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TABLES FOR ONE-SIDED STATISTICAL TOLERANCE LIMITS

BY

GERALD J. LIEBERMAN

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GERALD J. LIEBERMAN, PROJECT DIRECTOR
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TABLES FOR ONE-SIDED STATISTICAL TOLERANCE LIMITS

by

Gerald J. Lieberman

1. INTRODUCTION.

The quality of manufactured product is often specified by setting a range, the bounds of which are called tolerance limits. These limits have the property that a certain percentage, $100(1-\alpha)\%$, of the product may be expected to fall within these limits. If the quality of the items is normally distributed with known mean, μ, and known standard deviation, σ, two-sided tolerance limits are formed by adding to and subtracting from μ the quantity $K_{\alpha/2}\sigma$, where $K_{\alpha/2}$ is the normal deviate corresponding to $\alpha/2$. An upper one-sided tolerance limit is given by $\mu + K_\alpha\sigma$ and a lower one-sided tolerance limit is given by $\mu - K_\alpha\sigma$. Unfortunately, in practice, the values μ and σ are rarely known and estimates of these values, $\bar{x}$ and s, respectively, are used. However, where before it could be stated that 95% of a manufactured product lies below $\mu + 1.645\sigma$, the upper one-sided tolerance limit, this statement cannot be extended to the limit $\bar{x} + 1.645\, s$. The quantities $\bar{x}$ and s are random variables, and hence the limit depends upon the particular outcome of the sample. Different samples will lead to different limits. How close these limits are to $\mu + 1.645\sigma$ depends upon how good the estimates are.

It is evident then that the fraction of the items included between

a. $[\bar{x} - K_{\alpha/2}s\ ,\ \bar{x} + K_{\alpha/2}s\ ]$ for two-sided tolerance limits,

b. $[-\infty,\ \bar{x} + K_{\alpha}s\ ]$ for an upper one-sided tolerance limit,

c. $[\bar{x} - K_{\alpha}s\ ,\ \infty\ ]$ for a lower one-sided tolerance limit

will not always contain a specified proportion $1-\alpha$ of the manufactured items. However, it is possible to determine constants K such that in a large series of samples from a normal distribution, a fixed proportion γ of the intervals

a. $[\bar{x} - Ks\ ,\ \bar{x} + Ks\ ]$ for two-sided tolerance limits,

b. $[-\infty,\ \bar{x} + Ks]$ for an upper one-sided tolerance limit, and

c. $[\bar{x} - Ks\ ,\ \infty\ ]$ for a lower one-sided tolerance limit

will include $100(1-\alpha)\%$ or more of the distribution. The finite limits of these intervals are known as statistical tolerance limits. Thus, an upper one-sided statistical tolerance limit is given by $\bar{x} + Ks$ and has the property that the probability is equal to a preassigned value γ that the interval $[-\infty,\ \bar{x} + Ks]$ includes at least a specified proportion $1 - \alpha$ of the distribution. Note that in most practical situations γ is usually a large number close to 1. Statistical tolerance intervals should not be confused with confidence intervals for a parameter of the distribution. Confidence limits for the mean of a normal distribution are such that in a given fraction, say $.95$, of the samples from which they are computed, the interval bounded by the limits will include the true mean of the distribution. For confidence interval estimation, $.95$ is also called the confidence coefficient.

Many extensive tables of factors K for two-sided tolerance limits have been computed. A. H. Bowker presents such a table in [1] and very recently A. Lieberman issued a set of tables [2] which are an extension of Bowker's tables in that they can be used when s is computed from sample sizes other than n. Resnikoff [3] and Mitra [4] computed values of K when σ is estimated by the average range of subgroups. Tables of factors K for one-sided tolerance limits have never been computed and it is the purpose of this paper to present such tables.

