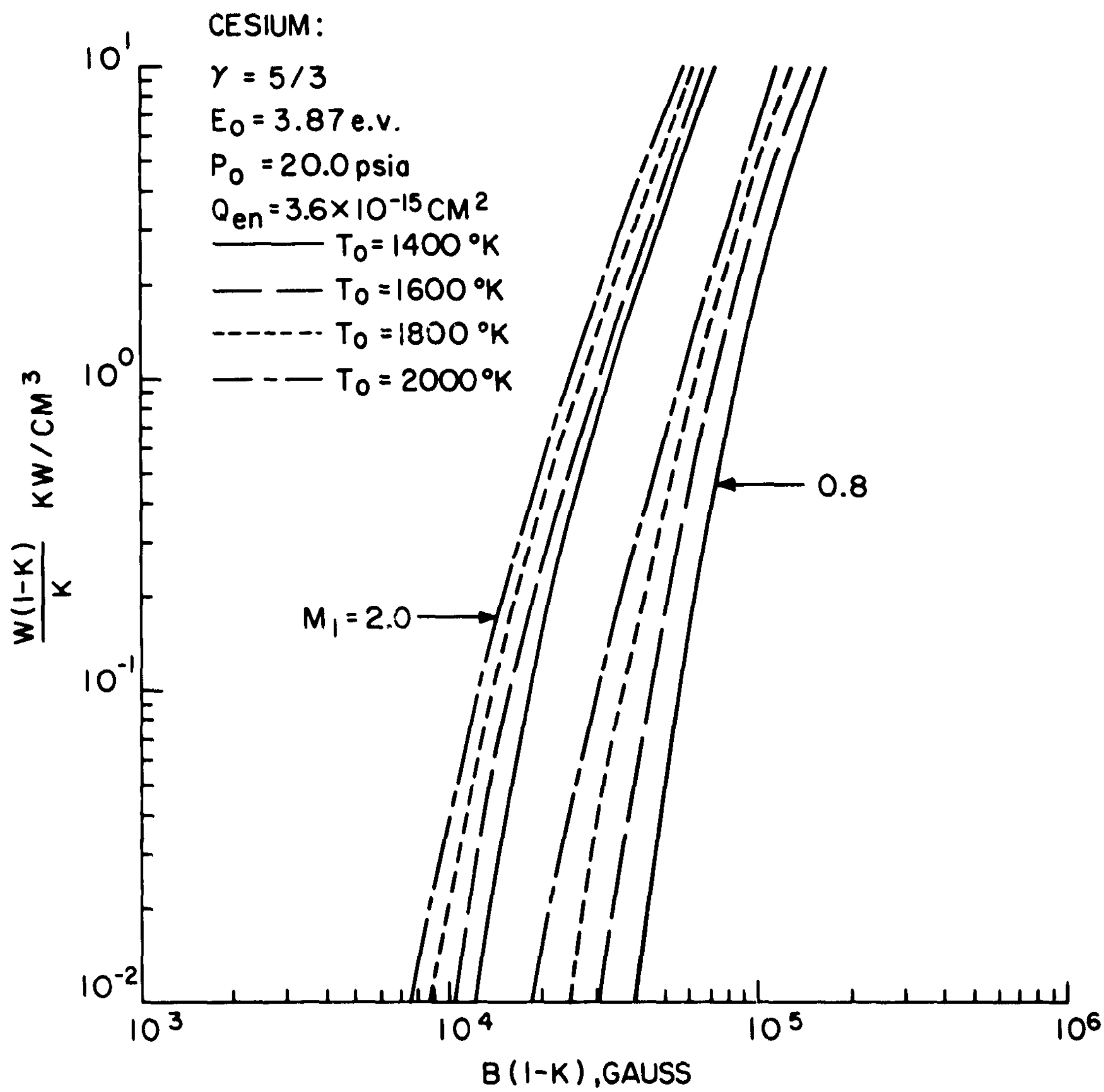

Figure 7. Power Density for Cesium, $Q_{en} = 3.6 \times 10^{-15}$ cm^2

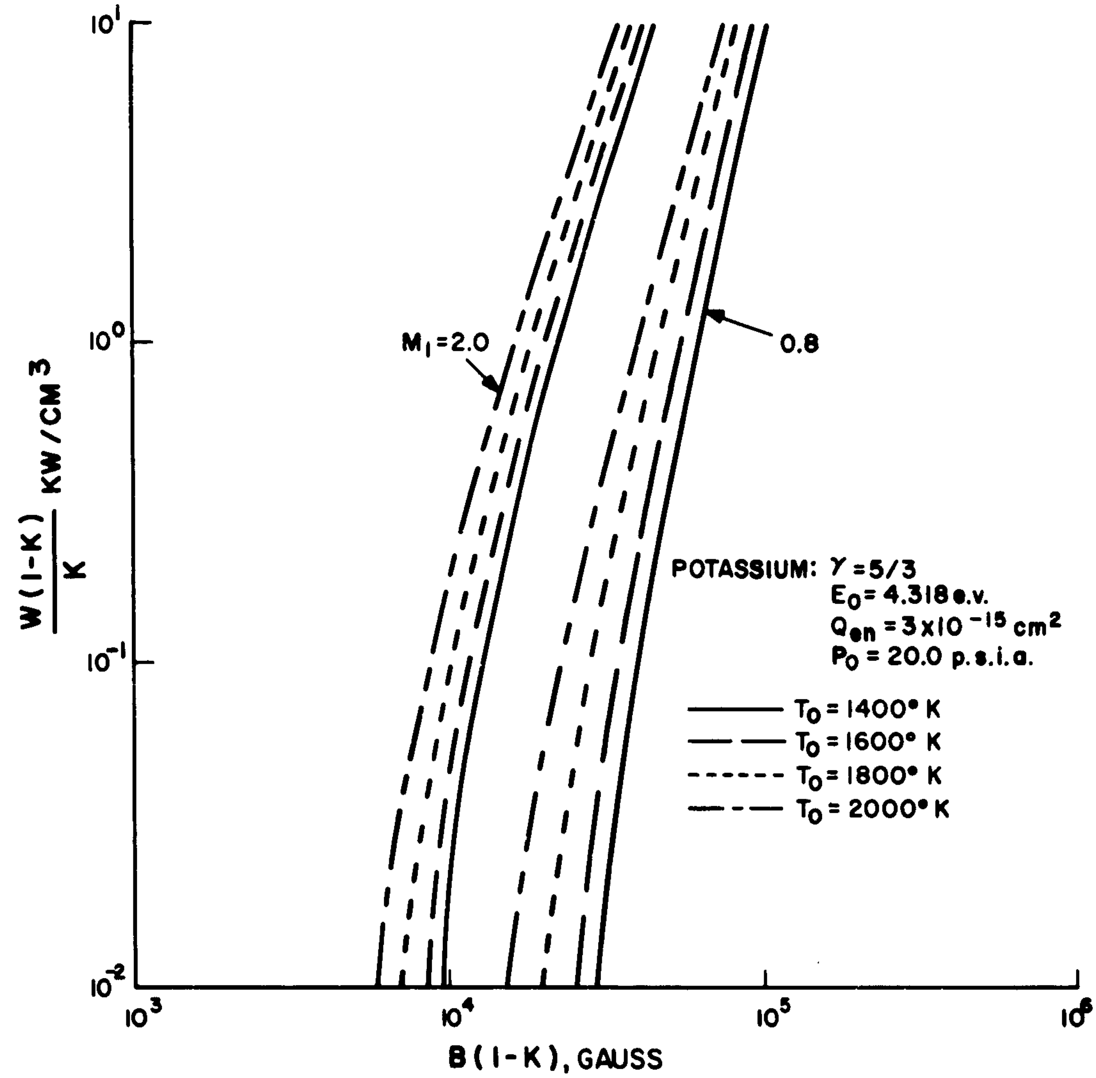

Figure 8. Power Density for Potassium, $Q_{en} = 3 \times 10^{-15}$ cm^2

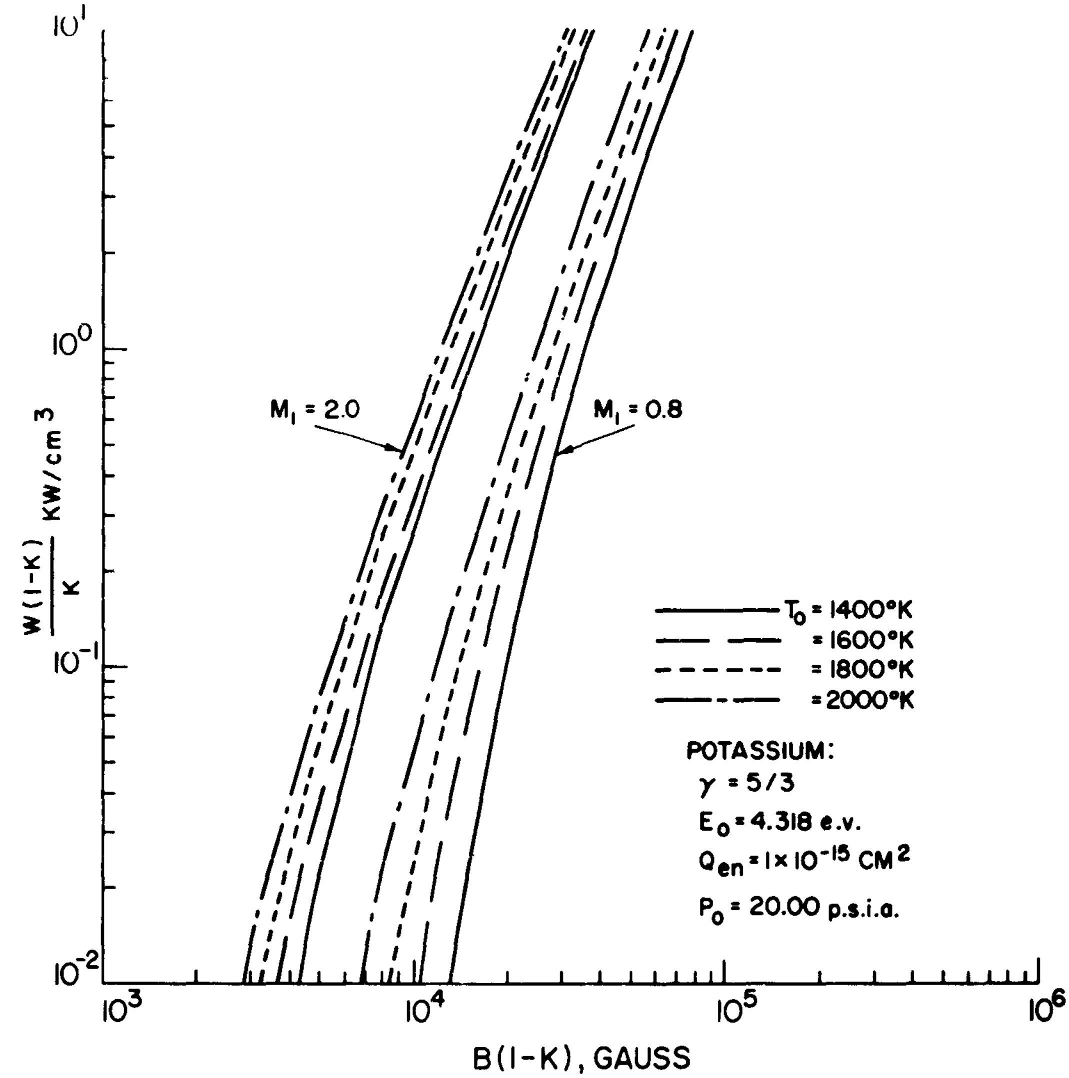

Figure 9. Power Density for Potassium, $Q_{en} = 10^{-15}$ cm^2

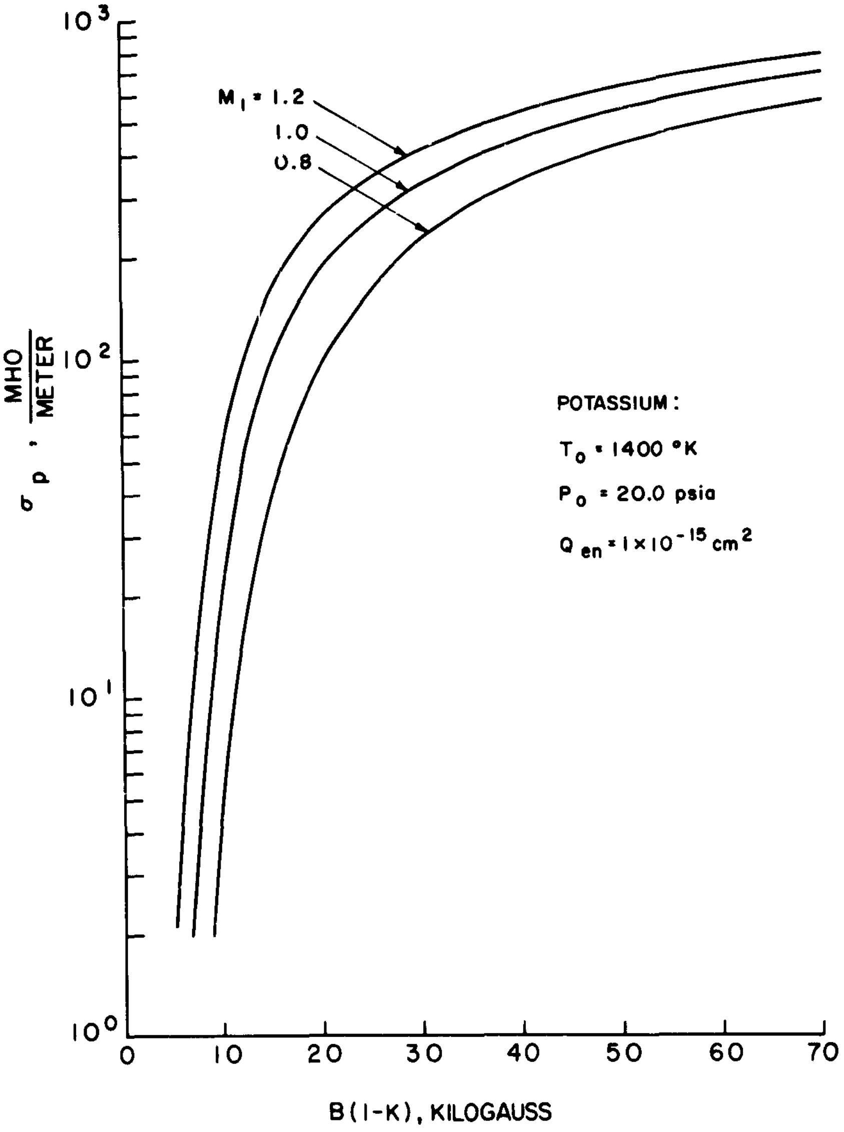