2. TABLES OF ONE-SIDED TOLERANCE LIMITS.

Factors K such that $\bar{x} + Ks$ and $\bar{x} - Ks$ are an upper one-sided tolerance limit and a lower one-sided tolerance limit, respectively, for a normal universe are given in Table 1 for $n = 3,4,\ldots,25,30,35,40,45,50$; for $\gamma = 0.75, 0.90, 0.95, 0.99$; and for $\alpha = 0.25, 0.10, 0.05, 0.01, 0.001$. $\bar{x}$ is the sample mean $(\bar{x} = (x_1 + x_2 + \ldots + x_n)/n)$ and s is the sample standard deviation

$$\left(s = \sqrt{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2 \;/(n-1)}\right).$$

For n greater than 50, the values of K can be obtained using the asymptotic normality property of $\bar{x} + Ks$. With this approximation, K is given by

$$K = \frac{K_\alpha + \sqrt{K_\alpha^2 - ab}}{a}$$

where

$$a = 1 - \frac{K_\gamma^2}{2(n-1)}$$

$$b = K_\alpha^2 - \frac{K_\gamma^2}{n}$$

and K_α and K_γ (which equals $-K_{1-\gamma}$ so that $K_\gamma^2 = K_{1-\gamma}^2$) are the normal deviates corresponding to α and γ respectively, i.e.

$$\int_{K_\alpha}^{\infty} e^{-\frac{1}{2}z^2}\, dz = \quad \text{and} \quad \int_{K_\gamma}^{\infty} e^{-\frac{1}{2}z^2}\, dz = \gamma$$

3. **EXAMPLE.**

A manufacturer of light bulbs would like to specify a single lower limit above which he can be assured, with probability 0.95, that at least 99% of his production will lie. A sample of 30 bulbs is taken and the sample mean and sample standard deviation are found to be 987.2 and 5.963 respectively. A value of $K = 3.064$ corresponding to $n = 30$, $\gamma = 0.95$ and $\alpha = 0.01$ is obtained from Table 1. The required lower tolerance limit is given by $\bar{x} - Ks = 987.2 - (3.064)(5.963) = 968.9$.

4. **CONSTRUCTION OF THE TABLES.**

For the case of a normal distribution with unknown parameters μ and σ, exact one-sided tolerance limits may be computed by the following procedure. Let $t_{n-1,\sqrt{n}K_\alpha}$ be a non-central t statistic with $n-1$ degrees of freedom and non-centrality parameter $\sqrt{n}K_\alpha$. K_α is defined by

$$\frac{1}{\sqrt{2\pi}} \int_{K_\alpha}^{\infty} e^{-z^2}\, dz = \alpha .$$

Determine t_0 such that $P\left(t_{n-1,\sqrt{n}K_\alpha} > t_0\right) = 1 - \gamma$. Let $K = t_0/\sqrt{n}$. Then if $\bar{x}$ and s are the sample mean and the sample standard deviation, respectively, based on a sample $x_1, x_2, \ldots x_n$ i.e

$$\bar{x} = \frac{\sum x_i}{n} \quad \text{and} \quad s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} ,$$

$\bar{x} + Ks$ and $x - Ks$ are upper one-sided tolerance limit and a lower one-sided tolerance limit, respectively Thus, the probability is γ that at least a proportion $1-\alpha$ of the distribution will be less than $\bar{x} + Ks$ (or greater than $\bar{x} - Ks$) .

The value t_0 may be obtained from tables of the distribution of the non-central t statistic. Such tables are those of Johnson & Welch [5] and Resnikoff and Lieberman [6]. The latter tables are perhaps more convenient for this application, and were used in the computation of Table 1.

TABLE 1.

Tolerance Factors for Normal Distributions

Factors K such that the probability is γ that at least a proportion $1-\alpha$ of the distribution will be less than $\bar{x} + Ks$ (or greater than $\bar{x} - Ks$), where $\bar{x}$ and s are estimates of the mean and the standard deviation computed from a sample of size n .