Figure 10. Influence of Magnetic Field upon Plasma Electrical Conductivity

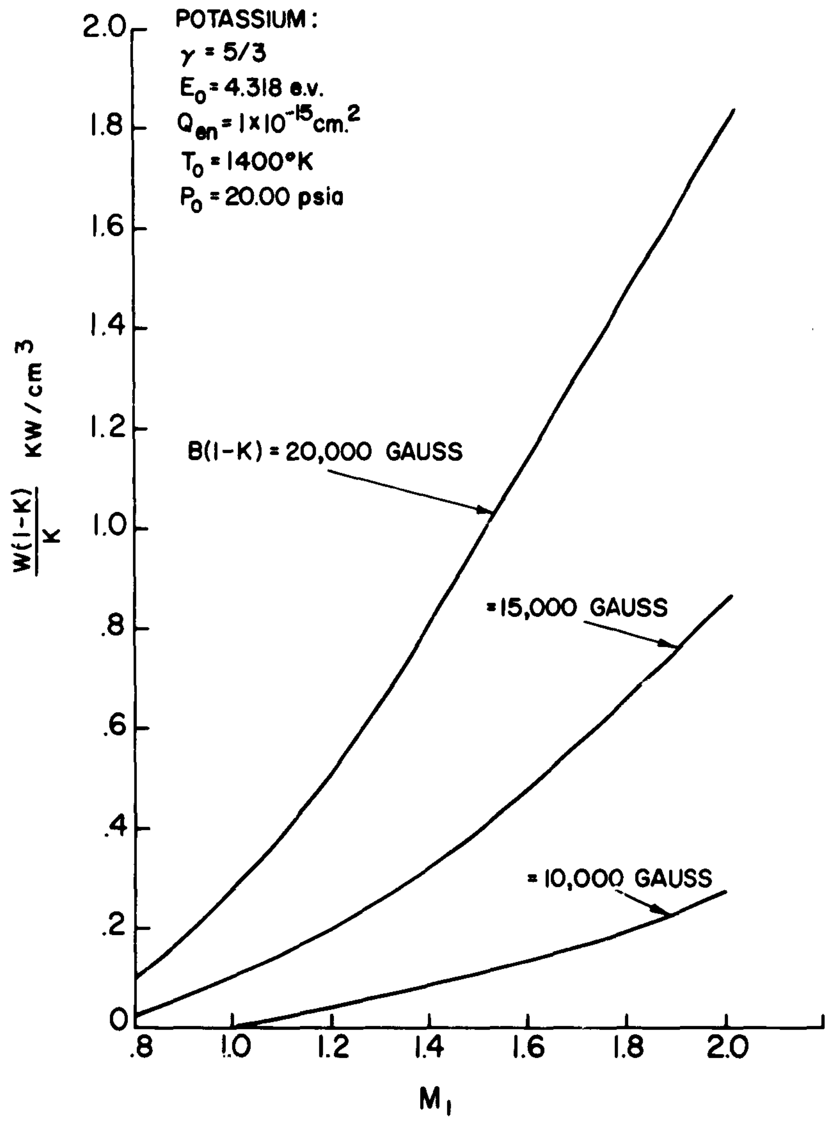

Figure 11. Influence of Mach Number for Power Density of MHD Generator for Potassium Vapor

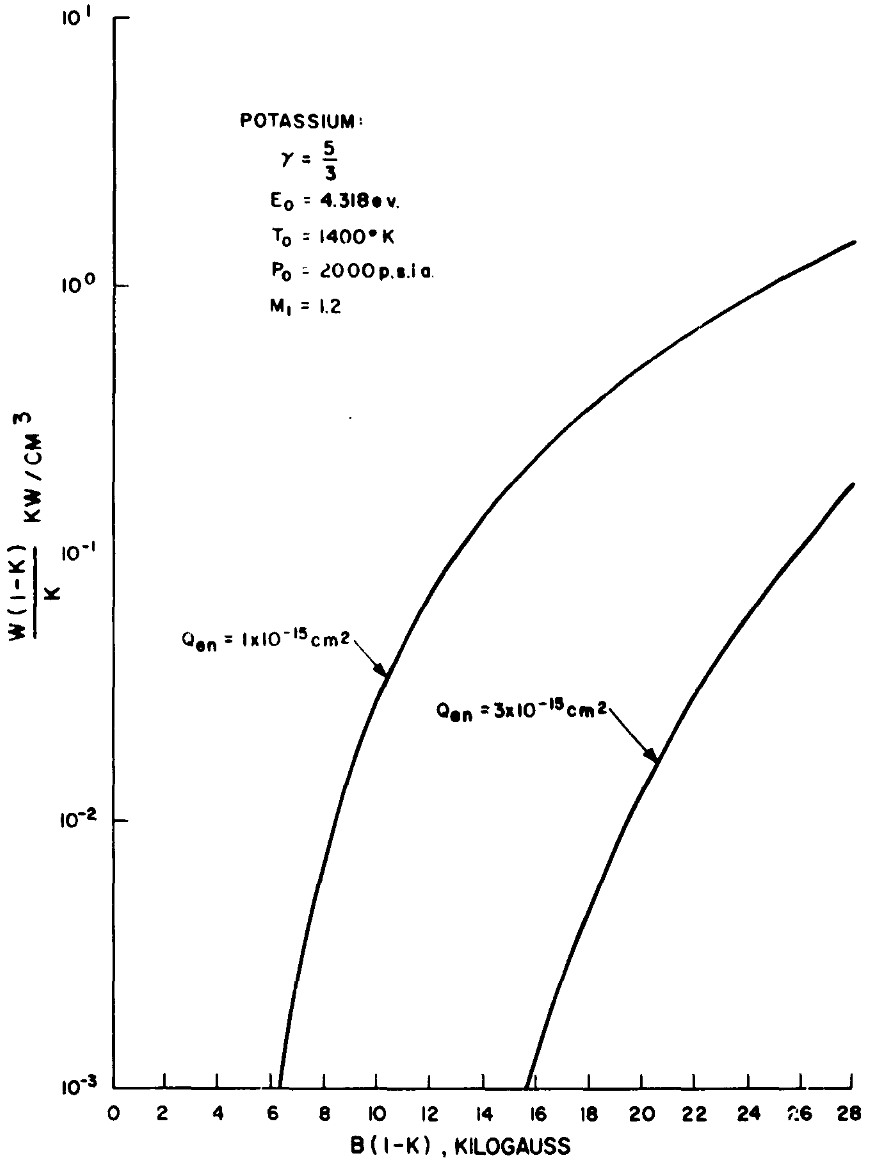

Figure 12. Effect of Electron Cross-Section Upon MHD Generator Performance for Potassium