	$\gamma = 0.75$					$\gamma = 0.90$				
n \ α	0.25	0.10	0.05	0.01	0.001	0.25	0.10	0.05	0.01	0.001
3	1.464	2.501	3.152	4.396	5.805	2.602	4.258	5.310	7.340	9.651
4	1.256	2.134	2.680	3.726	4.910	1.972	3.187	3.957	5.437	7.128
5	1.152	1.961	2.463	3.421	4.507	1.698	2.742	3.400	4.666	6.112
6	1.087	1.860	2.336	3.243	4.273	1.540	2.494	3.091	4.242	5.556
7	1.043	1.791	2.250	3.126	4.118	1.435	2.333	2.894	3.972	5.201
8	1.010	1.740	2.190	3.042	4.008	1.360	2.219	2.755	3.783	4.955
9	0.984	1.702	2.141	2.977	3.924	1.302	2.133	2.649	3.641	4.772
10	0.964	1.671	2.103	2.927	3.858	1.257	2.065	2.568	3.532	4.629
11	0.947	1.646	2.073	2.885	3.804	1.219	2.012	2.503	3.444	4.515
12	0.933	1.624	2.048	2.851	3.760	1.188	1.966	2.448	3.371	4.420
13	0.919	1.606	2.026	2.822	3.722	1.162	1.928	2.403	3.310	4.341
14	0.909	1.591	2.007	2.796	3.690	1.139	1.895	2.363	3.257	4.274
15	0.899	1.577	1.991	2.776	3.661	1.119	1.866	2.329	3.212	4.215
16	0.891	1.566	1.977	2.756	3.637	1.101	1.842	2.299	3.172	4.164
17	0.883	1.554	1.964	2.739	3.615	1.085	1.820	2.272	3.136	4.118
18	0.876	1.544	1.951	2.723	3.595	1.071	1.800	2.249	3.106	4.078
19	0.870	1.536	1.942	2.710	3.577	1.058	1.781	2.228	3.078	4.041
20	0.865	1.526	1.933	2.697	3.561	1.046	1.765	2.208	3.052	4.009
21	0.859	1.520	1.923	2.686	3.545	1.035	1.750	2.190	3.028	3.979
22	0.854	1.514	1.916	2.675	3.532	1.025	1.736	2.174	3.007	3.952
23	0.849	1.508	1.907	2.665	3.520	1.016	1.724	2.159	2.987	3.927
24	0.845	1.502	1.901	2.656	3.509	1.007	1.712	2.145	2.969	3.904
25	0.842	1.496	1.895	2.647	3.497	0.999	1.702	2.132	2.952	3.882
30	0.825	1.475	1.869	2.613	3.454	0.966	1.657	2.080	2.884	3.794
35	0.812	1.458	1.849	2.588	3.421	0.942	1.623	2.041	2.833	3.730
40	0.803	1.445	1.834	2.568	3.395	0.923	1.598	2.010	2.793	3.679
45	0.795	1.435	1.821	2.552	3.375	0.908	1.577	1.986	2.762	3.638
50	0.788	1.426	1.811	2.538	3.358	0.894	1.560	1.965	2.735	3.604

TABLE 1.

Tolerance Factors for Normal Distributions (continued).

n \ α	$\gamma = 0.95$ 0.25	0.10	0.05	0.01	0.001	$\gamma = 0.99$ 0.25	0.10	0.05	0.01	0.001
3	3.804	[illegible]	7.655	10.552	13.857					
4	2.619	4.163	5.145	7.042	9.215					
5	2.149	3.407	4.202	5.741	7.501					
6	1.895	3.006	3.707	5.062	6.612	2.849	4.408	5.409	7.334	9.540
7	1.732	2.755	3.399	4.641	6.061	2.490	3.856	4.730	6.411	8.348
8	1.617	2.582	3.188	4.353	5.686	2.252	3.496	4.287	5.811	7.566
9	1.532	2.454	3.031	4.143	5.414	2.085	3.242	3.971	5.389	7.014
10	1.465	2.355	2.911	3.981	5.203	1.954	3.048	3.739	5.075	6.603
11	1.411	2.275	2.815	3.852	5.036	1.854	2.897	3.557	4.828	6.284
12	1.366	2.210	2.736	3.747	4.900	1.771	2.773	3.410	4.633	6.032
13	1.329	2.155	2.670	3.659	4.787	1.702	2.677	3.290	4.472	5.826
14	1.296	2.108	2.614	3.585	4.690	1.645	2.592	3.189	4.336	5.651
15	1.268	2.068	2.566	3.520	4.607	1.596	2.521	3.102	4.224	5.507
16	1.242	2.032	2.523	3.463	4.534	1.553	2.458	3.028	4.124	5.374
17	1.220	2.001	2.486	3.415	4.471	1.514	2.405	2.962	4.038	5.268
18	1.200	1.974	2.453	3.370	4.415	1.481	2.357	2.906	3.961	5.167
19	1.183	1.949	2.423	3.331	4.364	1.450	2.315	2.855	3.893	5.078
20	1.167	1.926	2.396	3.295	4.319	1.424	2.275	2.807	3.832	5.003
21	1.152	1.905	2.371	3.262	4.276	1.397	2.241	2.768	3.776	4.932
22	1.138	1.887	2.350	3.233	4.238	1.376	2.208	2.729	3.727	4.866
23	1.126	1.869	2.329	3.206	4.204	1.355	2.179	2.693	3.680	4.806
24	1.114	1.853	2.309	3.181	4.171	1.336	2.154	2.663	3.638	4.755
25	1.103	1.838	2.292	3.158	4.143	1.319	2.129	2.632	3.601	4.706
30	1.059	1.778	2.220	3.064	4.022	1.249	2.029	2.516	3.446	4.508
35	1.025	1.732	2.166	2.994	3.934	1.195	1.957	2.431	3.334	4.364
40	0.999	1.697	2.126	2.941	3.866	1.154	1.902	2.365	3.250	4.255
45	0.978	1.669	2.092	2.897	3.811	1.122	1.857	2.313	3.181	4.168
50	0.961	1.646	2.065	2.863	3.766	1.096	1.821	2.296	3.124	4.096