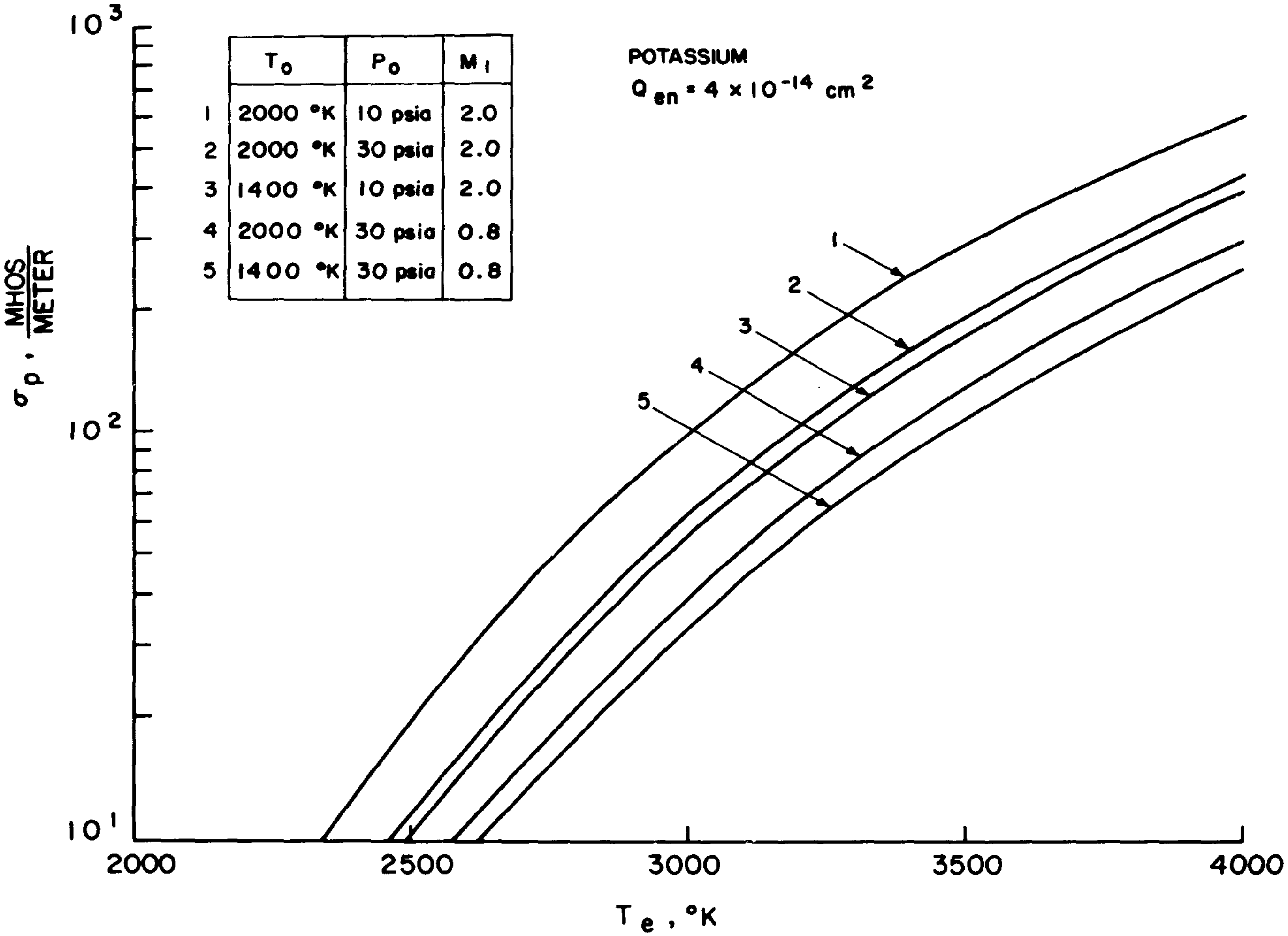

Figure 13. Relation Between Plasma Conductivity and Electron Temperature for Potassium Vapor

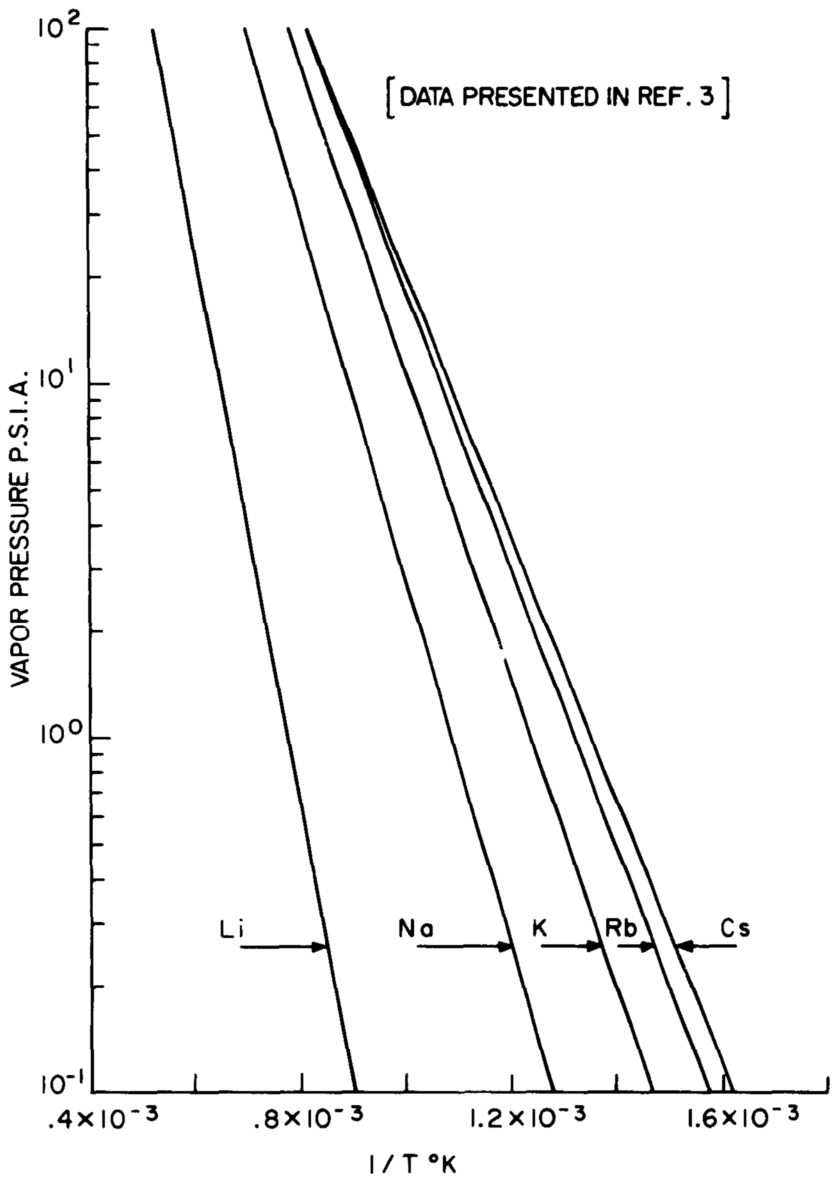

Figure 14. Vapor Pressure Curves for Alkali Metals

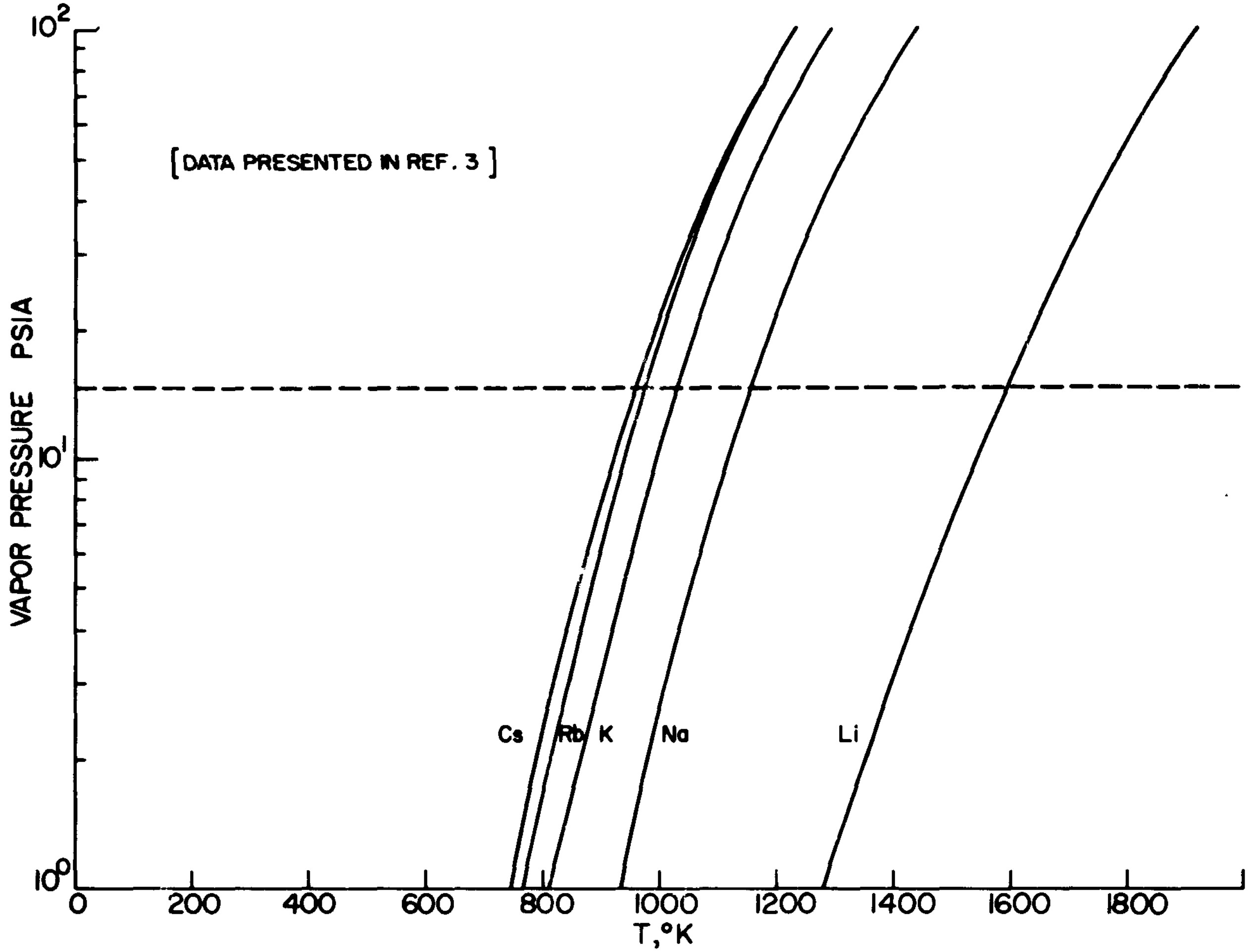

Figure 15. Vapor Pressure Curves of Alkali Metals

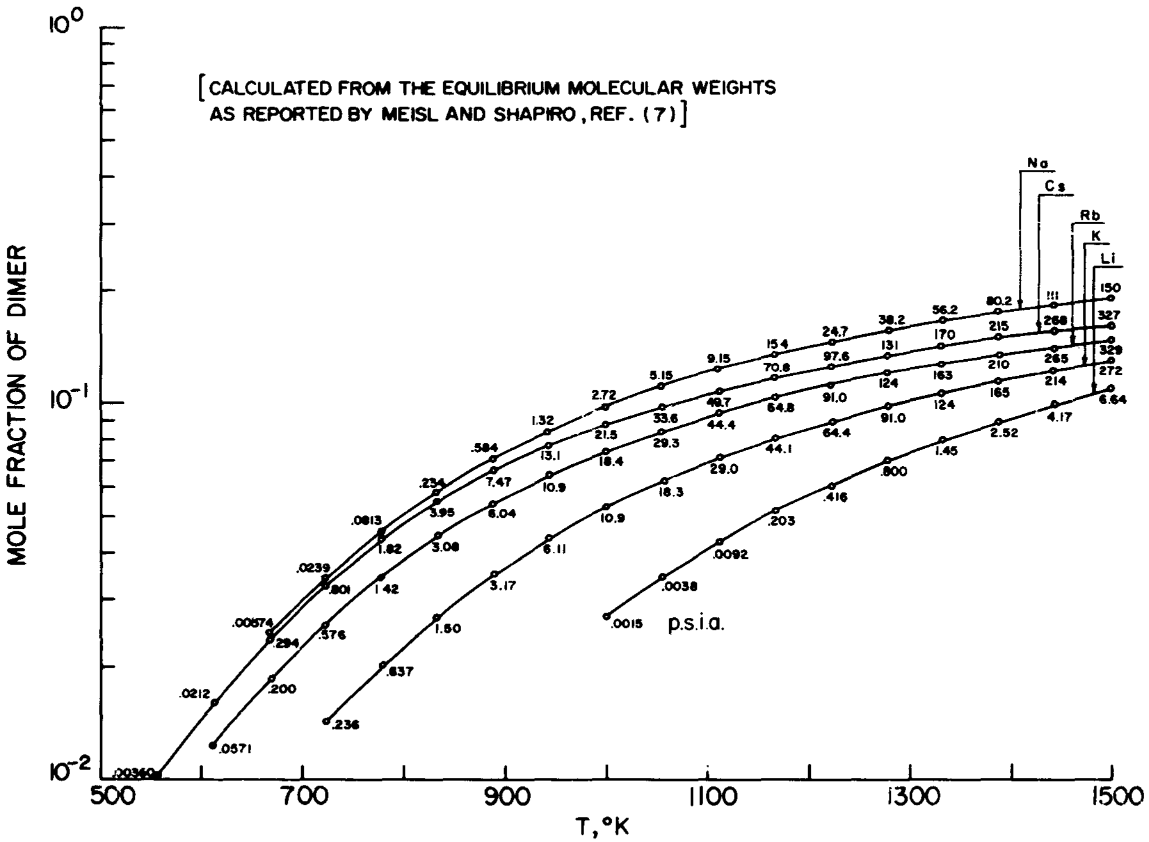

Figure 16. Equilibrium Dimer Concentration