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Attn: Quality Control Division
Directorate of Maintenance
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Attn: Directorate of Supply &
Services
3300 Jackson Avenue
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Commander
Shelby Air Force Depot
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Attn: Quality Control Division
Directorate of Maintenance
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P. O. Box 524
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Commander
Shelby Air Force Depot
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Services
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P. O. Box 310
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Cheli Air Force Station
Attn: Directorate of Supply & Services
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Dr. Adam Abruzzi
Dept. of Economics of Engineering
Stevens Inst. of Technology
Hoboken, New Jersey 1

Prof. Stephen G. Allen
244 Laurel Street
San Francisco, California 1

Prof. T. W. Anderson
Center for Advanced Study of
Behavioral Sciences
Stanford, California 1

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University of Nebraska
Lincoln 8, Nebraska 1

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Institute of Technology, USAF
Air University
Wright-Patterson Air Force Base
Ohio 1

Prof. Robert Bechhofer
Dept. of Mechanical Engineering
Cornell University
Ithaca, N.Y. 1

Mr. J. R. Beck
Kodak Processing Laboratory
925 Page Mill Road
Palo Alto, California 1

Professor R. E. Beckwith
Statistical Laboratory
Purdue University
Lafayette, Indiana 1

Prof. Maurice H. Belz
University of Melbourne
Carlton N. 3
Victoria, Australia 1

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Western Reserve University
Cleveland, Ohio 1

Mr. Carlton M. Beyer
Office of Guided Missiles
Office, Asst. Secretary of
Defense (R & E)
Washington 25, D.C. 1

Mr. P. M. Blunk
Box 532
Fair Oaks, California 1

Mr. Milton N. Bradley
2481 Davidson Avenue
New York, N.Y. 1

Prof. Russell Bradt
Dept. of Mathematics
University of Kansas
Lawrence, Kansas 1

Prof. Irving W. Burr
Dept. of Mathematics
Purdue University
Lafayette, Indiana 1

Prof. A. Charnes
Statistical Laboratory
Purdue University
Lafayette, Indiana 1

Mr. Allen Chop
162 - 8th Street
Oakland 7, California 1

Prof. Paul Clifford
New Jersey State Teachers College
Montclaire, New Jersey 1

Prof. Edward P. Coleman
Engineering Department
University of California
Los Angeles 24, California 1

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Quality Assurance, Dept. 551
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Albuquerque, New Mexico

Dr. Louis Court
Division 17
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Bureau of the Census
Washington 25, D.C. 1

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Prof. Georges Darmois
Director, Institute de Statistique
11 Rue Pierre Curie
Paris 5, France
c/o Office of Naval Research
London Branch
Navy No. 100, Fleet Post Office
New York, N.Y. 1