in Saturated Pure Alkali Metal Vapors [Calculated from the Equilibrium Molecular Weights as Reported by Meisl and Shapiro, ref. (7)]

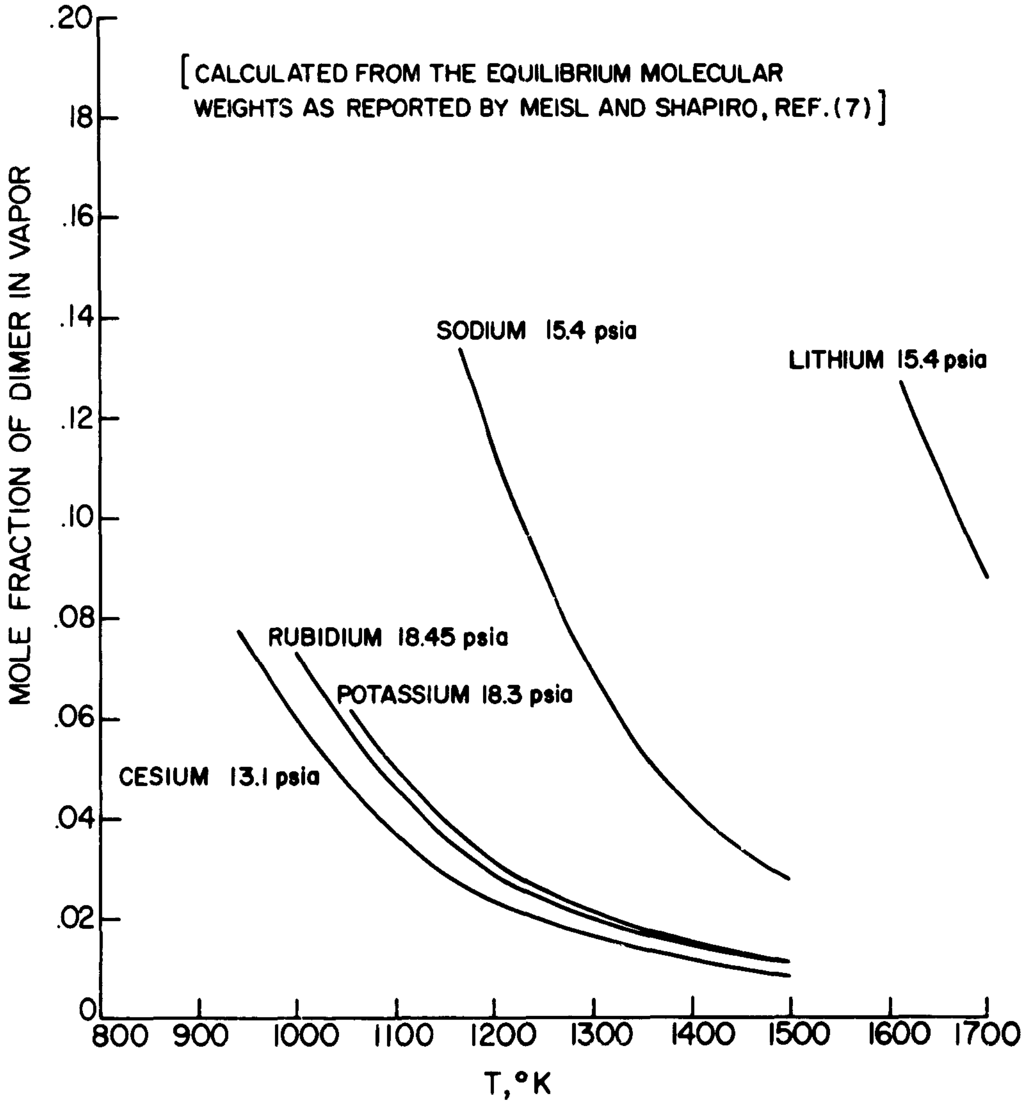

Figure 17. Equilibrium Dimer Concentration in Pure Alkali Metal Vapors Near Atmospheric Pressure