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Prof. Cyrus Derman
Dept. of Industrial Engineering
Columbia University
New York 27, N.Y. 1

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Sperry Gyroscope Co.
Great Neck, New York 1

Mr. H. F. Dodge
Bell Telephone Labs., Inc.
463 West Street
New York, N.Y. 1

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Mr. B.F. Doetsch
1519 E. Fox Street
South Bend 14, Indiana 1

Dr. Francis Dresch
Stanford Research Institute
1915 University Avenue
Palo Alto, California 1

Prof. Acheson J. Duncan
Dept. of Industrial Engineering
The Johns Hopkins University
Baltimore 18, Maryland 1

Prof. Meyer Dwass
Dept. of Mathematics
Northwestern University
Evanston, Illinois 1

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Attn: MCRR
Wright-Patterson Air Force Base,
Ohio 1

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Birmingham, Alabama 1

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Pasadena Annex
3202 E. Foothill Blvd.
Pasadena 8, California 1

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U. S. Census Bureau
Washington 25, D.C. 1

Mr. Bernard P. Goldsmith
Quality Control Engineer
Raytheon Mfg. Co.
55 Chapel Street
Newton 58, Mass. 1

Prof. Leo A. Goodman
Statistical Research Center
University of Chicago
1126 E. 59th Street
Chicago 37, Illinois 1

Prof. Eugene L. Grant
Civil Engineering Dept.
Stanford University
Stanford, California 1

Mr. Geoffrey Gregory
4, Osborne Grove
Gatley, Cheadle
Cheshire, England
c/o Office of Naval Research
London Branch
Navy No. 100, Fleet Post Office
New York, N.Y. 1

Prof. Frank M. Gryna, Jr.
Quality Control Consultant
Signal Corps Engineering Lab.
Fort Monmouth, New Jersey 1

Mr. Douglass Hawks, Jr.
Administrative Ass't.
American Bosch Arma Corp.
5555 So. Archer Avenue
Chicago 38, Illinois 1

Dr. Leon H. Herbach
Dept. of Mathematics
New York University
New York 53, N.Y. 1

Mr. Walter W. Hoy
Vought Aircraft, Inc.
Dallas, Texas 1

Dr. James R. Jackson
Management Sciences Research Project
65 Administration Bldg.
University of California
Los Angeles 24, California 1

Mr. Brent C. Jacob, Jr.
Chief Industrial Engineer
Chrysler Division-Chrysler Corp.
12200 E. Jefferson Avenue
Detroit 14, Michigan 1

Mr. J. P. Kearney
Quality Control Division
General Services Adm.
Room 6316 Region 3 Bldg.
Washington 25, D.C. 1

Prof. Tosio Kitagawa
Mathematical Institute
Faculty of Science
Kyusyu University
Fukuoka, Japan 1

Mr. Howard Laitin
2134 Homecrest Ave.
Brooklyn 29, N.Y. 1

Mr. Neil M. Leary
30-50 Thirtieth Street
Long Island City, New York 1

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Bell Telephone Labs.
Murray Hill, New Jersey 1

Mr. H. L. Lindgren
Joy Manufacturing Company
54 Main Street
Claremont, New Hampshire 1

Prof. Sebastian Littauer
Dept. of Industrial Engineering
Columbia University
New York, N.Y. 1

Dr. Eugene Lukacs
Department of Mathematics
Catholic University
Washington 17, D.C. 1

Dr. Clifford J. Maloney
Chief, Statistics Branch
Allied Sciences Division
Camp Detrick
Frederick, Maryland 1

Prof. G. W. McElrath
Dept. of Mechanical Engineering
University of Minnesota
Minneapolis 14, Minn. 1

Dr. Paul Meyer
Department of Mathematics
Washington State College
Pullman, Washington 1

Dean Paul E. Mohn
School of Engineering
University of Buffalo
Buffalo, N. Y. 1

Mr. Leo E. Morris
Quality Evaluation Laboratory
N.A.D. Bangor
Bremerton, Washington 1

Dr. R. B. Murphy
c/o Bell Telephone Labs., Inc.
463 West St.
New York 14, N.Y. 1

Mr. Monroe Norden
Engineering Statistical Group
Research Division
College of Engineering
New York University
401 West 205th Street
New York 34, N.Y. 1

Prof. E. G. Olds
Dept. of Mathematics
Carnegie Institute of Technology
Pittsburgh, Pa. 1

Mr. Cyril Peckham
Project Globe
University of Dayton
Dayton, Ohio 1

Dr. Paul R. Rider
Chief Statistician
Aeronautical Research Laboratory
Wright Air Development Center
Wright-Patterson Air Force Base
Ohio 1

Dr. Harry G. Romig
351 Alma Real Drive
Pacific Palisades, California 1

Dr. Alan J. Rowe
General Electric Co.
Production Control Service, Rm. 2401
570 Lexington Avenue
New York 22, N.Y. 1

Prof. Herman Rubin
Dept. of Mathematics
University of Oregon
Eugene, Oregon 1

Prof. Norman Rudy
Statistics Department
Sacramento State College
Sacramento, California 1

Prof. Henry Scheffé
Statistical Laboratory
University of California
Berkeley 4, California 1

Prof. Robert Schlaifer
Graduate School of Business Adm.
Harvard University
Boston 63, Mass. 1

Mr. R. H. Shaw
General Dry Batteries, Inc.
13000 Athens Avenue
Cleveland 7, Ohio 1

Prof. Seymour Sherman
Moore School of Electrical Eng.
University of Pennsylvania
Philadelphia 4, Pa. 1

Mr. Walter Shewhart
Bell Telephone Labs., Inc.
Murray Hill, New Jersey 1

Mr. J. Gordon Siddons
Quality Control Supervisor
Boeing Airplane Company
Seattle 14, Washington 1

Dr. Rosedith Sitgreaves
Teachers College
Columbia University
New York 27, N.Y. 1

Dr. Milton Sobel
Bell Telephone Labs.
555 Union Blvd.
Allentown, Pa. 1

Prof. Herbert Solomon
Teachers College
Columbia University
New York 27, N.Y. 1

Dr. M. D. Springer
Head, Statistical Analysis Branch
U.S. Naval Ordnance Plant
Indianapolis, Indiana 1

Mr. Selig Starr
Research & Development Dept.
U.S. Naval Powder Factory
Indianhead, Md. 1

Mr. Arthur Stein
Cornell Aeronautical Laboratory, Inc.
P. O. Box 235
Buffalo, N.Y. 1

Mr. M.P. Straubel
Lear, Inc.
110 Ionia Ave., N W.
Grand Rapids 2, Michigan 1

Mr. Dan Teichroew
National Cash Register Co
Product Development Dept.
1401 E. El Segundo Blvd.
Hawthorne, California 1

Miss Elizabeth Vaughan
2325 7th Street
Bremerton, Washington 1

Mr. Cesareo Villegas
Institute de Matematica y Estadistica
Av. J. Herrera y Reisrig
Montevideo, Uruguay 1

Mr. T. M. Vining, Chief
Engineering Statistics Unit
Chemical Corps Engineering Agency
Army Chemical Center, Maryland 1

Mr P. F. Wade, Statistician
Aluminum Company of Canada, Ltd.
Kingston, Ontario, Canada 1

Prof W. Allen Wallis
Committee on Statistics
University of Chicago
Chicago 37, Illinois 1

Prof. A. Walther
Technische Hochschule
Darmstadt, Germany 1

Mr. Harry Weingarten
Bureau of Ships, Code 260
Department of the Navy
Washington 25, D. C. 1

Mr. Joseph Weinstein
Physical Research Branch
Evans Signal Laboratory, SCEL
Belmar, New Jersey 1

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Standards Branch
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DCS/Logistics, U. S. Army
Washington 25, D. C. 1

Mr. J. W. Young
Quality Control
North American Aviation, Inc.
International Airport
Los Angeles 45, Calif. 1

Dr. M. A. Woodbury
Dept. of Mathematics
College of Engineering
New York University
New York 53, N. Y. 1

Mr. R. P. Zieke
Sr. Manufacturing Methods Engineer
Dept. 22200-Tool Project Eng.
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