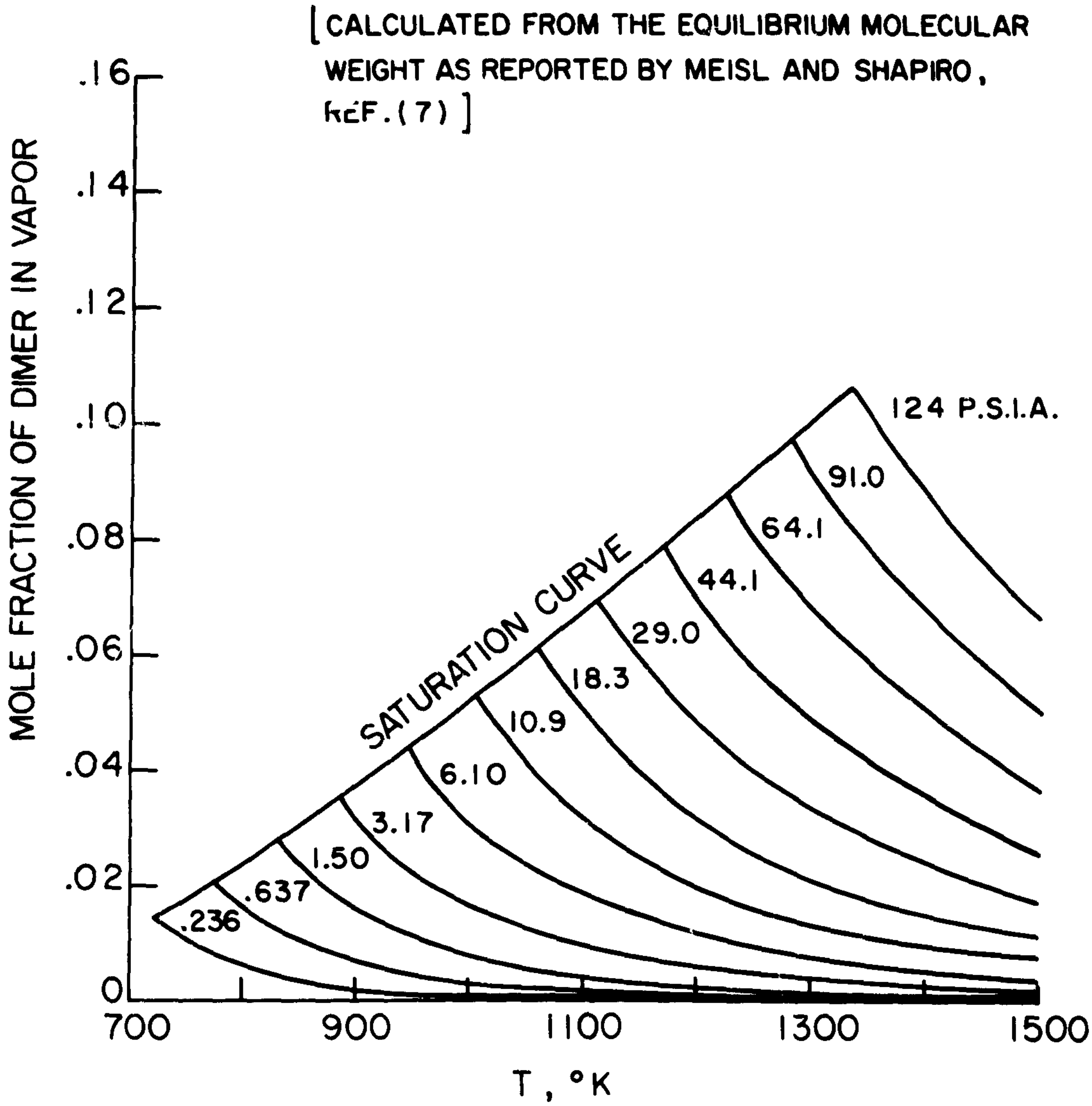

Figure 18. Equilibrium Dimer Concentration in Pure Potassium Vapor

GENERAL ELECTRIC

SPACE SCIENCES LABORATORY
MISSILE AND SPACE VEHICLE DEPARTMENT

TECHNICAL INFORMATION SERIES

AUTHOR	SUBJECT CLASSIFICATION	NO. R62SD94
Shair, F. H. Cristinzio, F.		DATE December, '62

TITLE Theoretical Performance for MHD Generators Utilizing Non-Equilibrium Ionization in Pure Alkali Metal Vapor Systems

ABSTRACT Each member of the alkali metal series has been investigated to determine the best system for a closed cycle MHD generator utilizing magnetically induced non-equilibrium ionization. MHD generator performances have been calculated for one-dimensional steady state flows with constant Mach number in which the dimer concentration is neglected. The calculations are for a generator with segmented electrodes.

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Abstract (cont.)

Potassium appears to be the best choice among the alkali metal series for a 1 megawatt generator operating at total temperature near 1600°K and total pressures around 20 psia. This choice is based upon considerations of the calculated generator performance, the vapor pressure, and the equilibrium concentration of dimer in the vapor.

The calculations show that the performance of an MHD generator utilizing a non-equilibrium condition of the electrons is primarily dominated by the elastic electron-neutral collision cross section. If the elastic electron collision cross section in potassium is $4\times10^{-14}cm^2$, then magnetic fields of 130,000 gauss will be required to obtain non-equilibrium conditions necessary to obtain power densities near $1 kw/cm^3$ when Mach no. is around 2. However, if the cross section in potassium is near $1\times10^{-15}cm^2$, then magnetic fields of 12,000 gauss will produce power densities near $1 kw/cm^3$ at a Mach of 2.

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LOCATION King of Prussia, Pennsylvania